Prices, Non-homotheticities, and Optimal Taxation

The Amplification Channel of Redistribution*

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We characterize theoretically and quantitatively the effects of price changes on optimal tax design in the presence of non-homothetic preferences. We find that, rather than offsetting price changes, the optimal tax system amplifies their redistributive effects. With a quantitative model matching observed non-homotheticities and the empirical elasticity of prices to market size in the United States, we find that due to the amplification channel, (i) the optimal tax schedule is more redistributive when accounting for non-homothetic spending patterns, (ii) observed heterogeneous inflation rates, which are lower for luxuries relative to necessities in the United States, generate a regressive tax response.

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1 Introduction

How do prices affect the optimal design of taxes? To a large extent, the optimal taxation literature has ignored the effects of prices on optimal redistribution policies. Indeed, the seminal result of Diamond and Mirrlees (1971b) states that optimal tax formulas can be derived as if prices were fixed at their equilibrium level, and leaves implicit the optimal response of taxes to price changes.[1] Yet, empirically price changes are ubiquitous, and they tend to correlate with household income. Recent work shows that heterogeneous inflation rates across products consumed by low- and high-income households played an important role for purchasing power inequality in the United States (e.g., McGranahan and Paulson (2005), Kaplan and Schulhofer-Wohl (2017), Jaravel (2019), Argente and Lee (2020), Klick and Stockburger (n.d.)). Despite the prevalence of price changes, to date we lack the tools to characterize their potential effects on optimal taxation.

In this paper, we develop a theoretical framework to analyze the effect of prices on optimal tax design, and we quantitatively estimate their impact. We provide an explicit characterization of the impact of prices on the marginal social value of transfers, on labor supply, and on labor supply elasticities. Furthermore, we show that when the response of the tax schedule to price changes is non-trivial, a feedback loop may emerge: taxes shift demand for goods, which can induce a further change in prices through general equilibrium adjustments (e.g., returns to scale), and a new response of the tax schedule. Equilibrium prices are still a sufficient statistic in our optimal tax formulas, but finding the new equilibrium prices requires characterizing this feedback loop and, in particular, the response of the supply side of the economy.

To facilitate comparison with prior work, we work with a standard, static Mirrlees model: agents have preferences over multiple consumption goods and leisure, and labor is the only factor of production; preferences are weakly separable between consumption and labor, as in the Atkinson-Stiglitz benchmark. This setting allows us to capture non-homothetic spending patterns across the income distribution while focusing on a single tax instrument for redistribution, the nonlinear income tax.

The main challenge is that the channels through which prices shape redistribution are not explicit,[2] as they appear only implicitly in the first-order conditions determining the optimal marginal tax rates. To overcome this issue and understand the economic forces at play, we use a comparative static approach: we derive the equations governing the derivatives of the income tax with respect to prices. This approach allows us to characterize the first-order responses of taxes to price changes in terms of observable statistics. Thus, it allows us to evaluate how redistribution policies should have responded to recent heterogeneous inflation trends, and to uncover the mechanisms through which prices operate.

We start by analyzing the "partial equilibrium" effects of prices, omitting the general equilibrium adjustments that result from shifts in aggregate demand for goods.[3] We show that the impact of prices on taxes is governed by the marginal propensity to spend (MPS) on the products experiencing a price change. Indeed, the MPSs determine the marginal price indices of households, i.e. the prices of the baskets of goods they would consume with an additional dollar of income.

The marginal propensity to spend is central to understand the impact of price changes on the value of redistribution and on labor supply. To illustrate, consider an increase in the price of a product for which the marginal propensity to spend increases with income, which we label a "luxury product". First, a price

[1]Standard optimal tax formulas are first-order conditions featuring endogenous variables that depend on prices, such as the marginal utility of disposable income.

[2]The added difficulty in our case is that non-homothetic demand systems do not yield closed form solutions in general.

[3]In the main text, we consider a benchmark case where utility is quasilinear (at initial prices) and the social welfare function is linear (i.e., the Pareto weights depend on the agents' productivity but not on their income). In Online appendix A3, we extend these result to non-linear social welfare functions (which can captures a decreasing social value of income) and more general labor supply functions. Our results are qualitatively similar in these more general cases.

increase on a luxury good raises the marginal price index of higher-income households relatively more than that of lower-income households. This means that higher income households can now buy less with an additional dollar of income. Therefore, the social value of a dollar transfer from higher-income to lower-income households increases. Second, as the decrease in marginal purchasing power is larger at higher income levels, the price increase generates a negative income effect, which is larger as income increases: a dollar transfer disincentivizes labor supply relatively more at higher income levels, and thus a price increase on a luxury good reduces the efficiency cost of taxation. Since both the cost of taxation and the social value of transfers to higher-income decreases, the marginal tax rates increase everywhere. As the new tax system has to be budget neutral, the burden of taxation decreases at the bottom of the distribution and increases at the top. These channels are at play even when it is feasible to compensate all agents at no fiscal cost – for example when the price of luxury goods increases while necessity goods become more affordable so that the average price change is nil. Perhaps surprisingly, we find that, far from compensating price movements, the optimal tax system amplifies their redistributive effects: an increase in the price of luxury goods induces more redistribution at the bottom of the income distribution; the opposite is true when necessities become more expensive. These channels do not operate when preferences are homothetic as all agents are equally impacted by price changes.[4]

Our analysis of the partial equilibrium problem thus reveals a meaningful interaction between the direct effects of prices and taxes, as price changes are not merely compensated for. While these direct effects operate irrespective of the supply side specification, we show that in general equilibrium, the elasticity of prices with respect to aggregate consumer demand across products becomes pivotal to evaluate the interplay between optimal taxes and prices. Empirically, the literature shows that product markets with larger demand tend to have higher productivity and lower prices due to several channels. For instance, higher demand increases the incentives to enter a market, to innovate, and to compete, which leads to lower marginal cost, lower markups, larger product variety, and lower consumer price indices. These channels have been documented in a recent empirical literature (e.g., Costinot et al. (2019), Jaravel (2019), Faber and Fally (2021)) as well as in a long-standing theoretical literature (e.g., Romer (1990), Aghion and Howitt (1992), Melitz (2003)).

To capture the key feature of product markets (the relationship between demand and consumer prices), we adopt a sufficient statistics approach. We assume that firms make no profit and summarize the supply side of the economy through a key statistic, the elasticity of prices with respect to a change in market size. The only restriction imposed by this assumption is that prices are set according aggregate quantities demanded rather than the exact distribution of consumers in each market. For this restriction to be met, it is sufficient for the "product" in each market to be a homothetic aggregate of subvarieties produced competitively or monopolistically. The prices derived in our model can be interpreted as price indexes for the subvarieties in each market derived from standard aggregator (e.g., CES, Kimball, translog demand system, etc.). As such, we cover a wide range of standard structural models of the supply side, including models with firm selection (Melitz (2003)), variable markups (Feenstra and Weinstein (2017)), and innovation (Bustos (2011)). For completeness, we also consider the standard Diamond-Mirrlees benchmark where goods are produced competitively and all profits are taxed and rebated to households in a lump sum fashion.

When product prices decrease as their market expands, we find that the redistributive effects of price

[4]We also consider the case of a concave social welfare function, which we report in the Online Appendix due to space constraints. The concavity of social preferences introduces a counterbalancing force: reducing the income of any agents makes transfer towards them more valuable. The amplification discussed above is therefore muted: when, for example, price changes favor high-income households, marginal tax rates decrease by less than in the linear case and lower-income households bear a relatively smaller fraction of the tax burden. However, we find that redistribution towards higher-income households still occurs at the new optimum and the welfare of households at the bottom of the distribution still declines.

changes and their amplification through taxes are strengthened in general equilibrium.[5] We show there are three channels governing the changes in aggregate demand across products, leading to amplification. To illustrate, consider an increase in the relative price of luxuries.[6] First, households reallocate their spending to other products through standard substitution effects, leading to a fall in demand for luxuries. Second, a relative increase in the price of luxuries has a negative income effect on higher income households, as luxuries constitute a larger portion of their consumption basket. Higher income households have a higher propensity to spend on luxuries, so the aggregate share of luxuries decreases through income effects. Third, there are several changes in optimal taxes. As explained above, it becomes more valuable to redistribute to lower income households: tax rates increase along the income distribution. Income is reallocated to lower income households, which amplifies the decline in the share of luxuries. The markets for luxuries shrink relative to other markets: the relative price of luxuries increases further through a supply side response, which generates a larger decline of the share of luxuries and more redistribution towards lower income households. Therefore, the government amplifies both the inflation of luxuries prices and their redistributive impact through changes in optimal tax rates.[7] In addition, by increasing tax rates, the planner lowers labor supply in the aggregate. As a result, all markets shrink, and all products become more expensive. As households' aggregate real income decreases, they shift their consumption towards necessities, which amplifies the hike in luxuries prices. Thus, optimal tax policy not only amplifies the redistributive effects of an increase in luxury goods' prices by reallocating income to poorer households, but also by reducing aggregate income.[8] These channels operate in any supply side model with elastic prices, including in the canonical Diamond-Mirrlees setting with non-constant return to scale production functions (although this effect remains implicit in the standard Diamond-Mirrlees tax formulas).

Finally, by reallocating consumption to different markets, prices and taxes may affect the average elasticity of prices to market size in the economy. For example, if more productive markets expand, then taxation becomes more costly and it is optimal to reduce distortions by lowering marginal tax rates. The impact of taxes on the average productivity of the economy is of first-order importance in our benchmark model, while it does not matter in the Diamond-Mirrlees specification. Indeed, as our model features variable prices and no profits, as a result of free entry, it is generally inefficient, which leaves a role for corrective taxation. In light of our reduced form specification, the inefficiency is best interpreted as an aggregate demand externality. Agents do not internalize that consuming more of a good expands its market, which improves efficiency, e.g. through endogenous innovation or stronger competition.[9] The correction can be implemented via commodity taxes and a wage subsidy. Commodity taxes efficiently allocate consumption to each market: if increasing demand for a product lowers its price by more than the average market size elasticity, then it is optimal to subsidize this product. The average demand externality across markets is then corrected by a flat wage subsidy. While the correction is by itself simple, our comparative statics approach reveals a non-trivial interaction between redistributive and corrective taxation: the corrective work subsidy is regressive, which tends to increase consumption

[5] When prices increase as the market expands (in contrast with our baseline case featuring increasing returns), the redistributive effects of prices are muted through general equilibrium effects.

[6] By increase in the relative price of luxuries, we mean an increase in the price of luxuries and a decrease in the price of all other goods, keeping the price of the average consumption basket constant.

[7] When the relative price of necessities increases, the effects are symmetric: the income tax becomes more regressive and the share of necessities declines.

[8] When the price of necessities increases, tax rates decrease. This stimulates labor supply, all products become cheaper and, as real income increases, consumption is reallocated towards luxuries. This further increases the relative price of necessities, which benefits higher income households.

[9] In the Diamond-Mirrlees specification, if an increase in demand reduces prices, it also decreases firms' profit and households' disposable income. The two effects cancel each other and there is no externality.

of luxury products and reduce their prices. When preferences are non-homothetic, these price changes affect the optimal redistributive tax through the channels previously described. Even when preferences are homothetic, if the social welfare function is concave, we find that higher-income households will contribute more to the financing of the subsidy, and that the response of the optimal tax schedule is non-trivial.

Building on these theoretical insights, in the quantitative section of the paper we evaluate the optimal response of taxes to the price changes observed in the data in recent years, and we examine more generally how our benchmark specification – with non homothetic preferences and downward sloping supply curves – affects optimal redistribution policies. We first implement our comparative static approach. While it only gives the first order response of taxes to price changes, it has two advantages: we can directly use recent causal estimates of the elasticity of prices to market size to evaluate supply side responses to shifts in demand, and we can non parametrically fit non-homothetic spending patterns. By linking the Consumer Expenditure Survey (CEX) and the Consumer Price Index (CPI) datasets, we obtain observed price changes and households' spending across 248 product categories for the period 2004 to 2015, covering the entire consumption basket of American households. Empirically, inflation was lower in product categories with higher income elasticities. We find that, in response, it is optimal to reduce redistribution and set lower marginal tax rates, with a fall in marginal tax rates of about 8 percentage points at the bottom of the income distribution (relative to the observed tax schedule).[10]

While we treat the price changes observed in the data as exogenous in our first analysis, empirically prices respond endogenously to shifts in demand, e.g. those stemming from growing income inequality. We use our comparative statics approach to characterize quantitatively the optimal response of the tax schedule to exogenous shifts in the income distribution, accounting for the endogenous response of prices. Using publicly available statistics on the income distribution from the U.S. Census from 2004 to 2015, we find that income was stagnant at the bottom of the distribution, and increased at faster and faster rates with higher incomes. This spread in the income distribution leads to a more redistributive tax schedule: marginal tax rates increase in U-shaped fashion to take advantage of the thinning mass of taxpayers in the middle of the distribution. However, as higher income households become richer, the markets for luxury goods expand, which lower their equilibrium prices. Due to the large empirical estimates of price elasticities to market size, this direct effect of income inequality on prices makes the optimal tax schedule less redistributive, which almost entirely offsets the redistributive response of optimal taxes to rising inequality. These results show that it is crucial to jointly study shifts in the skill distribution and their effect on prices to assess the effects of income inequality on optimal tax design.

Finally, we make parametric assumptions on non-homotheticities, using non-homothetic CES (nhCES) preferences as in Hanoch (1975), Matsuyama (2019), and Comin et al. (2021). We then study the quantitative importance of increasing returns to scale, non-homotheticities and price shocks for optimal tax rates and welfare across the skill distribution. By introducing parametric assumptions on preferences, these analyses are complementary with the analysis of first-order approximations, because they characterize how our new channels affect the optimum when accounting for potential non-linearities. They also allow us to characterize the quantitative importance of non-homotheticities for the optimal tax schedule.

First, we find that the wage subsidy used to correct for inefficiencies is quantitatively large. With an average price elasticity of 0.30, consistent with causal estimates, the net of tax wage increases everywhere by 43%. However, a naive implementation of this work subsidy, without considering the interaction

[10] Since empirical studies stress the importance of using granular data to properly measure inflation heterogeneity, we also estimate the impact of price changes in the subset of goods covered by the Nielsen scanner data. We find that the sensitivity of the tax rate to change in the prices is larger when we consider granular products rather than goods aggregated at a level comparable to the CEX. Our baseline results using the CEX-CPI data are therefore likely to underestimate the impact of price changes on optimal redistribution.

with redistributive motives, would significantly overstate the regressive impact of this corrective tax. Indeed, with a logarithmic social welfare function, it is optimal for the cost of the work subsidy to be predominantly paid by high-skill agents, hence marginal tax rates do not fall as much as with the naive correction: at the optimum marginal tax rates only fall by 4 percentage points at the bottom of the distribution, while a naive implementation would reduce them by 14 percentage points. This finding shows that understanding the interaction between corrective and redistributive motives is crucial to the overall design of tax systems in practice.

Second, relative to the optimal tax schedule with homothetic preferences, we find that non-homotheticities imply more redistribution. Relative to the optimum under homothetic preferences, marginal taxes increase over the full range of the income distribution. The increase is more pronounced at the bottom of the income distribution, with an increase in marginal tax rates of about 6pp for levels of earned income below $20,000. The increase is about 2pp at an income level of $100,000, and then gradually decreases, reaching levels close to zero above $300,000. Thus, the simulations show that non-homoheticities have a significant quantitative impact on optimal marginal tax rates. We show that this increase in redistribution can be explained by the change in equilibrium prices and in the marginal utility of redistribution across the skill distribution.[11]

Third, using the nhCES parametric framework, we confirm the results obtained with the first-order approximation about the impact of observed heterogeneous price shocks: it is desirable for the planner to redistribute more toward high-income households, by reducing marginal tax rates at the bottom of the income distribution.

In all simulations, a unifying mechanism operates: changes in equilibrium prices and the distribution of marginal propensities to consume govern the change in the optimal tax schedule. This mechanism explains both why the optimal tax schedule is more redistributive when we take into account non-homothetic spending patterns, and why observed inflation heterogeneity benefiting higher income households generates a regressive response of the optimal tax schedule.

Relative to prior work, the main contribution of this paper is to provide a theoretical and quantitative characterization of the impact of prices on optimal tax design. We thus relate to several strands of literature. First, in prior work the effect of prices on the tax schedule has remained implicit, as standard tax formulas depend on endogenous variables that depend on prices, such as the marginal utility of disposable income. Several papers have highlighted the implications of specific assumptions on consumers' preferences for tax design, including preference heterogeneity (e.g., Saez (2002), Diamond and Spinnewijn (2011)) and consumers' myopia (e.g., Allcott et al. (2019)). Instead, we show theoretically and quantitatively that prices play an important role even in the canonical setting where the utility function is separable between labor and all commodities, i.e. no indirect taxes need to be used, as in Atkinson and Stiglitz (1976). We explicitly characterize the impact of prices on the tax schedule, both in partial equilibrium and general equilibrium, providing decompositions isolating the economic forces at play. Second, our results contribute to a growing strand of the optimal taxation literature that has isolated the general equilibrium effects of taxes, focusing on wages (e.g., Rothschild and Scheuer (2013), Sachs et al. (2020)); we complement these analyses by characterizing the general equilibrium impact on prices in the presence of non-homotheticities. Third, although imperfect competition is not our focus, our work relates to a growing literature on optimal taxation in the presence of imperfect competition, in which endogenous prices or wages play a role for redistribution from firm owners toward workers (e.g., Boar

[11] As the relative price of the necessity bundle decreases, it is optimal to redistribute more to those with a higher marginal propensity to consume on necessities, which induces further tax changes and changes in labor supply, etc. The strength of these feedback loops depends on the parameters governing increasing returns and social preferences for redistribution, and we find them to be large in our calibration. We document the robustness of our results to alternative parameter values.

and Midrigan (2019), Eeckhout et al. (2021), Kushnir and Zubrickas (2020)).[12] Instead, we demonstrate the importance of non-homotheticities and show that prices play an important role even in the canonical setting with no profit or full profit taxation, as in Diamond and Mirrlees (1971b). We thus isolate a novel mechanism, the amplification of redistribution due to the interaction between price changes and non-homotheticities.

Furthermore, by studying price changes stemming from increasing returns to scale, this paper contributes to a growing literature on optimal tax design and endogenous productivity. Recent work highlights the role that taxes may have on entrepreneurial effort (e.g, Jaimovich and Rebelo (2017), Bell et al. (2018)) and draws implications for optimal taxation of top earners (e.g, Jones (2019), Bell et al. (2019)).[13] In contrast, we study productivity effects that are induced by changes in demand, through returns to scale, and which inherently interact with the income tax schedule. We find that the impact of taxes on productivity through demand and returns to scale is quantitatively large, implying substantial adjustments to the optimal tax schedule.

The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the optimal income and commodity taxation formula in terms of sufficient statistics. Section 4 uses the comparative static approach to characterize the sensitivity of optimal tax rates to price shocks, the elasticity of prices to market size, and shifts in the skill distribution. The quantitative analysis is carried out in Section 5. Supplemental results and proofs are reported in the Online Appendix.

2 Model

We consider an economy with n sectors, each producing a differentiated good i sold at price p_i. Labor is the only factor of production and the wage is normalized to 1. There is a mass 1 of households with different types θ distributed according to $\pi(\theta)$. We now describe in the demand and supply sides of the economy as well as the social planner problem.

Households. We use the standard notation where pre-tax labor income z, rather than hours worked, enters the utility function. Households have weakly separable preferences in consumption $\{c_1, ..., c_n\}$ and labor income z. Their utility is given by:

$$U(u(c_1, .., c_n), z, \theta),$$

where u is the sub-utility function of consumption. These are the standard Atkinson-Stiglitz preferences and consumption choices only depend on consumer prices and post tax income z^*.

[12] These papers highlight the importance of rents that accrue to firm owners, which can be redistributed through taxation of income, endogenous price changes and commodity taxes. In contrast, our results continue to apply in settings with no rents, i.e. with full profit taxation or zero profit. In particular, Kushnir and Zubrickas (2020) study optimal taxation with endogenous prices, decreasing returns to scale, positive firm profits, and homothetic utility. Their Appendix A.3 examines the case of non-homothetic preferences, but the impacts of non-homotheticities and prices remain implicit in their tax formulas through endogenous variables that depend on prices, such as the marginal utility of disposable income. In contrast, our results do not depend on the taxation of profits and provide a full characterization of the role of non-homotheticities. The intuition for their main result is that, when profits are not fully taxed, the social planner uses the price level as an additional redistributing tool: a decrease in the price level benefits low-productivity agents as they can afford more consumption, but hurts high-productivity agents through a decrease in firm profits. Instead, our price effects operate through non-homotheticities and changes in the marginal utility of income at different income levels.

[13] A limitation of this approach, from the perspective of optimal income and commodity taxation, is that there may exist distinct policy tools, such that the R&D tax credits, which may be sufficient to affect inventors' and entrepreneurs' incentives appropriately, effectively leaving the optimal tax problem unchanged. For an analysis of optimal R&D policy in a model with heterogeneous firms, see Akcigit et al. (2016).

The Atkinson-Stiglitz specification allows us to capture non-homothetic spending patterns across the income distribution, and thus the unequal effects of price changes, while focusing on a single tax instrument for redistribution, the nonlinear income tax. With more general preferences, it would be possible to use the consumer prices of certain goods to better discriminate between different taxpayers (e.g., Saez (2002)). However, we focus on characterizing how unequal price changes for consumption baskets along the income distribution affect the desirability of redistribution policies. In the interest of providing a streamlined analysis, it is sufficient to generate heterogeneous baskets of consumption through non-homothetic Atkinson-Stiglitz preferences rather than idiosyncratic preferences.[14]

Finally, we denote by v the indirect utility of the agent and by v_{z^*} the marginal utility of income. v depends on the agent type θ, on consumer prices and on the tax schedule. Aggregate demand for i across all households is denoted by C_i.

Pricing Function and Profits. The supply side of the economy affects households' welfare through the price of commodities (and potentially the rebated firms' profit). We use a flexible reduced-form specification summarizing prices as a function of aggregate quantities produced $\{Q_1, ..., Q_n\}$. The price p_i of the good produced in sector i is given by $p_i = \phi_i(Q_1, ..., Q_n)$. This specification captures a wide range of supply side models, i.e. production functions and market structures. For example, when the n goods are produced competitively, the price of i is determined by its marginal cost of production $p_i = mc_i(Q_1, ..., Q_n)$. Similarly, when Q_i is a homothetic aggregate of sub-varieties (e.g. CES, Kimball, translog) produced monopolistically by firms in sector i, the price of the aggregate will be a function of the vector $\{Q_1, ..., Q_n\}$. Thus, our specification encompasses a large set of structural models of the supply side, including models with firm selection (Melitz (2003)), variable markups (Feenstra and Weinstein (2017)), and endogenous innovation (Bustos (2011)).[15]

An important statistic for our analysis is the elasticity of the price p_i to market size. It captures how the price of i reacts to a change in the demand for j. We denote by A the matrix of market size elasticities, with $A_{ij} = -Q_j/p_i \, d\phi_i/dQ_j$. While there are no a priori sign restrictions on the matrix A, empirical studies (e.g., Costinot et al. (2019), Jaravel (2019), Faber and Fally (2021)) find that prices decrease in response to an increase in market size. Thus, A is defined such that $Q_i/p_i \partial p_i/\partial Q_i = -A_{ii}$, with A_{ii} positive in our benchmark specification.[16] This matrix will be crucial: when the tax rate is modified, it shifts households' income and thus aggregate demand for the n goods, which affects prices. The matrix A is again a reduced form object which can be linked to structural parameters of fully specified model. In the competitive case, A is given by the matrix of gradients of the marginal cost: $A_{ij} = -Q_j/p_i \, \partial mc_i/\partial Q_j$. With imperfect competition, A captures both the change in marginal cost and the potential change in mark-ups. A will be a sufficient statistic for the supply side of the economy:[17] since we have direct reduced-form estimates of the market size elasticity of prices,[18] we do not need to take a stand on the precise channels (competitiveness, technological change, etc.) through which demand for goods affects prices. We will sometime assume

[14] Indeed, the heterogeneous welfare impacts of price changes only depend on the heterogeneity in households' expenditure shares, whether they stem from idiosyncratic preferences or households' income levels.

[15] Our reduced-form specification excludes models where the identity of the consumers of i matters for p_i. For example, if a monopoly produces the n goods, prices may depend on the distribution of consumers across markets rather than on the aggregate demand for each good (e.g., Weyl and Fabinger (2013)). We also exclude dynamic growth models (e.g. Romer (1990), Aghion and Howitt (1992)): focusing on static models of innovation, as in Bustos (2011), facilitates the comparison to the workhorse Mirrlees model of optimal taxation, and the channels we uncover about the interplay between endogenous prices and non-homotheticities would continue to apply in dynamic models. For an analysis of optimal taxation with homothetic preferences in a dynamic growth model, see Aghion et al. (2013).

[16] Our results remain valid with negative A_{ii}.

[17] The fact that, conditional on equilibrium prices, the matrix A is a sufficient statistics for the optimal income and commodity taxes is a result of our analysis.

[18] We discuss the available estimates in Section 5.1.

that the matrix A is diagonal: prices only respond to demand in their own market. In this case, we denote the matrix of market size elasticities by Δ_α, with $\alpha_i = -Q_i/p_i \, d\phi_i/dQ_i$ (and α_i is in general non negative).

To complete our description of the supply side, we need to determine how profits are distributed to households. In our benchmark specification, we assume that firms make no profit and pay a fixed cost, which is consistent with entry models where entry is free. This assumption is i line with available empirical evidence (e.g., Jaravel (2019)) suggesting that the downward-sloping supply curves observed in the data is primarily explained by increased competition: as market size increases, more firms enter which increases competition and drives prices down through lower markups. In that case, the supply side can be entirely summarized through the functions ϕ_i and the matrix A. For completeness, we will also consider the case where profits are non zero but fully taxed by the government.

To illustrate our reduced form specification, we provide a simple structural example. We give more examples of such models in Online Appendix A4. In each market i, identical firms produce subvarieties q_i^k at cost $\chi_i(q_i^k)$. To enter the market, firms pay a fixed labor cost ξ_i. Agents have identical and symmetric translog preferences (with parameter γ_i)[19] for the different sub-varieties q_i^k.[20] The equilibrium price charged by firms $\tilde{p}_i$ as a function of total physical quantities produced $\tilde{Q}_i$ is given by:

$$\tilde{p}_i = \left(1 + (\gamma_i N_i)^{-1}\right) \chi_i' \left(\frac{\tilde{Q}_i}{N_i}\right) \quad \text{Pricing Equation}$$
$$\left(1 + (\gamma_i N_i)^{-1}\right) \chi_i' \left(\frac{\tilde{Q}_i}{N_i}\right) \frac{\tilde{Q}_i}{N_i} - \chi_i \left(\frac{\tilde{Q}_i}{N_i}\right) = \xi_i \quad \text{Entry Equation}$$

The equilibrium number of firms N_i is determined implicitly by the free entry condition, while the optimality condition from the firm side determines the price $\tilde{p}_i$. Note that with translog, the mark-up $(\gamma_i N_i)^{-1}$ decreases with N_i.[21] Since consumer preferences exhibit love of variety, the welfare relevant quantity is $Q_i = exp(-1/(2\gamma_i N_i))\tilde{Q}_i$. The variety adjusted consumer price index is similarly $p_i = exp(1/(2\gamma_i N_i))\tilde{p}_i$.[22] Therefore, the pricing function $\phi_i(Q_i)$ is given by:

$$p_i = \phi_i(Q_i) = \left(1 + (\gamma_i N_i(Q_i))^{-1}\right) \chi_i' \left(e^{\frac{1}{2\gamma_i N_i(Q_i)}} \frac{Q_i}{N_i(Q_i)}\right) e^{\frac{1}{2\gamma_i N_i(Q_i)}}$$

with $N(Q_i)$ implicitly given by the entry condition. The supply side is entirely summarized by the set of functions $\phi_i(Q_i)$. Finally, denoting $\rho > 0$ the curvature of marginal cost, the price elasticity α_i is:

$$\alpha_i = \frac{(\gamma_i N_i)^{-1}}{1 + (\gamma_i N_i)^{-1}} \frac{Q_i dN_i}{N_i dQ_i} + \frac{1}{2\gamma_i N_i} \frac{Q_i dN_i}{N_i dQ_i} - \rho \left(1 - \frac{1 + 2\gamma_i}{2\gamma_i N_i} \frac{Q_i dN_i}{N_i dQ_i}\right)$$

In this case, α_i captures the gains from mark-up reduction (the first term), from increased product diversity (the second) and the loss from decreasing returns (the third term).[23]

Government. The social planner has access to a full set of commodity taxes and to a non linear income tax. Although our agents have Atkinson-Stiglitz preferences, we will show that there is a role for com-

[19]For each agent θ, spending $e_i(\theta)$ in market i, the homothetic aggregate $c_i(\theta)$ is given by $ln(c_i(\theta)) = ln(e_i(\theta)) - 1/(2\gamma N_i) - 1/N_i \int ln(p_i(k))dk - \gamma_i/(2N_i) \int\int ln(p_i(k))(ln(p_i(k')) - ln(p_i(k)))dkdk'$ with $p_i(k)$ the price of sub variety k and N_i the number of sub variety.

[20]Note that preferences for the aggregated products can still be non-homothetic.

[21]Intuitively, with this demand system firms price at a markup over marginal cost; the demand elasticity depends on the number of varieties in the market: as more varieties enter, consumers perceive varieties as more substitutable, the elasticity of substitution increases and markups decrease.

[22]Note that we have $\tilde{p}_i\tilde{Q}_i = p_i Q_i$

[23]Denoting $\mu = (\gamma_i N_i)^{-1}$, we have $QdN/NdQ = (1 + 1/(2\gamma_i) + 1/(1 + \rho(1 + 1/\mu))N\, exp(-1/(2\gamma_i N)))^{-1} > 0$.

modity taxation since goods are potentially not priced efficiently.[24] As it may be implausible to assume that the government can tax all goods separately, we will briefly discuss the case where the planner only has limited commodity taxes in Section 3.2.

The planner's problem is standard. The planner maximizes the following social welfare function,

$$\int_{\theta_{min}}^{\theta_{max}} G(v(\theta))\pi(\theta)d\theta,$$

by setting consumer prices $\{q_1, .., q_n\}$ and the income tax $T(z)$ subject to three constraints. First, the producer prices $\{p_1, .., p_n\}$ are given by the functions ϕ_i, with $p_i = \phi_i(C_1, ..., C_n)$, with C_i the aggregate demand for i.[25]. Second, households optimally choose consumption and labor supply under $\{q_1, .., q_n\}$ and $T(z)$. We denote by $f(z)$ the resulting distribution of income (with $dz/d\theta f(z) = \pi(\theta)$) and by z^* disposable income (with $z^* = z - T(z)$). Finally, the government's budget constraint is given by $\sum(q_i - p_i)C_i + \mathbb{E}(T)$ when firms make no profit and by $\sum q_i C_i - \chi(C_1, .., C_n) + \mathbb{E}(T)$ when profits are fully taxed, with χ the cost of producing demanded quantities $\{C_1, ..., C_n\}$.

Missing Tax. Our benchmark specification with a potentially downward sloping supply curve and no profit does not have, in general, an efficient supply side. With more tax instruments, the social planner could regulate firms and improve the allocation. For example, in an entry model, the planner could directly choose the number of firms in each market to minimize the total cost of production (which includes the variable cost of production and the entry cost). The planner can always regulate supply in a revenue neutral fashion, and for a given industrial policy τ that depends on aggregate quantities, there is a new reduced-form pricing function $p_i = \phi_i^\tau(C_1, ..., C_n)$ which depends implicitly on the regulatory regime.[26] In that sense, industrial policies and redistribution are separable: for a given regulatory rule of the supply side, we take the induced pricing function (and the market size elasticity) as given and derive the optimal redistributive policy.[27] Our results will therefore be valid whether or not industrial policies are optimal, or missing altogether.

Notation. We use standard notation throughout the paper. ζ is the compensated labor supply elasticity: when agents face a linear budget constraint $q \cdot c = (1-\tau)z + I$, $\zeta = (1-\tau)/z \, dz^h/d(1-\tau)$, with z^h the Hicksian supply of labor. $\tilde{\zeta}$ is the compensated labor supply elasticity corrected for non-linearities in the budget constraint: $\tilde{\zeta} = \zeta/(1 + z\zeta T''/(1-T'))$. Similarly, η is the income effect with a linear budget constraint and $\tilde{\eta}$ the corrected income effect. Regarding spending patterns, $e_i = q_i c_i(z^*, q)$ denotes the agent's expenditure on i, $\partial_{z^*} e_i$ the marginal propensity to spend on i, E_i is aggregate spending on i and $\partial_{z^*} E_i$ the average marginal propensity to spend on i. Finally, S is the matrix of cross price derivative of the aggregate hicksian demand function, with $S_{ij} = \mathbb{E}(\partial_{q_j} c_i^h)$, and $\mathcal{S}$ the matrix of price elasticities $\mathcal{S}_{ij} = q_j/$

[24]For example in the case where profit is fully taxed, the price function could differ from the marginal cost of production. More interestingly, we will show in section 3.1 that commodity taxes are in general needed when firms make no profit.

[25]The pricing relationship ϕ_i depends on aggregate quantities produced Q_i, but we use the equilibrium relationship $Q_i = C_i$.

[26]In the simple example with translog preferences, the optimal regulation policy can be achieved with a tax τ_c on variable costs and an entry subsidy τ_e set such that the regulatory policy is budget neutral: $\tau_c \chi_i = \tau_e \xi_i$. The pricing function becomes $p_i = (1+\tau_c)(1 + (\gamma_i N_i)^{-1})\chi_i' \, exp(1/(2\gamma_i N_i))$, where N_i is implicitly given by the new entry condition $(1+\tau_c)(1+(\gamma_i N_i)^{-1})\chi_i' Q_i / N_i - \chi_i = \xi_i$. The optimal τ_c only depends on Q_i and minimizes p_i. Therefore, with optimal regulation we have a new pricing function $p_i = \tilde{\phi}_i(Q_i, \tau_c(Q_i))$. In a more complex model with firm selection and spillovers across markets, the optimal policy depends on the full vector $Q = \{Q_1, ..., Q_n\}$ and can be implemented through entry subsidies and variety specific taxes. Given these instruments τ, the regulation problem is simply to minimize $\sum p_i(Q, \tau) Q_i$ at fixed Q. The resulting prices again depends only on aggregate quantities.

[27]In the quantitative analysis, we use the estimated market size elasticity in the United States between 2004 and 2015, which depends implicitly on the regulatory regime in that period.

$C_i S_{ij}$.

3 Optimal Taxation: First-Order Approach

In this section, we derive first the optimal commodity taxes and then the optimal income tax schedule.[28] As shown in Saez (2002), for a type θ agent, a change in the consumer price of i, dq_i can be compensated with the non linear transfers $dT(z,\theta) = -c_i(z,q,\theta)dq_i$. This compensation not only keeps the agent welfare constant, it also leaves labor supply unchanged.[29] This compensation is not implementable in general since consumption depends on the unobserved type of the agent. With our preferences, however, consumption only depends on consumer prices and on disposable income $z^* = z - T(z)$ and the compensation can be implemented through a change in the labor tax $dT(z) = -c_i(z^*, q)dq_i$. This simple result will be useful in this section to summarize the effects of consumer price changes by the fiscal cost of the non-linear compensation.

3.1 Optimal Commodity Tax

We first derive the optimal commodity tax in our benchmark case with no profit. Since consumer price changes can be compensated with the income tax, the social planner sets the commodity tax to maximize government revenue, taking into account the cost of compensation. Since the derivation is simple, we go through it step by step.

Consider a small change in the consumer price of i, dq_i. The price change is compensated through $dT(z) = -c_i(z^*, q)dq_i$. As explained in the preamble, the compensated price change does not generate a labor supply response, so the impact on government revenue is:

$$\underbrace{dq_i C_i}_{\text{mechanical effect}} + \underbrace{\sum (q_i - p_i) dC_i}_{\text{households' behavioral response}} - \underbrace{\sum dp_i C_i}_{\text{firms' response}} + \underbrace{\mathbb{E}(dT(z))}_{\text{cost of the compensation}}$$

The increase in q_i first mechanically raises revenues from the tax on i by $C_i dq_i$. Since dq_i is compensated, aggregate consumption reacts through a substitution effect: $dC/C = \mathcal{S}dq/q$. The supply side responds to this shift in demand, and producer prices are adjusted through the matrix of market size elasticities A: $dp/p = -A\mathcal{S}dq/q$. Households' responses and the supply side response generates a second change in the revenue of commodity taxes. Finally, since agent are compensated for the consumer price increase, revenue from the income tax decreases: $\mathbb{E}(dT) = -C_i dq_i$. Therefore, the total effect on government revenue is given by:

$$\left(qC' - pC'(Id - A)\right) \mathcal{S} \frac{dq}{q}.$$

Since this is true for any consumer price change dq, $qC - (Id - A')pC$ has to belong to the kernel of $\mathcal{S}'$, which is the vector qC: the optimal commodity tax are determined up to a scaling constant β, $(1-\beta)qC = pC - A'pC$. Without loss of generality, we choose the scaling so that on average commodity taxes raise no revenue.[30]

[28] Online Appendix A1 reports the proofs for this section.

[29] The result is intuitive: this change in the tax schedule compensates the agent for the change in prices given her consumption basket at her current income level, and it also compensates her for the change in prices that she would face given her optimal consumption choices at higher or lower income level. Given that her current labor supply was optimal before the price change, it remains optimal after the price change and the non-linear compensation.

[30] We do so for two reasons. First, by construction, the average commodity tax is zero. If it was instead positive, consumer prices would be on average higher than producer prices, which is an implicit income tax. Second, if the revenue from the tax was

Proposition 1. *Consider our benchmark economy where there is no profit and the market size elasticity of prices is A. We define the average market size elasticity α by:*

$$\alpha = \frac{\sum \left(p_i C_i \sum_j A_{ij} \right)}{\sum p_i C_i},$$

which captures the elasticity of the average price $\sum p_i C_i$ with respect to a proportional increase of all market sizes. The optimal and budget neutral commodity taxes t_i (with $q_i = (1 + t_i) p_i$) are given by:

$$1 + t_i = \frac{1}{1 - \alpha} \left(1 - \sum A_{ji} \frac{p_j C_j}{p_i C_i} \right).$$

In particular, when A is diagonal, we have $1 + t_i = (1 - \alpha_i)/(1 - \alpha)$ and $t_i = 0$ when the α_i are equal across markets.

The commodity tax is purely corrective: its role is to make agents internalize the social benefit of consumption. When demand for i increases, the effect on the price of j is given by A_{ij}. Equation 1 simply states that if increasing demand for i lowers prices by more than α (the average market size elasticity), i is subsidized. Conversely, it is taxed if the price reduction is less than the average. This correction is needed because there is an implicit aggregate demand externality in our model where firms make no profit.

The externality is nil when firms' profit is fully taxed and prices are equal to marginal cost: if an increase in demand reduces prices, it also decreases firms' profit. Profit is rebated to households, so lower prices indirectly reduce households' disposable income. The two effects cancel each other and there is no externality. In our benchmark economy, lower prices only have a positive impact so the externality does not cancel. Note that the planner does not use commodity taxation to take advantage of the average price impact, this will be done through the income tax. Commodity taxes are used to internalize differential price effects across markets: through substitution effects, consumption is reallocated towards more responsive markets.[31]

When profits are fully taxed, prices are simply set to the marginal cost of production. For completeness we restate this standard result in the following proposition:

Proposition 2. *Consider an economy where profits are fully taxed. We denote by $\chi(Q_1, ..., Q_n)$ the total cost of production. At the optimal allocation, consumer prices satisfy: $q_i = \partial_{Q_i} \chi(C_1, ..., C_n)$.*

In our benchmark economy with no profit, our optimal commodity tax result can also be expressed in terms of total cost. Since there is no profit, there is an implicit fixed cost of production. In equilibrium, firms' revenue equals their total cost (variable plus fixed). We have that $p_i = \phi_i(Q_1, ..., Q_n)$ and $-Q_j / p_i \partial_{Q_j} \phi_i = A_{ij}$, so the partial derivative of the total cost (which is equal to total revenue $p_i C_i$ in equilibrium) with respect to Q_i is given by $p_i - \sum A_{ji} p_j C_j / C_i$. Proposition 1 therefore simply states that consumer prices should be proportional to "total marginal costs", where the notion of total costs incorporates the utilization of the fixed factor.[32]

negative, the government would have to raise funds to finance the subsidy (or rebate the revenue optimally if it was positive) and there would be a non trivial interaction between redistribution and corrective commodity taxation. Imposing zero revenue on average cleanly separates the redistributive and corrective motives.

[31] The entry model in Online Appendix A3 provides a concrete example. In that simple model, when markets are larger, more firms enter and mark-up decreases. The effect of market size on mark-ups is not reflected in producer prices. Thanks to commodity taxes, consumer prices capture the marginal effect on mark-ups so consumption is redirected towards markets where mark-ups are more responsive to aggregate demand.

[32] For example, in an entry model, the fixed factor would be given by the number of firms times the cost of entry.

3.2 Optimal Income Tax

We now turn to the optimal income tax. We denote by λ the Lagrange multiplier on the government budget constraint. To characterize the schedule we consider the standard perturbation of Saez (2001): a small change of marginal tax $d\tau$ in a neighborhood dz of z and a change in tax $dzd\tau$ above. We have four effects to consider: the mechanical change in revenue, the welfare effect of the tax change, the fiscal externality of the labor supply responses and the fiscal externality of shifts in aggregate consumption and producer price adjustments.

Mechanical and Welfare Effects. This effect is standard: every households above z pays an additional $dzd\tau$ in taxes. Their welfare loss is $v_{z^*}dzd\tau$ which is valued $G'v_{z^*}dzd\tau/\lambda$ by the planner. The total effect is:

$$\mathbb{E}_{z'>z}\left(1 - G'v_{z^*}/\lambda\right) dzd\tau$$

Labor Supply Effects. This is again standard. The change in tax rate at z generates a compensated wage effect on labor supply and the change in tax above z creates an income effect. The change in government revenue is then given by:

$$-f(z)\frac{T'}{1-T'}z\tilde{\zeta}dzd\tau - \mathbb{E}_{z'>z}\left(\frac{T'}{1-T'}\tilde{\eta}\right) dzd\tau$$

Price and Demand Effects. The small change in the tax schedule affects households' disposable income both mechanically and through labor supply responses. This shifts aggregate demand for goods and producer prices are adjusted. The impact on government revenue, through the receipts of the commodity taxes is given by:

$$\sum(q_i - p_i)dC_i - \sum dp_iC_i$$

The price adjustment is $dp_i/p_i = -\sum A_{ij}dC_j/C_j$ so we can rewrite the equation as:

$$\sum_i \left(q_i - p_i\left(1 - \sum_j A_{ji}\frac{p_jC_j}{p_iC_i}\right)\right) dC_i = \sum \alpha q_i dC_i$$

The equality uses our result on the optimal commodity taxes. This can be further simplified: using the budget constraint of households, we have that $\sum q_i dc_i$ is exactly the change in disposable income. The impact on government revenue is then given by:

$$-\alpha\left(f(z)z\tilde{\zeta} + \mathbb{E}_{z'>z}\left(1+\tilde{\eta}\right)\right) dzd\tau$$

Since commodity taxes optimally distributes consumption across markets, when they are in place it is as if there was a unique market where the market size elasticity of price is the average elasticity α.

Summing these four effects gives the first order conditions for the optimal tax rate. We denote by g the pareto weights $g = G'v_{z^*}/((1-\alpha)\lambda)$, where the $1-\alpha$ normalization is such that $\mathbb{E}(g) = 1$ when there are no income effects.

Proposition 3. *When firm makes no profit in an economy where the average market size elasticity is α, the optimal* *non linear income schedule is characterized by:*

$$\frac{T'}{1-T'} = -\alpha + \frac{1-\alpha}{z\tilde{\zeta}f(z)}\left\{\mathbb{E}_{z'>z}\left(1-g\right) - \frac{1}{1-\alpha}\mathbb{E}_{z'>z}\left(\left(\alpha + \frac{T'}{1-T'}\right)\tilde{\eta}\right)\right\}$$

When income effects are negligible, the optimal schedule satisfies:

$$\frac{T'}{1-T'} = -\alpha + \frac{1-\alpha}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(1-g\right)$$

When profits are fully taxed, the optimal schedule is:

$$\frac{T'}{1-T'} = \frac{1}{z\tilde{\zeta}f(z)}\left\{\mathbb{E}_{z'>z}\left(1-g\right) - \mathbb{E}_{z'>z}\left(\frac{T'}{1-T'}\tilde{\eta}\right)\right\}$$

When the average market size elasticity is positive ($\alpha > 0$), labor supply is subsidized and tax rates are lower.[33] Intuitively, lower tax rates incentivize work, so aggregate income is higher and households consume more which reduces prices. Note that if the endogenous quantities (the pareto weights, the income distribution and the labor supply elasticities) remain constant as α varies, then the formula tells us that the tax rate at $\alpha > 0$ is such that $1 - T' = (1-T')_{\alpha=0}/(1-\alpha)$. In that case, the planner implements a uniform wage subsidy $1/(1-\alpha)$ on top of the standard non linear tax. There would be no interaction between the corrective tax (the wage subsidy) and the redistributive motive. In fact, prices and all endogenous quantities are likely to vary as α changes, which we characterize in the remainder of the paper.[34]

These observations highlight an important limitation of the standard optimal tax formula: the effect of prices is completely implicit, so it provides little insight on how they affect optimal redistribution. In particular, when profits are fully taxed, the first order conditions are exactly the same whether prices are fixed or not. In the next section, we provide a thorough characterization of the role of prices using a comparative static approach addressing these limitations.

Optimal Taxation without Commodity Taxes. Up to now, we have assumed that commodity taxes were very flexible, such that the government could tax differently every good in the economy. This might be an implausible assumption. Here, we briefly discuss how the optimal tax rate is modified when this is not the case. To simplify, we consider the extreme opposite case: the planner cannot set different taxes on different goods, i.e. there are no commodity taxes. Suppose, for clarity, that there are no income effects and that the matrix A is diagonal. The optimal tax rate then satisfies:

$$\frac{T'}{1-T'} = -\tilde{\alpha} + \frac{1-\tilde{\alpha}}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(1-g\right) - \underbrace{\sum_j(\tilde{\alpha}_j - \tilde{\alpha})\left(\partial_{z^*}e_j + \frac{1}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(\partial_{z^*}e_j\right)\right)}_{\text{Reallocation across markets}}$$

[33]Note that this correction could be done through consumer prices using uniform commodity taxes. Indeed, an homogeneous reduction in prices is equivalent to a wage subsidy, so the optimal income tax could be given by the formula of Proposition 3 with $\alpha = 0$ if all consumer prices are multiplied by $1-\alpha$.

[34]Rothschild and Scheuer (2014) and Kushnir and Zubrickas (2020) also considered aggregate externalities from returns to scale, and showed that it leads to a proportional adjustment to the entire income tax schedule. Relative to these papers, our contribution is threefold. First, we characterize the role of non-homotheticities and endogenous prices in Section 4. Second, we are the first to show that the response of the optimal tax schedule is non-trivial if the social welfare function is concave, even when preferences are homothetic: higher-income households will contribute more to the financing of the subsidy (see Section 4.4 for the theoretical characterization and Section 5.3.1 for the quantification). Third, we quantify the impact on the optimal tax schedule using a model matching causal estimates of increasing returns to scale (see Section 5).

The optimal tax is the sum of two terms. The first term is the same as in our analysis with commodity taxes. It captures the standard redistributive motive and the uniform wage subsidy $1/(1-\tilde{\alpha})$, with $\tilde{\alpha}$ the average reduction in prices when agents' income increases.[35] Without commodity taxes, however, the allocation of consumption across markets is inefficient: there is too much consumption of low α_i products. The second term precisely expresses how the planner utilizes the income tax to reallocate consumption more efficiently.

Let us describe this reallocation term. First, $\tilde{\alpha}_i$ is the average response of prices when the market for i expands, taking into account the equilibrium response of demand.[36] It takes a simple form when the elasticity of substitution is constant (denoted here by σ):

$$\tilde{\alpha}_i = \frac{\alpha_i}{1-\sigma\alpha_i} / \left(\sum \frac{s_j}{1-\sigma\alpha_j}\right)$$

where $s_j = p_j C_j / \sum p_j C_j$ is the share of j. Since $\tilde{\alpha}_i$ increases with α_i, reallocating consumption from low to high α_i markets lowers prices on average and improves efficiency. Second, increasing the tax rate at z decreases demand for i by $\partial_{z^*} e_i + 1/z\tilde{\zeta} f(z)\mathbb{E}_{z'>z}(\partial_{z^*} e_i)$. Therefore, the reallocation term lowers the income tax at z if households above z spend more on high α_i goods than the average household. To see this more clearly, note that we can rewrite the reallocation term as:

$$\sum_j (\tilde{\alpha}_j - \tilde{\alpha}) \left(\partial_{z^*} e_j - \partial_{z^*} E_j + \frac{1}{z\tilde{\zeta} f(z)} \mathbb{E}_{z'>z}\left(\partial_{z^*} e_j - \partial_{z^*} E_j\right)\right).$$

When preferences are non-homothetic, the planner uses the income tax to address the lack of commodity taxes. Redistributing income to high α_i consumers, indirectly reallocates consumption from low to high α_i markets. With homothetic preferences, by contrast, the reallocation term is nil since redistributing income no longer affects the relative market sizes.

Finally, note that when the α_i are equal to α across markets, we have $\tilde{\alpha}_i = \tilde{\alpha} = \alpha$ and we recover the income tax of Proposition 3. Indeed, commodity taxes are superfluous in this case.

4 Understanding the Impact of Prices and Non-homotheticities

In this section, we use a comparative statics approach to understand the mechanisms through which optimal tax rates respond to prices, market size effects and shifts in the income distribution in the presence of non-homotheticities.[37] To streamline the analysis, we make several simplifying assumptions. First, we impose an additively separable utility function, assume that the wage elasticity of labor supply is constant and that there are no income effects at initial prices. These assumptions are common in the optimal taxation literature and provide a useful benchmark, but can easily be relaxed.[38] Second, we assume that the matrix of market size elasticity is constant. This is a practical assumption: our goal is to bring the formula we derive to the data, and existing evidence suggests that a constant elasticity provides a good

[35] We have $\tilde{\alpha} = [s_1 .. s_n]'(Id + \Delta_\alpha \mathcal{S})^{-1}[\alpha_1 \partial_{z^*} E_1 / s_1 .. \alpha_n \partial_{z^*} E_n / s_n]$, or in terms of the $\tilde{\alpha}_i$ defined below, $\tilde{\alpha} = \sum \tilde{\alpha}_i \partial_{z^*} E_i$. $\tilde{\alpha}$ plays the same role here as in our previous analysis, as it quantifies the benefits of increasing agents' income.

[36] We have $\tilde{\alpha}_i = [s_1 .. s_j .. s_n]'(Id + \Delta_\alpha \mathcal{S})^{-1}[0 .. \alpha_i / s_i .. 0]$ where s_j the share of j. As the market for i expands, the price of i decreases by α_i / s_i. As price changes are compensated (with the income tax), this generates shifts in demand for all goods though substitution effects. Each round, prices adjust by $\Delta_\alpha \mathcal{S}[0 .. \alpha_i / s_i .. 0]$ so the equilibrium change in prices is $(Id + \Delta_\alpha \mathcal{S})^{-1}[0 .. \alpha_i / s_i .. 0]$.

[37] Online Appendix A2 reports the proofs for this section.

[38] In Online Appendix A3, we show that the results of this section hold when the elasticity of labor supply is not constant (see Proposition A5).

fit to the data.[39] Finally, in the first subsections below we assume that the social welfare function is linear (i.e., $G(v(\theta)) = \lambda_\theta v(\theta)$); we show how to relax this assumption in Online Appendix A3.[40]

4.1 Response to a Price Change in Partial Equilibrium

We first focus on the response of the optimal income tax schedule to an exogenous increase in the price of good i, q_i. We consider here the *partial equilibrium* response: we omit the general equilibrium adjustment of prices that results from shifts in aggregate demand for goods. This is useful for two reasons. First, it provides the full response of the tax schedule in the standard case where production functions are linear (i.e. $A = 0$). In this case, price changes are fully exogenous, as producer prices are inelastic to shifts in demand. Second, it is a stepping stone to characterize the general equilibrium responses, which will depend on the partial equilibrium response and on the endogenous change in consumer prices due to the reallocation of demand, which we analyse in Section 4.2. We start by deriving the response of the tax rate faced by an agent with ability θ, which should be interpreted as the change in tax rate at the $F(z(\theta))$ percentile of the income distribution.

Proposition 4. *The response of the optimal tax rate at θ to a marginal increase in the price of good i is given by:*

$$\begin{aligned}
\frac{q_i\partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\} &= \frac{1-\alpha}{z\tilde{\zeta}f(z(\theta))}\underbrace{\mathbb{E}^g_{z>z(\theta)}\left(\partial_{z^*}e_i - \mathbb{E}^g\left(\partial_{z^*}e_i\right)\right)}_{\text{value of redistribution}} \\
&+\frac{1}{z\tilde{\zeta}f(z(\theta))}\underbrace{\mathbb{E}_{z>z(\theta)}\left(\left(\frac{T'}{1-T'}+\alpha\right)z\tilde{\zeta}(1-T')\partial_{z^*z^*}e_i - \mathbb{E}\left(\left(\frac{T'}{1-T'}+\alpha\right)z\tilde{\zeta}(1-T')\partial_{z^*z^*}e_i\right)\right)}_{\text{income effects}} \\
&-\frac{1}{1-\alpha}\left(\frac{T'}{1-T'}+\alpha\right)\underbrace{\mathbb{E}\left(\left(\frac{T'}{1-T'}+\alpha\right)z\tilde{\zeta}(1-T')\partial_{z^*z^*}e_i\right)}_{\text{value of public funds}}.
\end{aligned}$$

With $\partial_{z^*}e_i$ the marginal propensity to spend on good i and $\mathbb{E}^g(X) = \mathbb{E}(gX)$ the average welfare value of a variable X.

The derivative of the tax rate with respect to consumer prices is the sum of three terms, which corresponds to three channels of the impact of prices on redistribution. First, an increase in prices affects the value of redistribution. At the initial prices, the social value of a dollar transfer to an agent with income z is given by the pareto weight $g(z)$. With this additional dollar, the agent spends $\partial_{z^*}e_i$ on good i. When the price of i increases, the purchasing power of the agent is therefore reduced at the margin by $\partial_{z^*}e_i$. Since an agent at z can buy less with a an additional dollar, the value of a dollar transfer is reduced by $g\partial_{z^*}e_i$, and taxes are increased.[41]

[39] Section 5 provides a review of available estimates.

[40] The comparative statics approach we use in this section allows us to provide an explicit characterization of the impact of non-homotheticities and market size effects on optimal taxation, in terms of observable statistics. With non-homotheticities, it is not possible to obtain an explicit solution of the integral equation characterizing the optimal tax schedule in partial or general equilibrium. First, non homothetic demand systems do not yield closed form expressions for both demand functions and marginal utility of income – with some exceptions e.g. Stone-Geary preferences, which are too limited to capture the impact of price changes observed in the data. Second, in general, the Pareto weights will depend on agents' disposable income and thus on on the income tax $T(z)$. With homothetic utility, quasilinear preferences in consumption, and a linear social welfare function, a closed form solution can be obtained in some cases (e.g., Eeckhout et al. (2021)).

[41] An equivalent way to understand the impact of prices on the value of redistribution is to note that $\partial_{z^*}e_i$ corresponds to the shift in the marginal price index of the agent. The social value of a dollar transfer to an agent is reduced by the increase in the price index of the agent since it determines how the agent's utility increases with this additional dollar.

The change in a household's price index is not enough, in isolation, to understand how redistribution is impacted by prices. What matters is the change in the individual price index *relative to* the average change in price indices. Increasing q_i reduces the purchasing power of all agents but this reduction is not necessarily uniform when preferences are non-homothetic. The planner decreases the tax rate at θ – to redistribute more to agent with income $z(\theta') > z(\theta)$ – if and only if a dollar transfer above θ buys relatively more welfare than below θ, that is if the price index increases relatively less above than below. In terms of spending patterns, this means that if the marginal propensity to spend on good i increases with income (or equivalently that spending on good i is convex), then the tax rate will be raised everywhere in response to an increase in q_i. The converse is true when the marginal propensity decreases. When preferences are homothetic, there is no effect on the tax rate since the change in price index is uniform along the income distribution.

The second and third channel through which the tax rate is impacted by price changes are tied to labor supply. An increase in the price of good i generates an income effect on labor supply. The change in income effect is given in our formula by $-z\tilde{\zeta}(1-T')\partial_{z^*z^*}e_i$. To understand this term, suppose that the marginal propensity to spend on good i decreases (so $\partial_{z^*z^*}e_i < 0$). An increase in the price of i implies that the agents' price index increases relatively less at higher income. Therefore, after the price change, the (marginal) purchasing power of the agent increases with income: a dollar transfer to an agent makes work more valuable – since she now has a higher real wage – and stimulates labor supply through an income effect. Formally, when $\partial_{z^*}e_i$ decreases (increases), an increase in q_i convexifies (concavifies) the indirect utility of consumption and therefore generates a positive (negative) income effect. When the income effect is positive, it increases the cost of taxation at θ: raising the tax rate at θ lowers the income of all agents with $\theta' > \theta$, and reduces their labor supply. Through this mechanism, the tax rate should be lowered at θ.[42] As above, this channel is inoperative when preferences are homothetic, with $\partial_{z^*z^*}e_i = 0$.

Since price changes generate income effects, they also impact the value of public fund – the social value of a lump sum transfer to all taxpayers.[43] The price change increases or decreases the marginal value of public fund depending on whether the income effects are on average positive or negative. When income effects are positive (that is, when q_i increases and the marginal propensity to spend on good i is decreasing), the value of a lump sum payment increases. Indeed, without income effects, a lump-sum transfer to all agents costs exactly one dollar of public fund, as it induces no behavioral response. Positive income effects reduce the cost of a lump sum payment: a higher lump-sum payment increases agents' disposable income, they work more which generates a positive fiscal externality. So when income effects increase with the price change, public funds become more valuable. Higher tax rates therefore become more valuable as they generate more revenue, which is then transferred with a lump sum to all agents.

This first characterization of the effect of prices on the tax rate has the advantage of making explicit the mechanisms through which prices affect redistribution. The impact of prices on the tax rate can be decomposed through three channels: the direct effect of prices on the social value of income, their effect on labor supply, and their effect on the value of public fund. A limitation is that these effects appear difficult to sign without making assumptions on the monotonicity of $\partial_{z^*z^*}e_i$. In the following proposition, we rearrange the formula[44] and derive unequivocal results on the optimal response of the tax schedule, requiring only an assumption on the sign (rather than the monotonicity) of $\partial_{z^*z^*}e_i$.

[42]The response of redistribution depends on whether the income effect induced by the price change is higher above or below θ. The expression given in Proposition 4 makes it difficult to sign the overall effect. We will provide a complete characterization in Proposition 5. For now, one can note that, as $(T'/(1-T')+\alpha)z\tilde{\zeta}(1-T')$ likely increases (this term represents the per capita cost of a change in the tax rate proportional to $1-T'$ at θ), the overall effect will be to lower taxes when q_i increases and the marginal propensity to spend on good i decreases.

[43]Formally, this is defined as the lagrange multiplier on the government budget constraint λ.

[44]For the simplification, we use the optimality of the initial tax schedule, as discussed in the proof in Online Appendix A2.

Proposition 5. *The response of the optimal tax rate at θ to a marginal increase in the price of good i can be reexpressed as (recall that $\partial_{z^*} E_i = \mathbb{E}(\partial_{z^*} e_i)$):*

$$\frac{q_i \partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\} = \frac{1-\alpha}{z\tilde{\zeta} f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*} e_i - \partial_{z^*} E_i\right) - \left(\frac{T'}{1-T'} + \alpha\right)\left(\partial_{z^*} e_i - \partial_{z^*} E_i\right)$$

When the marginal propensity to spend on i is decreasing ($\partial_{z^* z^*} e_i < 0$), we have:

$$\frac{q_i \partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\} < 0 \quad \forall \theta$$

When the marginal propensity to spend on i is increasing ($\partial_{z^* z^*} e_i > 0$), we have:

$$\frac{q_i \partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\} > 0 \quad \forall \theta$$

Moreover, if $\partial_{z^* z^*} e_i < 0$ ($\partial_{z^* z^*} e_i > 0$), then $z\tilde{\zeta} q_i \partial_{q_i}\left\{T'/(1-T')\right\} f$ is decreasing (increasing) at the bottom of the distribution and increasing (decreasing) for $\theta \geq \theta_i$, with θ_i such that $\partial_{z^*} e_i[\theta_i] = \partial_{z^*} E_i$.

The advantage of this new characterization is twofold. First, it allows us to quantify the effect of prices on the tax rate as a function of *observable* quantities. For example, we do not need to specify pareto weights to evaluate the impact of prices. Second, we can unequivocally sign the impact of prices on taxes.

When the marginal propensity to spend on good i decreases (i.e, i is a "necessity" good), the tax rate decreases everywhere in response to an increase in q_i. The tax burden decreases at the top of the distribution and increases at the bottom: the planner redistributes to higher-income households. This result might seem surprising, because the optimal tax schedule amplifies the redistributive effects of price changes instead of offsetting them, but Proposition 4 explains why. When i is a necessity good, the social value of a dollar transfer decreases less for higher-income than lower-income households, and the income effects increase more at the top.

Note that Proposition 5 defines a partial ordering over consumer goods. If the marginal propensity to spend on good i increases faster than the marginal propensity to spend on j,[45] an increase in the price of i leads to more redistribution than an increase in the price of j: $q_i \partial_{q_i} T' > q_j \partial_{q_j} T'$. The last item in Proposition 5 shows that the change in the value of redistribution is decreasing at the bottom of the distribution and increasing in the upper half: even if there is a large fall in the marginal tax rate in the middle of the distribution, the goal is to redistribute income to higher-income households.

While social preferences do not appear in our formulas, they still play a crucial role. Note that the derivative of the tax rate $\partial_{q_i} T'$ is of order $(1-T')^2$. The stronger the preference for redistribution, the higher the (initial) tax rate, and the lower the sensitivity of the tax rate to changes in prices. This result is not specific to prices. For any exogenous changes (be it a change in the skill distribution, in the elasticity of labor supply or in tastes for redistribution), the optimal tax rate will be less sensitive to these changes if it is initially set at a high level. To illustrate, assume that the marginal propensity to spend on i is constant above θ_0. Then an increase in q_i does not generate any income effect on labor supply above θ_0 and only impacts the tax rate through changes in the value of redistribution. Denoting $\bar{g}(\theta) = \mathbb{E}(g \mid \theta' > \theta)$ the average Pareto weight for households with ability larger than θ, we have:

$$\frac{q_i \partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\} = \frac{\bar{g}(\theta)}{1-\bar{g}(\theta)}\left(\frac{T'}{1-T'} + \alpha\right)\left(\partial_{z^*} e_i - \partial_{z^*} E_i\right)$$

[45] That is, $\partial_{z^*} e_i - \partial_{z^*} e_j$ is increasing.

If $\partial_{z^*} e_i$ is larger than average above θ_0, then an increase in q_i reduces the value of a transfer to high income households and tax rates are set higher at the top of the distribution. Note, however, that if $\bar{g}(\theta)$ is small then the increase in tax rates is small. Intuitively, if the planner does not value the welfare of higher ability households, price changes have no effects on top tax rates if they do not change the cost of taxation through labor supply.

The formulas of Proposition 5 can be adapted when social preferences are Rawlsian. In that case, we have:

$$\frac{q_i \partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\} = \left(\frac{T'}{1-T'} + \alpha\right)\left(\mathbb{E}\left(\partial_{z^*} e_i \,|\, z' > z(\theta)\right) - \partial_{z^*} e_i\right)$$

Even in the extreme case were the social planner only values the welfare of the poorest agent, an increase in the price of necessities leads to more redistribution towards higher income households. This is entirely due to the impact of the price change on labor supply. An increase in the price of necessities generates a positive income effect on labor supply and decreases the income tax. The inverse is true for luxuries.

4.2 Response to a Price Change in General Equilibrium

We now turn to the general equilibrium response, which we characterize in three steps. We first characterize how aggregate demand for goods responds to consumer price changes, taking into account the partial equilibrium response of taxes derived in the previous section. We then derive the response of the optimal tax schedule. Finally, we provide simple examples illustrating the economic forces at play.

4.2.1 Characterizing the Aggregate Demand Response

We now characterize the response of aggregate demand to a price change, including the partial equilibrium response of taxes, and provide a decomposition into income and substitution effects. We show that the income effect can be further decomposed into an aggregate income channel and a reallocation channel. These intermediate steps are necessary to understand the general equilibrium response of the tax schedule, since changes in aggregate demand induce further price changes, which themselves induce further changes in optimal tax rates and more changes in aggregate demand.

Proposition 6. *The elasticity $\mathcal{C}_{ji}$ of aggregate demand for good j, C_j, to an increase in price q_i, accounting for the (partial equilibrium) response of optimal taxes and keeping all other consumer prices q_j fixed, is:*

$$\mathcal{C}_{ji} = \underbrace{-\frac{1}{(1-\alpha)E_j}\mathbb{E}\left(z\zeta\left(\partial_{z^*}E_j + q_j\tau_j^{nh}\right)\left(\partial_{z^*}E_i + q_i\tau_i^{nh}\right)\right)}_{\text{Income Effect}} + \underbrace{\frac{q_i}{C_j}\frac{\partial C_j^h}{\partial q_i}}_{\text{Substitution Effect}}$$

where $\partial C_j^h/\partial q_i = \mathbb{E}(\partial c_j^h/\partial q_i)$ is the cross-price derivative of Hicksian demand and τ_i^{nh} captures the impact of non-homothecities on the sensitivity of aggregate demand to prices with:

$$q_i\tau_i^{nh} \equiv (1-\alpha)(1-T')\left(\frac{1}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(\partial_{z^*}e_i - \mathbb{E}\left(\partial_{z^*}e_i\right)\right) + \partial_{z^*}e_i - \mathbb{E}\left(\partial_{z^*}e_i\right)\right).$$

Under mild technical assumptions,[46] *τ_i^{nh} is everywhere positive when i is a luxury good, negative when i is a necessity and nil when the marginal propensity to spend on i is constant.*

[46] For example, $z(\underline{\theta}) = 0$ and $\tilde{\zeta}f\left(1 + \frac{z\tilde{\zeta}'}{\tilde{\zeta}} + \frac{zf'}{f}\right) < 1$, which is verified in the data with a large margin.

Proposition 6 decomposes the derivative of aggregate consumption with respect to prices into an income effect and a substitution effect. The substitution effect is standard: it is the aggregation of the individual substitution effects, given by the price derivatives of Hicksian demand. The income effect term captures both the direct effect of prices on income and their indirect effect through the income tax adjustments of Proposition 5. It operates through an aggregate income channel (captured by $\partial_{z^*} E_j$) and, when preferences are non homothetic, through a reallocation channel (captured by τ_j^{nh}).

To spell out the economic forces at play and provide a heuristic proof for Proposition 6, consider the impact of a small increase in the marginal tax rate at θ, $d\tau$, on spending on good j, E_j:

$$dE_j = -\frac{z\zeta}{1-\alpha}\left(\partial_{z^*} E_j + q_j \tau_j^{nh}\right) d\tau \tag{1}$$

When preferences are homothetic ($\tau_j^{nh} = 0$), an increase in tax rates decreases aggregate demand for good j by the average marginal propensity to spend j times the reduction in total income.[47] This is the aggregate income channel. When good j is a luxury, the fall in aggregate demand is amplified since the tax increase redistributes income to poorer households, who have a smaller marginal propensity to spend on j. This reallocation channel is captured by $\tau_i^{nh} > 0$. Inversely, when i is a necessity, the non homotheticities dampen the sensitivity of consumption to taxes, with $\tau_i^{nh} < 0$.

These aggregate income and reallocation channels also characterize the income effects of prices (and the induced tax change) on consumption. Indeed, as explained in Section 3, a price change dq_i generates the same income effects as a change in taxes $dT(z) = e_i(z^*, q)dq_i/q_i$.[48] In addition, marginal tax rates are optimally adjusted by $\partial_{q_i} T'$. Thus, agents' reaction to the price change is equivalent to their reaction to a shift in tax rates given by $\partial_{z^*} e_i + q_i \partial_{q_i} T'/(1-T')$. Direct algebra shows that $\partial_{z^*} e_i + q_i \partial_{q_i} T'/(1-T') = \partial_{z^*} E_i + q_i \tau_i^{nh}$, so plugging $d\tau = \partial_{z^*} E_i + q_i \tau_i^{nh}$ in equation 1 gives the formula of Proposition 6.

Using Proposition 6, we can assess the sign of the aggregate demand response to various price shocks. The response of aggregate demand for j to an increase in its own price q_j is always negative. Furthermore, the income effect generated by an increase in the price of a luxury is negative for all luxuries. Indeed, a price increase for a luxury generates a negative income effect and induces a tax increase, as shown in Proposition 5. As a result, there is a fall in both richer households' income and aggregate income, which unambiguously decreases demand for luxury goods: the aggregate income channel and the reallocation channel both decrease the consumption of luxuries.

Turning to the response of the consumption of necessities, the impact of a luxury price increase is ambiguous. On the one hand, aggregate real income decreases: as prices and taxes increase, real labor income decreases and the aggregate income channel lowers the consumption of all goods. On the other hand, higher tax rates reallocate income to poorer households, which generates a shift of aggregate consumption from luxuries to necessities. The impact on aggregate demand for necessities depends on which effect – lower aggregate income or reallocation – dominates. Formally, the sign of $\partial_{z^*} E_j + q_j \tau_j^{nh}$ – which governs the sensitivity of aggregate demand to price changes – is ambiguous for necessities.[49]

While the aggregate response of consumption to prices can be ambiguous, we find that the response of aggregate expenditure shares to price changes is unambiguous under mild conditions. Specifically, under the assumption that $\partial_{z^*} E_j - s_j$ (with s_j the expenditure share of j) is positive for luxuries and negative for necessities, the aggregate income channel and the reallocation channel work in the same direction for both the share of luxuries and necessities. Note that this assumption simply states that the aggregate share of

[47] Which is here given by $z\zeta/(1-\alpha)$.

[48] Or equivalently, an increase in marginal tax rates $dT' = (1-T')\partial_{z^*} e_i dq_i/q_i$.

[49] Similarly, the impact of an increase in the price of necessities is ambiguous, as it leads to lower tax rates.

luxuries increases when aggregate income increases.[50]

Let us first consider the case of homogeneous inflation for all consumer prices: $dq_i/q_i = 1$ for all i. In this case, we have $\sum \partial_{q_i} T' dq_i = 0$. As all households are equally affected by the price change, there is no need to change the tax schedule. To streamline notations, we denote by $\mathbb{E}_z(\cdot)$ the income weighted mean of a variable.[51] The change in the aggregate share of j, $s_j = q_j C_j / \sum q_i C_i$ is given by:

$$\frac{ds_j}{dq} = -\frac{\zeta}{1-\alpha}\left(\partial_{z^*} E_j - s_j + \mathbb{E}_z(\, q_j \tau_j^{nh})\right).$$

When preferences are homothetic, $ds_j/dq = 0$: since a homogeneous price shock is equivalent to a wage reduction, the demand for all goods simply decreases by $\zeta/(1-\alpha)$ percent, which is the decline in real income. When preferences are non homothetic, this effect also operates, but as households become poorer, they reallocate their income away from luxuries and towards necessities: ds_j/dq is negative for luxuries, positive for necessities.[52]

Next, we consider an increase in the relative price of i: $dq_i/q_i = 1 - s_i$ and $dq_j/q_j = -s_i$. This price change partials out the effect of homogeneous inflation, as the price of the average consumption basket is kept constant. The change in the aggregate share of j, is given by:

$$\frac{ds_j}{dq} = \underbrace{s_j \frac{q_i \partial C_j^h}{C_j \partial q_i}}_{\text{Substitution Effect}} \underbrace{- \frac{\zeta}{1-\alpha}\mathbb{E}_z\left(\left(\partial_{z^*} E_j - s_j + q_j \tau_j^{nh}\right)\left(\partial_{z^*} E_i - s_i + q_i \tau_i^{nh}\right)\right)}_{\text{Income Effect}}$$

When preferences are homothetic the income effect is zero and the share of j reacts to prices only through a substitution channel. With a constant elasticity of substitution σ, for example, the share of all $j \neq i$ increases proportionally to $s_i \sigma$.

When preferences are non homothetic, the income effect is non trivial. If the price of a necessity increases, the income effect is negative for all necessities and positive for all luxuries.[53] Indeed, an increase in the price of necessities relative to luxuries has a negative income effect on lower income households, as necessities constitute a larger portion of their consumption basket. Lower income households have a higher propensity to spend on necessities, so the aggregate share of necessities decreases through income effects. With a constant elasticity of substitution, the share of any luxury good increases proportionally more than the share of any necessity good in response to an increase in the price of a necessity: income is reallocated away from necessities. In addition, the change in taxes is $q_i \partial_{q_i} T' < 0$, which amplifies this reallocation of income away from necessities. The impact of the tax adjustment on the share of j is given by:

$$-\frac{\zeta}{1-\alpha}\mathbb{E}_z\left(\left(\partial_{z^*} E_j - s_j + q_j \tau_j^{nh}\right)\frac{q_i \partial_{q_i} T'}{1-T'}\right),$$

which is negative when both i and j are necessities and positive if j is instead a luxury. Intuitively, in response to an increase in the relative prices of necessities, tax rates decrease as shown in Proposition 5. Since income is redistributed to higher income households, this accentuates the overall reallocation

[50]That is, when the income of every agent is increased by one dollar.

[51]More precisely $\mathbb{E}_z(X) = \mathbb{E}(zX)/\mathbb{E}(z)$.

[52]Indeed, τ_j^{nh} is positive for luxuries, and negative for necessities. Note that instead of making an assumption on the sign of $\partial_{z^*} E_j - s_j$, we could directly assume that the individual share of a luxury product i, $e_i(z^*)/z^*$ increases with z^*. To see this, note that $\partial_{z^*} E_j - s_j + \mathbb{E}_z(\, q_j \tau_j^{nh}) = \mathbb{E}_z(\partial_{z^*} e_i - e_i/z + q_i \partial_{q_i} T'/(1-T'))$ which is positive for luxuries with increasing individual shares and negative for necessities with decreasing individual shares.

[53]The opposite is true if the price of a luxury increases.

towards luxuries.

To summarize, in this subsection we have found that, for a homogeneous price increase, there is no tax adjustment and the aggregate share of necessities increases through standard income effects. For relative price increases, the change in taxes amplifies reallocation towards necessities when the price of luxuries increases and towards luxuries when the price of necessities increases. Consequently, if price elasticities with respect to market size are positive, when the inflation rate of necessities is larger the optimal tax adjustment leads to a larger divergence of the relative prices of necessities, and therefore to stronger redistribution towards higher income households in general equilibrium (compared with the partial equilibrium response from section 4.1).

4.2.2 Optimal Tax Response to Exogenous Price Changes in General Equilibrium

Having characterized the effect of prices on the demand side of the economy, both in terms of labor supply and consumption, we can now derive the general equilibrium response of the optimal income tax to exogenous producer price changes. For example, these price changes can be interpreted as technology shocks that shift the cost of production. To simplify the exposition, we consider here the case where the price of i only depends on the demand for i. Without spillovers, optimal commodity taxes are left unchanged up to a scaling constant: if $q_i = (1-\tau_i)p_i$, then $(1-\tau_i)/(1-\tau_j) = (1-\alpha_i)/(1-\alpha_j)$ at all prices. The general case with spillovers and varying price elasticities is reported in Online Appendix A2.

Proposition 7. *Consider an economy without spillovers ($p_i = \phi_i(C_i)$). The response of the optimal tax rate to an exogenous increase in the producer price of i, denoted dp_i^*, is:*

$$\frac{dT'}{dp_i^*} = \underbrace{\frac{1-\alpha_i}{1-\alpha}\frac{\partial T'}{\partial q_i}}_{\text{Mechanical Price Effect}} + \underbrace{\sum_j \frac{\partial T'}{\partial q_j}\frac{dq_j^e}{dp_i^*}}_{\text{Endogenous Price Effect}} - \underbrace{\frac{1-T'}{(1-\alpha)}\frac{d\alpha}{dp_i^*}}_{\text{Endogenous Market Size Effect}}$$

where the consumer price response, $1/q_j dq_j/dp_i^*$ is given by:

$$\left[\frac{1}{q_j}\frac{dq_j}{dp_i^*}\right] = (Id + \Delta_\alpha \mathcal{C})^{-1}\left[\frac{\alpha_j}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)\frac{C_i}{C_j} + \mathbb{1}_{i=j}\frac{1-\alpha_i}{1-\alpha}\frac{1}{q_i}\right],$$

where $[\cdot]$ denotes a vector with entry indexed by j.[54] *The endogenous price change dq_j^e/dp_i^* is simply defined as $dq_j^e/dp_i^* = dq_j/dp_i^* - \mathbb{1}_{i=j}(1-\alpha_i)/(1-\alpha)$. Finally, the endogenous market size effect, $d\alpha/dp_i^*$, is given by*

$$\frac{1}{1-\alpha}\frac{d\alpha}{dp_i^*} = \sum s_j\left(\frac{\alpha}{\alpha_i} - 1\right)\left(\frac{1}{q_j}\frac{dq_j}{dp_i^*} - \mathbb{1}_{i=j}\frac{\alpha_i}{1-\alpha}\frac{1}{q_i}\right)$$

with $s_i = q_i C_i / \sum q_j C_j$ the aggregate share of j.

Proposition 7 shows that the tax response to price changes depends on three channels in general equilibrium. First, there is a mechanical price effect. When p_i marginally increases, given that the subsidy on i is $1-\tau_i = (1-\alpha_i)/(1-\alpha)$, the consumer price mechanically increases by $1-\tau_i$. This mechanical price effect lowers (increases) tax rates everywhere when i is a necessity (luxury) good.

[54] For the derivative of the tax rate to be well defined in general equilibrium, we need the eigenvalues of $-\Delta_\alpha \mathcal{C}$ to be less than one, which is verified in our quantitative analysis. Note that if that were not the case, the equilibrium would not exist even without the price shocks. Indeed, in that case, arbitrarily subsidizing the consumer price of a subset of goods would generate a producer price decrease large enough so that the subsidy would pay for itself and infinite quantities of goods could be produced.

Second, the tax system responds to endogenous price changes arising from labor supply and aggregate consumption. The endogenous price changes work through two channels. First, since q_i is reduced by $(1-\tau_i)/q_i$ percent, the response of consumption is given by the cross-price elasticities matrix $\mathcal{C}$. As explained in Proposition 6, an increase in the relative price of a luxury (necessity) leads to a reallocation of income towards necessities (luxuries), while homogeneous inflation leads to a reduction in the shares of luxuries. This response of aggregate consumption generates a further hike in consumer prices given by $\Delta_\alpha \mathcal{C}$, which generates a further reallocation of consumption, and so on. This feedback loop is captured by the matrix $(Id + \Delta_\alpha \mathcal{C})^{-1}$. Second, the increase in the producer price p_i induces a mechanical drop in government revenue proportional to C_i.[55] The intercept of the tax schedule is increased and consumption of each good j is reduced by $\mathbb{E}(\partial_{z^*} c_j)/C_j \times C_i$ percent. All consumer prices are then adjusted through the feedback loop.

Finally, optimal taxes respond to an endogenous market size effect. When the market size elasticities α_i differ across markets, a consumer price increase in more elastic markets lowers the aggregate elasticity α.[56] A higher α calls for a higher wage subsidy and leads to a fall in the tax rate.

Simple Examples in a Two-Good Economy. To illustrate the channels in Proposition 7, we discuss some examples. Consider a two-good economy with a luxury good h and a necessity good l. The elasticity of substitution between the two goods is σ. We analyze the case where the price elasticities are constant across markets ($\alpha_h = \alpha_l = \alpha$). As seen in Proposition 1, commodity taxes are optimally set to zero in this case ($q_h = p_h$ and $q_l = p_l$), we only need to determine the change in the income tax. First note that in a two-good economy, we have $q_h \partial_{q_h} T' = -q_l \partial_{q_l} T'$. Higher luxury prices and lower necessity prices have the same impact on taxes. The optimal adjustment of tax rates is therefore entirely determined by the relative price change of the luxury good. For any producer price change dp^*, we have:

$$\frac{dT'}{dp^*} = \frac{q_h \partial T'}{\partial q_h} \left(\frac{1}{q_h} \frac{dq_h}{dp^*} - \frac{1}{q_l} \frac{dq_l}{dp^*} \right)$$

When the relative price of luxuries increases, tax rates rise everywhere along the income distribution. The larger the increase in relative price, the larger the rise in tax rates.

Let us first consider a shock to the relative producer price of h. Formally, $dp_h^*/p_h = (1-s_h)$, $dp_l^*/p_l = -s_h$.[57] In equilibrium, the relative consumer price of h is:

$$\frac{1}{q_h} \frac{dq_h}{dp^*} - \frac{1}{q_l} \frac{dq_l}{dp^*} = \overbrace{\frac{1}{1-\alpha\sigma}}^{\substack{\text{Amplification} \\ \text{through Substitution}}} \times \overbrace{\frac{1}{1-\frac{\alpha\zeta}{1-\alpha}\Omega}}^{\substack{\text{Amplification} \\ \text{through Income Effects}}}$$

$$\text{with} \quad \Omega = \frac{1}{s_h s_l (1-\alpha\sigma)} \left(\mathbb{E}_z \left((\partial_{z^*} E_h - s_h + q_h \tau_h^{nh})^2 \right) + \frac{\frac{\alpha\zeta}{1-\alpha}}{1 - \frac{\alpha\zeta}{1-\alpha}} \mathbb{E}_z (\partial_{z^*} E_h - s_h + q_h \tau_h^{nh})^2 \right)$$

The increase in the relative price of h is amplified through two channels in general equilibrium. First, as the relative price of h increases, agents substitute the necessity good for the luxury good. The market for l expands relatively to h so the relative price of h further increases which creates more substitution.

[55] A simple way to see this is that, at fixed consumer prices, an increase in p_i mechanically reduces the proceeds of the commodity taxes.

[56] Note that an increase in the producer price of i mechanically increases α if α_i is larger than average.

[57] Recall that $s_h = q_h C_h / (q_h C_h + q_l C_l)$, and $q = p$ when $\alpha_h = \alpha_l$.

Each round, the relative price of h increases by $\alpha\sigma$ and the overall amplification is $1/(1-\alpha\sigma)$. This is the only channel of amplification when preferences are homothetic, since the shares of h and l remain constant as income shifts.

When preferences are non homothetic, as discussed in Proposition 6, the share of h further decreases through income effects. This amplification is denoted by Ω and operates through two channels. The first term in Ω, corresponds, as we have seen, to the reduction in the share of h in response to an increase in the relative price of h. Indeed, when the luxury good becomes more expensive, higher income households are more affected, since h represents a larger share of their expenditure: their real income loss is larger. As they also consume more of the good at the margin, the direct impact of the price increase is to reduce the share of h. In addition, tax rates increase everywhere, as it is more valuable to redistribute to lower income households. This optimal adjustment of the schedule[58] amplifies the reallocation of income towards the necessity good. As the relative share of h[59] decreases, the relative price of h increases when α is positive, and agents further reallocate their income towards the necessity.

The second term captures the effect of homogeneous inflation on the reduction in the share of h. As the relative price of h increases, labor supply decreases and aggregate income therefore decreases.[60] As aggregate income falls, the market size for both the necessity and the luxury goods decreases, which increases their prices. This lowers households' real income and they therefore reallocate their income towards the necessity good. As we have seen, the reduction in the share of h when inflation is homogeneous is given by $\mathbb{E}_z(\partial_{z^*}E_h - s_h + q_h\tau_h^{nh})$. Again, the reduction in the share of h in response to homogeneous inflation increases the relative price of h when α is positive. Note that the optimal adjustment in tax rates accentuates both channels. By redistributing income from higher to lower incomes, the planner directly amplifies reallocation of income towards necessities when the relative price of luxuries increases. By increasing tax rates, the planner reduces aggregate labor supply which generates more inflation, and, as households become poorer on average, more reallocation towards necessities.

Redistribution towards lower income households is therefore amplified, through general equilibrium effects, when the relative price of h increases. We have:

$$\frac{dT'}{dp^*} = \frac{1}{(1-\alpha\sigma)(1-\frac{\alpha\zeta}{1-\alpha}\Omega)}\ \frac{q_h\partial T'}{\partial q_h}.$$

As can be expected, the amplification is stronger when the price elasticity α, the elasticity of substitution σ, and the labor supply elasticity ζ are larger. Moreover, the amplification is stronger when non-homothecities are more pronounced, as they accentuate reallocation towards necessities through income effects in Ω.[61]

Next, we consider an homogeneous increase in the price of the luxury and necessity: $dp_h^*/p_h = dp_l^*/p_l$. In partial equilibrium, this price change has no effect on tax rates. As explained in Proposition 7, this price increase reduces government revenue, which induces a second price change $\alpha/(1-\alpha)\partial_{z^*}E_j/s_j$.

[58] Given by $\partial_{q_h}T'$.

[59] The coefficient $s_h s_l$ in the formula captures the decrease in the share of h relative to l as $ds_h/s_h - ds_l/s_l = ds_h/(s_h s_l)$.

[60] Keeping taxes fixed, an increase in the relative price of h reduces income by $\zeta\mathbb{E}_z(\partial_{z^*}e_h - s_h)$. This is positive if $\partial_{z^*}E_h \geq s_h$. In addition, the increase in tax rates further reduces labor supply. The total reduction in aggregate income is given by $\zeta\mathbb{E}_z(\partial_{z^*}E_h - s_h + q_h\tau_h^{nh})$.

[61] More precisely comparing two economies A and B where $\partial_{z^*}e_h^A - \partial_{z^*}e_h^B$ is increasing and A and B are otherwise identical, then we have $\Omega^A \geq \Omega^B$ and the amplification through income effects is stronger.

Ignoring this second change, the increase in the relative price of h would be:

$$\frac{1}{(1-\alpha\sigma)\left(1-\frac{\alpha\zeta}{1-\alpha}\Omega\right)} \times \frac{\frac{\alpha\zeta}{1-\alpha}}{1-\frac{\alpha\zeta}{1-\alpha}} \frac{\mathbb{E}_z(\partial_{z^*} E_h - s_h + q_h \tau_h^{nh})}{s_h s_l}.$$

An increase in inflation reduces real income and therefore decreases the share of h. As a result, the relative price of h increases (through market size effects) and this increase is amplified through the substitution and income effects described above. Now incorporating the price change resulting from the drop in government revenue, the total price change is given by:

$$\frac{1}{q_h}\frac{dq_h}{dp^*} - \frac{1}{q_l}\frac{dq_l}{dp^*} = \frac{1}{(1-\alpha\sigma)\left(1-\frac{\alpha\zeta}{1-\alpha}\Omega\right)} \times \left(\frac{\alpha}{1-\alpha}\frac{\partial_{z^*} E_h - s_h}{s_h s_l} + \frac{\frac{\alpha\zeta}{(1-\alpha)^2}}{1-\frac{\alpha\zeta}{(1-\alpha)}} \frac{\mathbb{E}_z(\partial_{z^*} E_h - s_h + q_h \tau_h^{nh})}{s_h s_l} \right).$$

The loss in government revenue is paid for through a lump sum tax. As a result, households' income decreases and consumption shifts towards necessities. The relative price of the luxury good increases, which corresponds to the first term in the formula above. In addition, since consumption of both good decreases, inflation is stronger: both prices increases by $1/(1-\alpha)$, which explains the second term in the formula.

Thus, while homogeneous price increases, resulting from a technology shock or an increase in markups, have no impact on tax rates in partial equilibrium, we find that they lead to more redistribution in general equilibrium. Households reduce their labor supply and therefore reallocate their income towards the necessity product, which increases the relative price of the luxury product. It then becomes optimal to redistribute to lower income households, even though aggregate income decreases overall.

4.3 Price Effects in the Diamond-Mirrlees Model

In this section, we consider the effects of prices on optimal redistribution in a standard model of the supply side where goods are produced competitively and all profits are taxed, as in Diamond and Mirrlees (1971b). We find that the mechanisms at play are very similar to those discussed above, which highlights that the lessons from our analysis are not specific to our benchmark model of the supply side.

Recall that prices are determined by the vector of marginal cost of production $mc(Q_1,...Q_n)$, which potentially depends on the full vector of quantities produced. We denote by A the matrix of marginal costs elasticity, with $A_{ij} = Q_j \partial_{Q_j} mc_i / p_i$. As before, A summarizes how prices react to changes in aggregate demand. The function $\chi(Q_1,...,Q_n)$ denotes the total cost of production of $\{Q_1,...,Q_n\}$. To analyze an exogenous price change for good i, we consider a technological shock that affects the marginal cost of i and the average cost χ. To stay consistent with our previous notations, the derivatives with respect to this technology shock are denoted by $d\cdot/dp_i^*$. In particular, the exogenous change in average cost is $d\chi/dp_i^*$.

Corollary 1. *In an economy with neo-classical production functions, with a full tax on profit, (1) the partial equilibrium response of the optimal tax rate to an increase in the price of good i are given by the formulas of Proposition 5 by setting $\alpha = 0$. (2) The general equilibrium response of the optimal tax rate to an increase in the price of good i is:*

$$\frac{dT'}{dp_i^*} = \frac{\partial T'}{\partial q_i} + \sum_j \frac{\partial T'}{\partial q_j}\frac{dq_j^e}{dp_i^*}$$

where $\partial T'/\partial q_i$ is as in Proposition 5 with $\alpha = 0$, and the prices dq_j^e/dp_i^* solve:

$$\left[\frac{1}{q_j}\frac{dq_j^e}{dp_i^*} + \mathbb{1}_{i=j}\frac{1}{q_i}\right] = (Id - A\mathcal{C})^{-1}\left[\frac{1}{q_i}\frac{dq_j^0}{dp_i^*} + \mathbb{1}_{i=j}\frac{1}{q_i}\right],$$

and dq_j^0/dp_i^* is the impact of profit redistribution on consumption and prices:

$$\frac{1}{q_i}\frac{dq_i^0}{dp_i^*} = -\sum A_{ij}\frac{\mathbb{E}(\partial_{z^*}e_j)}{C_j}d\chi/dp_i^*$$

The matrix $\mathcal{C}$ is defined as in Proposition 6 with $\alpha = 0$.

The first part of Corollary 1 shows that the partial equilibrium response of the optimal tax rate in a Diamond-Mirrlees economy is identical to Proposition 5, setting $\alpha = 0$. This result is important in itself as it shows that, beyond the specificities of our supply-side specification, the design of the income tax *cannot* be thought through without accounting for price effects. Consumer prices affect redistribution policies even in the standard Diamond-Mirrlees case, through the channels discussed in Proposition 4. An increase in the price of luxuries leads to more redistribution towards lower income households and the opposite is true when necessities become more expensive.

The second part shows that, in the Diamond-Mirrlees economy, the optimal response of taxes to prices works through the same channels as in our benchmark specification in general equilibrium. There are two differences with Proposition 7. First, there is no change in the "average market size elasiticity" (our α), so the optimal tax schedule is no longer affected by this channel. Second, profits are taxed and rebated to households in a lump sum fashion, hence it is necessary to know the change in profit due to the shift in average cost $-d\chi/dp_i^*$.

Let us revisit our two-good example. Consider an increase in the relative price of h. For consistency, we assume that $A = \alpha\, Id$[62] and that the price change induces no change in total cost ($d\chi/dp^* = 0$). We then have:

$$\frac{1}{q_h}\frac{dq_h}{dp^*} - \frac{1}{q_l}\frac{dq_l}{dp^*} = \frac{1}{1-\alpha\sigma} \times \frac{1}{1-\alpha\zeta\,\Omega}$$

$$\text{with} \quad \Omega = \frac{1}{s_h s_l(1-\alpha\sigma)}\left(\mathbb{E}_z\left((\partial_{z^*}E_h - s_h + q_h\tau_h^{nh})^2\right) + \frac{\alpha\zeta}{1-\alpha\zeta}\mathbb{E}_z(\partial_{z^*}E_h - s_h + q_h\tau_h^{nh})^2\right).$$

The amplification works through the same channels as in the benchmark case. However, the income effect term depends on $\alpha\zeta$ rather than $\alpha\zeta/1-\alpha$: the amplification through income effects is not as large in the Diamond-Mirrlees economy when $\alpha > 0$. Indeed, when demand decreases, prices increase but profits are also larger. Since profits are rebated to households, this dampens the fall in demand.

4.4 Implications and Extensions

In this section, we briefly discuss some important implications and extensions of our comparative statics results. The formal results and a longer discussion can be found in Online Appendix A3, due to space constraints.

Market Size Effects and Redistribution In Proposition A1, we reexamine the impact of market size effects on optimal tax policies. As prices become more sensitive to market size (e.g., α increases), the planner

[62] For example, $\chi(Q_h, Q_l) = \chi_h Q_h^{1+\alpha} + \chi_l Q_l^{1+\alpha}$.

implements a larger wage subsidy. As explained in Proposition 3, the subsidy incentivizes labor supply, aggregate income increases, and all prices decrease. A naive interpretation of Proposition 3 suggests that this corrective wage subsidy can be implemented independently from redistributive policies: since the wage subsidy is $1/(1-\alpha)$, the derivative of the tax rate with respect to α would then simply be given by $dT'/d\alpha = -(1-T')/(1-\alpha)$. Our comparative statics results unveil a subtler interaction. A higher wage subsidy is equivalent to a homogeneous reduction in prices. As seen in section 4.2, this implies that the share of luxuries increases while the share of necessities decreases. The relative price of luxuries therefore decreases. This triggers a readjustment of optimal redistribution policies, with more redistribution toward higher income households. To see this explicitly, consider the two-good example of section 4.2, with $\alpha_h = \alpha_l = \alpha$ (with h the luxury, l the necessity). We find that commodity taxes are optimally set to zero and that the response of the income tax to an increase in α is:

$$\frac{dT'}{d\alpha} = \overbrace{-\frac{1-T'}{1-\alpha}}^{\text{Wage Subsidy}} + \overbrace{\frac{q_h \partial T'}{\partial q_h}\left(\frac{1}{q_h}\frac{dq_h}{d\alpha} - \frac{1}{q_l}\frac{dq_l}{d\alpha}\right)}^{\text{Redistribution Induced by Price Changes}}$$

$$\text{with} \quad \frac{1}{q_h}\frac{dq_h}{d\alpha} - \frac{1}{q_l}\frac{dq_l}{d\alpha} = -\frac{1}{1-\alpha}\frac{\frac{\alpha\zeta}{1-\alpha}}{1-\frac{\alpha\zeta}{1-\alpha}}\frac{\mathbb{E}_z(\partial_{z^*}E_h - s_h + q_h\tau_h^{nh})}{s_h s_l(1-\alpha\sigma)\left(1-\frac{\alpha\zeta}{1-\alpha}\Omega\right)}$$

Thus, when prices become more elastic to demand, the tax schedule becomes more regressive not only because of the corrective wage subsidy but also because of the change in redistribution policies it induces. When $\partial_{z^*}E_h - s_h \geq 0$,[63] the wage subsidy reduces the relative price of the luxury good. Note that this reduction is exactly the same as the one generated by an exogenous decrease in all producer prices of $1/(1-\alpha)$ percent (which corresponds to the increase in the wage subsidy). The price decrease is then amplified by general equilibrium effects[64] and further decreases tax rates according to $\partial_{q_h}T'$, as it becomes more valuable to redistribute towards higher income households. The interaction between corrective and redistributive taxation is therefore non trivial and works through prices. When prices instead become less sensitive to demand, the wage subsidy is lowered, the relative price of the necessity good decreases, which makes redistribution towards lower income households socially more valuable. A general lesson is that corrective and redistributive taxation cannot be conducted independently when prices are elastic.

Inequality and Prices. Up to now, we have considered exogenous producer price changes (e.g., stemming from technological shocks affecting the marginal cost of production). Empirically, however, growing income inequality is a significant source of endogenous price dispersion, through the demand shifts it generates. In Proposition A2, we analyse how income inequality impacts redistribution policies, both directly and indirectly through prices. At fixed prices, an increase in income inequality, modelled as an exogenous shift in the distribution of abilities θ, usually leads to a more redistributive tax schedule. For example, the top tax rate increases as the upper tail of the income distribution becomes fatter. If we instead observe a polarization of the income distribution, with relatively fewer middle-income households, marginal tax rates increase in U-shaped fashion to take advantage of the thinning mass of taxpayers in the middle of the distribution. These policies, however, do not fully redistribute income: the average income at the top still increases, so the markets for luxury goods expand, which lowers their equilibrium prices. As the price of luxuries decreases, it becomes optimal to redistribute towards higher income households. Thus, we find that higher income inequality leads to less redistributive policies with endogenous prices.

[63] Or, alternatively, when the share of luxuries $e_h(z^*)/z^*$ increases along the income distribution.

[64] As before, this amplification is larger when the elasticity of substitution σ, the initial price elasticity α and non-homothecities are stronger.

To see this explicitly, consider our two-good example. Assume that the income distribution is Pareto above θ^*, with a shape parameter $1/\bar{a}$,[65] that the marginal propensity to spend on the luxury h is constant above θ^*, equal to $\partial_{z^*}\bar{e}_h$, and that the planner does not values the welfare of top income households. We denote by $\bar{\tau}$ the top tax rate and consider an increase in $\bar{a}$.[66] The change in top tax rate at fixed prices is given by $\bar{a}/(1-\bar{\tau})\partial\bar{\tau}/\partial\bar{a} = (\alpha + (1-\alpha)\bar{\tau})$. As is well known, top tax rates increase when the upper tail is fatter. Tax rates are left unchanged below θ^*. With endogenous prices, we find that the change in tax rates becomes:

$$\frac{\bar{a}}{1-T'}\frac{dT'}{d\bar{a}} = (\alpha + (1-\alpha)\bar{\tau})\,\mathbb{1}_{\theta>\theta^*} + \frac{1}{1-T'}\frac{q_h \partial T'}{\partial q_h}\left(\frac{\bar{a}}{q_h}\frac{dq_h}{d\bar{a}} - \frac{\bar{a}}{q_l}\frac{dq_l}{d\bar{a}}\right)$$

To the extent that the price of the luxury good decreases, tax rates will be everywhere lower than with fixed prices. Indeed, the price of the luxury good decreases through two channels. First, the additional income of households above θ^* is not fully redistributed: as these households spend more on the luxury good, the market for h expands and its price decreases. Second, as aggregate income increases, aggregate consumption shifts towards the luxury good. We have already discussed the second effect so we focus here on the first, which is quantitatively more relevant.[67] Denoting $\bar{s}_z$ the income share of top households, we have:

$$\frac{\bar{a}}{q_h}\frac{dq_h}{d\bar{a}} - \frac{\bar{a}}{q_l}\frac{dq_l}{d\bar{a}} < \overbrace{-\frac{\bar{a}}{1-\bar{\tau}}\frac{\partial\bar{\tau}}{\partial\bar{a}}\,\frac{\bar{a}\,\bar{s}_z}{1-\bar{a}}\,\frac{\frac{\alpha\zeta}{1-\alpha}}{1-\frac{\alpha\zeta}{1-\alpha}}\frac{\partial_{z^*}\bar{e}_h - \partial_{z^*}E_h}{s_l s_h(1-\alpha\sigma)\left(1-\frac{\alpha\zeta}{1-\alpha}\Omega\right)}}^{\text{Price change induced by high-income households}}$$

A larger $\bar{a}$ directly increases top households' income by $\bar{a}/(1-\bar{a})$ percent. Despite the higher top tax rate, the income of top households increases.[68] The formula shows that the decrease in the relative price of h will be larger when top households consumes more of h at the margin than the average household (when $\partial_{z^*}\bar{e}_h - \partial_{z^*}E_h$ is large), and that the decrease in the price of h can be large compared to the increase in top tax rate, for example when $\bar{a}$ is close to one (the increase in top income can be order of magnitude larger than the increase in top tax rate) or when $\alpha\sigma$ is close to one (the amplification through general equilibrium effects is large). Therefore, the policy response to income inequality cannot a priori neglect price effects.[69]

Analysis with a Non-Linear Social Welfare Function. In Proposition A3 and A4, we allow the social welfare function to be non-linear. This implies that the value of a dollar transfer now depends on agents' disposable income and not solely on their type (this captures, in particular, a decreasing marginal social value of disposable income). With a concave social welfare function, an increase in tax burden or in prices

[65] That is, $1-F(z) = C^* z^{-\frac{1}{\bar{a}}}$ for $z > z(\theta^*)$.

[66] With an adjustment of the scale parameter C^* such that the mass remains the same above θ^*.

[67] The change in prices given by change in aggregate income and inflation is similar to the one analysed in section 4.2 and is negative. It is given by:

$$-\frac{\bar{a}}{1-\bar{\tau}}\frac{\partial\bar{\tau}}{\partial\bar{a}}\frac{\bar{a}\bar{s}_z}{1-\bar{a}}\frac{\left(\frac{\alpha/\bar{a}}{1-\alpha}+\frac{\alpha\zeta}{1-\alpha}\right)\left(\partial_{z^*}E_h - s_h + \frac{\frac{\alpha\zeta}{1-\alpha}}{1-\frac{\alpha\zeta}{1-\alpha}}\mathbb{E}_z(\partial_{z^*}E_h - s_h + q_h\tau_h^{nh})\right)}{s_l s_h(1-\alpha\sigma)\left(1-\frac{\alpha\zeta}{1-\alpha}\Omega\right)}$$

[68] Proportionally to $\zeta\bar{a}/(1-\bar{\tau})d\bar{\tau}/d\bar{a}\ \bar{a}/(1-\bar{a})$.

[69] This is especially true since we find that the sensitivity of the income tax to a change in the distribution of ability and to prices can be of similar magnitude. For example, in the Rawlsian case, if the distribution of income is Pareto with shape parameter $1/a$, we have $a/(1-T')\ \partial T'/\partial a = (\alpha + (1-\alpha)T')$, while the response of the tax rate to an increase in q_h is $q_h/(1-T')\ \partial T'/\partial q_h = (\alpha + (1-\alpha)T')\ \ (\mathbb{E}(\partial_{z^*}e_h \,|\, z' > z) - \partial_{z^*}e_h)$.

has a direct "income effect" on Pareto weights, in addition to the valuation effects derived in Proposition 4: increasing the tax burden of an agent directly increases the social value of a transfer to this agent. This means in particular that when the relative price of a necessity increases, it is no longer optimal to implement the tax change of Proposition 7. Indeed, decreasing tax rates everywhere increases the tax burden of lower income households. There is therefore an incentive for the social planner to compensate lower-income households for their higher burden, and the regressivity of the tax change of Proposition 7 is muted.

However, we find that households are never fully compensated for price changes at the optimum, even with a concave social welfare function. Intuitively, fully compensating agents for a price change leaves their disposable income unchanged. This neutralizes the "income effect" of prices on Pareto weights, but leaves unchanged the valuation effects of prices and their effect on labor supply, derived in Proposition 4. Consider for example an increase in the relative price of necessities. Fully compensating agents cannot be optimal, as there is an incentive to redistribute to higher income households (as income effects on labor supply increase and the value of a social transfer to higher income households increases). A concave social welfare function therefore mutes the regressivity of the income tax schedule of Proposition 7, but does not overturn it.

Finally, we also find that, with a non-linear social welfare function, a non trivial interaction between corrective and redistributive taxation arises even when preferences are homothetic. Indeed, when markets become more sensitive to price changes, implementing the wage subsidy of Proposition A1 increases the tax burden of lower income households, which has to be partially compensated through redistributive policies.

5 Quantitative Analysis

In this section, we examine the quantitative importance of our theoretical results about increasing returns, non-homotheticities and price shocks for the optimal tax schedule. We first present the setting and main specifications (Section 5.1). We then implement our comparative static approach, studying a general first-order approximation (Section 5.2). Finally, we make additional parametric assumptions to study the optimal tax schedule and the feedback loops between redistribution and endogenous prices (Section 5.3).

5.1 Setting

Starting from the model from Section 2, we consider a standard additively separable specification (e.g., Saez (2001)):

$$U(z^*, z, \boldsymbol{p}, \theta) = v\left(z^*, \boldsymbol{p}\right) - \psi\left(\frac{z}{\theta}\right),$$

where $\psi\left(\frac{z}{\theta}\right) \equiv \frac{1}{1+\frac{1}{\varepsilon_z}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon_z}}$ is the isoelastic utility cost of earning z given ability θ, and $v\left(z^*, \mathbf{p}\right)$ is the indirect utility function given prices and disposable income. Following the nonparametric evidence of Kleven and Schultz (2014), we set $\varepsilon_z = 0.214$; for robustness, we consider $\varepsilon_z = 0.33$ as in the meta-analysis of Chetty (2012). We calibrate the skill distribution $f(\theta)$ nonparametrically to match the income distribution at the observed tax schedule, using data from Hendren (2020).

Returns to scale. As in Section 2, returns to scale are governed by the parameter α. There is an emerging empirical consensus that increasing demand leads to higher productivity and lower prices (in the long run), and recent papers provide causal estimates for α. Using a shift-share instrument with Nielsen data in the U.S., Jaravel (2019) finds that when demand increases by one percent, consumer prices for continued products fall by 0.42 percent. When accounting for changes in product variety, the consumer price index falls by 0.62 percentage points. Leveraging data on durable good industries in the Chinese manufacturing sector and an IV design based on potential market size, Beerli et al. (2020) estimate that increasing market size by one percent leads to a TFP increase of 0.46 percent. Using trade shocks as instruments, Bartelme et al. (2019) estimate sector-level economies of scale and find statistically significant scale elasticities in every 2-digit manufacturing sector, with an average of 0.13.[70] Given this range of estimates, we set $\alpha = 0.30$ in our baseline specification and study sensitivity.[71]

For the comparative static analysis in Section 5.1, α can be viewed as the "local" returns to scale. When studying the optimal tax schedule in Section 5.3, we specify the global relationship between the price p_i of the good produced in sector i and equilibrium quantities in that sector, setting $p_i = \gamma_i Q_i^{-\alpha} \quad \forall i \in \mathcal{I}$.[72]

Nonhomotheticities. We set the indirect utility function $v(z^*, \mathbf{p})$ to be either homothetic or non-homothetic in the analysis below to isolate the quantitative impact of non-homotheticities on the optimal schedule.

A non-homothetic utility function introduces curvature in the agent's indirect utility from consumption, which affects the social marginal utility of disposable income. Therefore, we normalize the curvature of utility at fixed prices, so that we mechanically reach the same optimum with homothetic and non-homothetic utility under constant returns to scale.[73] This approach ensures that the comparison between the homothetic and non-homothetic specifications captures the channel of interest, namely differences stemming from endogenous prices and their impact on the marginal utility of disposable income across the skill distribution, rather than assumptions about curvature *per se*. Our results are thus comparable to the benchmark models of Mirrlees (1971) and Saez (2001), with no additional curvature and no additional income effects absent returns to scale, despite the introduction of non-homothetic utility.

For the comparative static analysis, we directly use the formulas derived in Section 4. In partial equilibirum, we only need to know the local marginal propensities to consume across goods for agents across the income distribution, $\partial_{z^*} e_i$. We measure marginal propensities to consume non-parametrically from expenditure shares across 248 product categories. This dataset covers the full consumption basket of American households, linking the CPI price dataset to the consumption patterns of the Consumer Expenditure Survey (CEX) to the Consumer Price Index (CPI), following Jaravel (2019).

As shown in Proposition 7, the demand elasticity of substitution σ between products plays an important role for the feedback loops in general equilibrium. We take estimates from the literature as bounds for the elasticity of substitution between our product categories. Based on estimates of the elasticity of substitution between goods and services, two broad categories of consumption which are likely to be less substitutable than our 248 categories, we set $\sigma = 0.6$ as a lower bound (see Comin et al. (2021) and

[70] Other papers provide empirical evidence for returns to scale in different settings. Acemoglu and Linn (2004) provide empirical evidence that market size influences entry of new drugs and U.S. pharmaceutical innovation. Weiss and Boppart (2013) show that TFP growth is higher in more income-elastic sectors, using national accounts data covering the entire U.S. economy. Analyzing Nielsen scanner data across local markets, Handbury (2019) finds that the products and prices offered in markets are correlated with local income-specific tastes. Focusing on housing and local amenities, Diamond (2016) and Couture et al. (2020) find that amenities adjust endogenously to an increase in local demand and lower the price index.

[71] The closest empirical evidence to discipline our model is provided by Jaravel (2019), who looks directly at consumer prices rather than TFP. For continued products, the estimate for α varies between 0.23 and 0.458, depending on the set of controls, and $\alpha = 0.30$ cannot be rejected in any of the specifications. The estimates are larger when product entry is accounted for, hovering between 0.38 and 0.67 depending on the specification.

[72] We use the observed equilibrium to calibrate the set of parameters γ_i, as discussed in Online Appendix B.1.2.

[73] We work with a "deflated indirect utility function" $\widetilde{v}(z^*, \boldsymbol{p})$, defined such that

$$\widetilde{v}(z^*, \boldsymbol{p}) = v^{-1}(v(z^*, \mathbf{p}), \mathbf{p}_{CRS}),$$

where $\mathbf{p}_{CRS}$ are the prices prevailing under constant returns (which are normalized to one in the simulations, without loss of generality). We have $\widetilde{v}(z^*, \mathbf{p}_{CRS}) = z^*$,which is identical to the homothetic specification. Online Appendix B.2.2 discusses the properties of the deflated non-homothetic indirect utility function.

Cravino and Sotelo (2019)). Given estimates on the substitutability between products within the same detailed product category, we take $\sigma = 2$ as our upper bound (e.g., Broda and Weinstein (2006), Broda and Weinstein (2010), DellaVigna and Gentzkow (2019), and Handbury (2019)).

To study the optimal tax schedule beyond the comparative static approach, we need parametric assumptions on the utility function. As further described in Subsection 5.3.2 , we use non-homothetic CES preferences as in Hanoch (1975), Matsuyama (2019), and Comin et al. (2021).

Social preferences for redistribution. For the comparative static approach in Section 5.2, the formulas derived in Section 4 show that social preferences for redistribution can be recovered from the initial tax schedule. Taking the observed tax schedule as optimal obviates the need for specifying the social welfare function explicitly. For the analysis of the optimal tax schedule in Section 5.3, the planner's social welfare function, $G\left(U(\theta, \mathbf{p})\right)$, is assumed to be CRRA, with a relative risk aversion coefficient of one in our baseline specification and 0.5 for sensitivity.

5.2 Comparative Statics

Using the comparative static approach introduced in Section 4, we first examine the quantitative response of the tax schedule to exogenous price shocks, and then turn to its response to exogenous shifts in the skill distribution.

5.2.1 The Response of the Tax Schedule to Exogenous Price Shocks

Starting from the observed tax schedule, we implement the formulas from Section 4 characterizing the response to a price change. We obtain observed price shocks for the period 2004 to 2015 over 248 product categories covering the full consumption basket of American households, linking the CPI price dataset to the consumption patterns of the CEX. Empirically, inflation is lower in product categories with higher income elasticities: how large is the impact on the optimal tax schedule? To characterize the response of the tax schedule to these price changes, we first proceed in partial equilibrium (as in Section 4.1) and then in general equilibrium (as in Section 4.2).

Partial and general equilibrium results. Figure 1 presents the results. We report the changes in the tax schedule in response to price shocks, depending on the value of σ and contrasting the responses in partial and general equilibrium.

Using Proposition 5, we obtain the partial equilibrium response, which does not depend on σ. This response is large, with a fall in marginal tax rates of about 6pp at the bottom of the income distribution; the marginal tax rates gradually converge back to the observed tax schedule at the top. Because inflation is lower in the product categories for which higher-skill agents have a higher marginal propensity to consume, it is optimal for the planner to redistribute toward them, which can be done most efficiently by reducing marginal tax rates at the bottom of the income distribution.[74] Because the fall in the empirical price index increases with income, it is optimal for redistribution to be increasing in agents' skills. This result shows that inflation inequality generates a sizable regressive response of the tax schedule in partial equilibrium.

Moreover, using Proposition 7, we find that the response of the tax schedule is amplified in general equilibrium. With $\sigma = 0.6$, the planner reduces marginal tax rates by an additional two percentage points at the bottom of the income distribution. With $\sigma = 2$, the amplification is much larger and the optimal marginal tax rate is reduced to only 10% at the bottom of the income distribution. Indeed, in general

[74] This mechanism is standard: high marginal tax rates at the bottom are paid by all agents earning higher levels of income, without distorting their marginal incentives to work, and all revenue is rebated to the lower-income households through the intercept of the tax schedule.

equilibrium consumers reallocate their expenditures toward the goods that become relatively cheaper, which amplifies the price changes through increasing returns and further reduces the relative price of products with a high income elasticity. These endogenous price changes create an additional motive for the social planner to redistribute toward higher-skill agents, which leads to further price changes, and so on. These results show that the general equilibrium response of prices plays a quantitatively important role for optimal tax policy.

The role of non-linear social preferences. Next, Figure 2 quantifies the role of the curvature of the social welfare function for the response of the tax schedule, illustrating the theoretical insights from Section 4.4. While Figure 1 gives the results with linear social welfare weights, we now introduce curvature by taking the inverse optimum weights at the observed tax schedule as our empirical social welfare function. In that case, the degree of optimal redistribution toward the rich is slightly muted, because the social value of redistributing income toward high-skill agents falls endogenously as they get more transfers. With $\sigma = 0.6$, marginal tax rates increase by about 2.5pp at the bottom of the income distribution with the empirical nonlinear social welfare function, compared with the baseline case with linear Pareto weights.

To further investigate the role of the curvature of the social welfare function, we specify the social welfare function as $G(u, \theta) = \lambda(\theta)\frac{u^{1-\gamma}}{1-\gamma}$, with γ the CRRA coefficient and setting $\lambda(\theta)$ such that the observed tax schedule is optimal. With stronger curvature, e.g. a social welfare function with a CRRA coefficient of 0.5 or 1, redistribution toward the rich falls further. The result that marginal tax rates fall in response to the price shocks is attenuated but not overturned: the marginal tax rates remain about one to five percentage points below the observed tax schedule in the first six deciles, and gradually converge to the observed schedule at higher percentiles.

Likewise, with $\sigma = 2$ Figure A1 shows that redistribution toward the rich is reduced the more curved the social welfare function is. With the empirical social welfare function, the fall in marginal tax rates remains large, over 10 percentage points at the bottom of the income distribution.

Overall, these results show that non-linearities in social preferences for redistribution may play an important role for the optimal response of the tax schedule. With the empirical non-linear social welfare function, the fall in taxes remains substantial for both values of σ.[75] In unreported analyses, we find that the planner continues to increase (rather than decrease) marginal tax rates in response to lower inflation for high-income households even when the curvature of the social welfare function is much higher, e.g. with a CRRA coefficient of 10.

The response of the tax schedule to inflation inequality from scanner data. In the Online Appendix, Figure A2 shows that it is important to measure changes in prices at a detailed level to draw the implications of price changes for the optimal tax schedule. We repeat the previous exercise using the Nielsen Homescan Consumer Panel dataset instead of the CEX-CPI data. The Nielsen dataset covers only fast-moving consumer goods,[76] but it has the advantage of being available at a much higher level of granularity than the product categories from the CEX-CPI linked dataset and allows for the measurement of changes in product variety. We use the Nielsen data to illustrate the role of aggregation bias and product variety for the optimal tax policy response to inflation inequality. Price changes and changes in product variety are measured from 2004 to 2015 at different levels of aggregation.

Figure A2 shows the implied change in the tax schedule in partial equilibrium, depending on the level of aggregation used to measure the heterogeneity in consumption baskets, and thus in the inflation rates, across the income distribution. Because differences in inflation rates across the income distribution arise primarily within detailed product categories, rather than across, it is important to use granular data to avoid aggregation bias.

[75] Online Appendix Figure A1 shows that similar results apply with alternative values for the labor supply elasticity.

[76] Data coverage in Nielsen represents about 15% of total expenditure and 40% of expenditures on goods.

To illustrate the magnitude of this effect, we first focus on price indices computed at different levels of aggregation, using products available across consecutive years, and that do not account for changes in product variety. With the 994 most detailed product categories, called "product modules", we find that it is optimal for marginal tax rates to fall by about 12 percentage points at the bottom of the income distribution. With 109 larger product categories, called "product groups", the differences in inflation across the income distribution are attenuated and, accordingly, the fall in the tax schedule is only about 7.5 percentage points at the bottom of the distribution. With the ten broad "departments", measured inflation inequality is much smaller and the fall in tax rates is under 2 percentage points. These differences would be amplified further in general equilibrium, accounting for endogenous price changes and changes in demand.

Furthermore, we introduce a correction for changes in product variety, using a CES price index as in Feenstra (1994) and Broda and Weinstein (2010). In the data, product variety expands faster in product categories purchased by high-income households, which further reduces the price index faced by high-skill agents. Consequently, the fall in optimal marginal tax rates is amplified. At the bottom of the distribution, marginal tax rates fall by an additional 2.5 percentage points.

In sum, the results from Figure A2 highlight that, to be able to adjust the tax schedule optimally, it is important to measure inflation and spending patterns across the income distribution using granular data.

The response of the tax schedule to inflation inequality in the Diamond-Mirrlees model. Finally, we apply the comparative static approach formula to the Diamond Mirrlees model, where goods are produced competitively and all profits are taxed, as in Section 4.3. Appendix Figure A3 reports the partial-equilibrium response of the tax schedule to price changes measured in the CEX-CPI linked dataset. The response is similar to our benchmark model but is magnified, with a fall in marginal tax rates of 10pp percentage points at the bottom of the distribution (instead of 6pp in Figure 1, where the work subsidy reduces the sensitivity of taxes to the price shock). This result shows that some of the lessons from the quantitative analysis are not specific to our benchmark model of the supply side.

5.2.2 The Response of the Tax Schedule to Exogenous Shifts in the Skill Distribution

In this section, we characterize quantitatively the optimal response of the tax schedule to exogenous shifts in the income distribution, accounting for the endogenous response of prices. Using the publicly available statistics on the income distribution from the U.S. Census, we recover the shifts in the skill distribution from the observed shifts in the income distribution from 2004 to 2015.[77] Empirically, income is stagnant at the bottom of the distribution, and increases at faster and faster rates with higher incomes.

Partial and general equilibrium results. Using Proposition A2, Figure 3 reports the optimal response of marginal tax rates. We first consider the direct, partial equilibrium response to the change in the skill distribution, with fixed prices. As characterized by Proposition A2, as the income distribution becomes more spread out, the value of redistribution at higher incomes falls, which pushes for a more redistributive tax schedule, with higher marginal tax rates. Because of the shifts in the skill distribution, there is relatively more mass at the top and bottom of the skill distribution, hence the distortionary cost of taxation is higher in this range, while it is reduced in the middle of the distribution. To increase redistribution efficiently, it is therefore optimal to raise marginal tax rates especially in the middle of the income distribution. Thus, Figure 3 shows that optimal marginal tax rates increase by about 2.5pp at the bottom of the distribution, by about 5pp in the middle, and by 1pp at the very top.

Furthermore, general equilibrium effects are at play through prices, as characterized in Proposition 7. The direct effects on prices of the shifts in inequality is amplified through income and substitution

[77] We use the historical series available at https://www.census.gov/data/tables/time-series/demo/income-poverty/historical-income-households.html (Table H-2).

effects, as well as changes in optimal tax rates. These effects tend to reduce optimal tax rates, because the observed shift in the income distribution lowers the price of products with a high income elasticity. Because higher-income agents have a higher marginal propensity to spend on these goods, it is optimal to redistribute more toward them by lowering marginal tax rates, through the same channels as in Section 4.1. Quantitatively, with $\sigma = 0.6$ optimal tax rates are reduced by a few percentage points, relative to the optimum in partial equilibrium, throughout the distribution. The fall in marginal tax rates is larger with $\sigma = 2$, reaching about -4pp at the bottom of the distribution. In this case, marginal tax rates fall below the observed tax schedule at the bottom of the income distribution.

Finally, Figure 3 also shows the combined impact of the shift in the skill distribution and exogenous changes in prices. Price changes are measured from 2004 to 2015 as in Section 5.2.1, except that the estimation accounts at the same time for the shift in the skill distribution and the induced price changes, i.e. we estimate the "residual" price shocks to match observed price changes. Taking into account these residual price shocks leads to a substantial reduction in optimal tax rates at the bottom of the distribution. Indeed, as in Section 5.2.1, price shocks increase the value of redistribution at the top. Quantitatively, the direct price effects, which imply more redistribution toward higher-skill agents, more than offset the motive for increased redistribution toward low-skill agents from the shift in the skill distribution. Taking into account all effects, the optimal tax schedule becomes less redistributive. These results show that it is important to jointly study shifts in the skill distribution and price shocks.

The role of non-linear social preferences. Appendix Figure A4 shows the role of the curvature of the social welfare function. Both with exogenous prices (as in panel (i)) and endogenous prices (as in panels (ii) and (iii)), the response of the tax schedule is muted by additional curvature. Indeed, curvature tends to mute the motives for redistribution created either directly by the shift in the skill distribution or by the endogenous price response. We find that the changes in the optimal tax schedule remain substantial even with non-linear social welfare functions.

5.3 Optimal Tax Schedule

In this subsection, we show the quantitative importance of increasing returns to scale and non-homotheticities for optimal tax rates and welfare across the skill distribution. We first document the impact of increasing returns to scale in a homothetic model, then isolate the impact of non-homotheticities. Finally, we study the response of the tax schedule to exogenous price shocks. By introducing parametric assumptions on preferences, these analyses are complementary with the first-order approximations of Section 5.2, because they characterize how our new channels affect the optimum when accounting for potential non-linearities. Online Appendix B.4 provides a complete discussion of the solution algorithm.

5.3.1 The Interaction between Returns to Scale and Redistributive Motives

We first investigate the impact of returns to scale on the optimal tax schedule under homothetic utility, i.e. with $v(z^*, \mathbf{p}) \equiv \frac{z^*}{p}$. We find that the interaction between the corrective tax and redistributive motives is quantitatively large.

We consider a setting with a single sector, such that α can be interpreted as "aggregate" returns to scale. In a multi-sector setting where increases in productivity in one sector occur at the expense of productivity in another sector, the rows of matrix A_{ij} sum up to zero and $\alpha = 0$; in such a setting, with homothetic utility the sectoral returns to scale leave the tax schedule unaffected, as shown in Proposition 3. With aggregate returns α, the "naive" interpretation of Proposition 3 is that, relative to the CRS tax schedule, the planner should uniformly subsidize nominal wages $1 - T'$ at a constant rate $1/(1-\alpha)$ throughout the distribution.

The solid blue line in Figure 4 shows the baseline optimal tax schedule under CRS and a logarithmic social welfare function. The optimal marginal tax rates start around 68% at the bottom of the income distribution, fall gradually to 58% at the 80th percentile, and then increase toward 68% at the top. The dashed blue line depicts the tax schedule with the naive correction for increasing returns to scale, with $\alpha = 0.30$, whereby the net-of-tax wage is increased by 43% everywhere.[78]

The solid red line shows the optimal tax schedule under returns to scale with a logarithmic social welfare function. The fall in marginal tax rates is much smaller than with the naive correction. This result shows that the curvature of the social welfare function plays a quantitatively important role in determining the correction for increasing returns to scale, i.e. there is an important interaction with redistributive motives. It is optimal for the cost of the work subsidy to be predominantly paid by high-skill agents, hence marginal tax rates do not fall as much as with the naive correction. In Appendix Figure A5, we show that the interaction remains quantitatively large with other parameter values for the labor supply elasticity and the CRRA coefficient of the social welfare function.

By contrast, with linear Pareto welfare weights, set to match welfare weights at the CRS optimum, the "naive" correction is correct. To isolate the role of non-homotheticities independently of the curvature of the social welfare function, we take the specification with Pareto weights as our baseline in the next subsections. The Pareto weights are set as $\lambda(\theta) \equiv \left(U_{optim}(\theta)\right)^{-\widetilde{\sigma}}$, where $U_{optim}(\theta)$ is the solution of the optimal taxation problem with homothetic utility, constant returns to scale ($\alpha = 0$), and the CRRA parameter $\widetilde{\sigma}$ for the social welfare function.

5.3.2 The Role of Non-Homotheticities

We now turn to a specification with non-homothetic utility, using non-homothetic CES (nhCES) preferences as in Hanoch (1975), Matsuyama (2019), and Comin et al. (2021).

Parametric Assumptions. The indirect utility function $v\left(z^*, \boldsymbol{p}\right)$ is given by $v \equiv v\left(z^*, \boldsymbol{p}\right) \equiv F(\mathbf{Q})$, where $\mathbf{Q}$ is the consumption vector of the agent over the set of products $i \in \mathcal{I}$. Indirect utility v is implicitly defined by:

$$\sum_{i \in \mathcal{I}} \left(\Omega_i v^{\varepsilon_i}\right)^{\frac{1}{\sigma}} Q_i^{\frac{\sigma-1}{\sigma}} = 1.$$

NhCES preferences have convenient features, in particular $\frac{\partial \log\left(Q_i/Q_j\right)}{\partial \log(v)} = \left(\varepsilon_i - \varepsilon_j\right)$ and $\frac{\partial \log\left(Q_i/Q_j\right)}{\partial \log\left(p_j/p_i\right)} = \sigma \quad \forall i, j \in \mathcal{I}$. This tractable specification allows us to separately examine the impact on the tax schedule of the "utility elasticities" $\{\varepsilon\}_{i \in \mathcal{I}}$, which govern non-homothetic spending patterns, and the elasticity of substitution σ.

For tractability, in our calibration we consider two products, labelled "high quality" and "low quality" products. In line with evidence on the substitutability between products within the same detailed product category (Broda and Weinstein (2006), Broda and Weinstein (2010), DellaVigna and Gentzkow (2019), and Handbury (2019)), we set $\sigma = 2$. We then specify the elasticities $\{\varepsilon\}_{H,L}$ to match the dissimilarity index of consumption shares observed across the income distribution in the Consumer Expenditure Survey in 2014;[79] we obtain $\varepsilon_L = -7$ and $\varepsilon_H = -1.5$, implying that low-income households have a large marginal propensity to spend on the low-quality goods.

Baseline simulation. Figure 5 characterizes the impact of non-homotheticities in our baseline specifica-

[78] These results show that it is important to take into account returns to scale for optimal tax design: the effect on optimal tax rates is large. In practice, the adjustment could be made through the tax schedule or through other tax instruments, for example VAT.

[79] We compute the dissimilarity index at the level of the product categories available in the CEX interview files, called UCCs. We focus on 2014 as the data on the observed tax schedule from Hendren (2020) is available for that year.

tion relative to the homothetic case, with $\alpha = 0.3$ and Pareto weights from the logarithmic social welfare function. Panels (a) and (b) shows the effect of introducing non-homotheticities on optimal marginal tax rates. Due to non-homotheticities, marginal tax rates increase over the full range of the income distribution. The increase is larger at the bottom of the income distribution, with an increase in marginal tax rates of about 6pp for levels of earned income below \$20,000. The increase is about 2pp at an income level of \$100,000, and then gradually decreases, reaching levels close to zero above \$300,000. Thus, the simulations show that non-homoheticities have a significant quantitative impact on optimal marginal tax rates.

Panels (c) through (e) of Figure 5 investigate the mechanism explaining the change in marginal tax rates, which operates through the change in equilibrium prices and in the marginal utility of redistribution across the skill distribution. Panel (c) reports the equilibrium prices, normalized to one at the observed tax schedule. In the homothetic specification with increasing returns, the price index increases by about 3.6% at the optimal tax schedule, because preferences for redistribution induce higher taxes than at the observed schedule, which reduces labor supply and market size and thus drives an increase in the price. With non-homotheticities, prices of the high-quality and low-quality products diverge: the price of the high-quality good increases by 14%, while the low-quality product becomes 10% cheaper. Indeed, additional redistribution (relative to the observed schedule) leads to an increase in the relative market size of the product which has a higher marginal propensity to consume from low-income households, i.e. the low-quality product in our specification. This result shows that the response of the optimal tax schedule to non-homotheticities lead to large endogenous price changes in equilibrium.

Panel (d) shows that the induced change in the marginal utility of disposable income across agents is substantial. While under homothetic utility the marginal utility is about 0.965 ($= 1/p$) throughout the distribution, with non-homotheticities the marginal utility is 0.99 at the bottom, falls gradually to 0.85 around \$150,000, and then increases slightly. The fall in marginal utility is largest for the agents with the highest marginal propensity to consume on the high-quality good, which in equilibrium occurs for earned income levels around \$150,000 in our simulation. Panel (e) combines each agent's marginal utility of disposable income with Pareto weights and shows a steeper decline in welfare weights across the distribution with the non-homothetic specification, because of the price effects.

Finally, panel (f) summarizes the willingness to pay of agents for the optimal tax schedule under non-homotheticitic preferences, relative to the optimal schedule under homothetic preferences.[80] The equivalent variation is close to 15% in the bottom decile of the income distribution and decreases monotonically throughout the distribution, turning negative in the 7th income decile. In the top decile, the welfare loss from the new schedule, and its induced price effects, is about 9%. These estimates show that adjusting the tax schedule for non-homotheticities generates substantial distributional effects, with large welfare gains at the bottom of the distribution. Although panels (a) and (b) depicted an increase in marginal tax rates at the bottom of the distribution, overall the change in the tax schedule benefits low-income households more. Indeed, setting higher marginal tax rates at the bottom of the income distribution raises the overall amount of redistribution in a more efficient way than increasing marginal tax rates at the top, and the induced price effects benefit agents with a high average spending share on the low-quality product.

Thus, the baseline simulation shows that non-homotheticities can have meaningful quantitative implications for optimal taxation. The results account for all feedback loops between the desirability of

[80] We study the equivalent variation defined by:

$$\widetilde{v}\left(z_H^*(\theta) + EV(\theta), \mathbf{p}_H\right) - \psi\left(\frac{z_H(\theta)}{\theta}\right) = u_{NH}(\theta),$$

where H denotes the equilibrium each under the optimal tax schedule with homothetic preferences, while NH corresponds to the equilibrium with non-homothetic preferences.

redistributing more and the induced price changes. As the relative price of the low-quality product decreases, it is optimal to redistribute more to those with a higher marginal propensity to consume, which induces further tax changes and changes in labor supply, etc. The strength of these feedback loops depend on the parameters governing increasing returns and social preferences for redistribution, which we turn to next.

Sensitivity to increasing returns. Figure 6 reports the simulation results with larger increasing returns, setting $\alpha = 0.4$, close to the baseline estimate of 0.42 in Jaravel (2019). The results and channels described for the baseline specification are all amplified by the larger increasing returns. Optimal marginal tax rates increase by 11.5 percentage points at the bottom of the income distribution (panel (b)). The price of the high quality good increases by 22%, while the price of the low-quality good falls by 18% (panel (d)). The new tax schedule and the induced price effects create welfare gains of 35% at the bottom of the skill distribution, and welfare losses of 16% at the top.

Sensitivity to preferences for redistribution. With $\alpha = 0.30$, Figure 7 investigates the effects with weaker preferences for redistribution. The Pareto weights are taken from the optimal schedule with constant returns to scale and a social welfare function with a CRRA coefficient of 0.5, rather than 1 as previously. With this specification, the impact of non-homotheticities on the optimal tax schedule is muted. The marginal tax rate increases by 3.75pp at the bottom of the distribution (panel (b)), the price of the high quality product increases by about 3.75%, while the price of the low-quality product falls by about 3.5% (panel (d)). The willingness to pay for the tax schedule accounting for non-homotheticities remains meaningful, especially at the bottom of the income distribution, with a welfare gain of 12% in the bottom decile and a welfare loss of about 3% in the top decile.

These results illustrate the interplay between social preferences for redistribution and endogenous prices. A weaker taste for redistribution endogenously leads to smaller changes in market size, hence smaller price changes in equilibrium and a smaller adjustment to optimal marginal tax rates.

In sum, the optimal tax schedule is sensitive to non-homotheticities because redistribution induces changes in relative prices and hence in the value of further redistribution. Another mechanism whereby price effects can arise is exogenous productivity shocks; this channel also has implications for the optimal tax schedule, which we document next.

5.3.3 The Impact of Exogenous Price Shocks

We now characterize the response of the optimal tax schedule to exogenous price shocks. As in Section 5.2, this analysis is motivated by the observed heterogeneous price changes across product categories in the United States, with lower inflation in product categories purchased by high-income households.[81] The simulations account for feedback loops created by large price changes, and thus complement the first-order approximations in Section 5.2. To assess the quantitative relevance for the optimal schedule, we consider an exogenous productivity shock (to parameters γ_i), whose direct partial equilibrium effect is to reduce the price of the high-quality good by 2.5% and to increase the price of the low-quality good by 2.5%.[82]

Baseline simulation. Figure 8 reports the results under the baseline parametrization. The exogenous price shock leads to lower taxes: marginal tax rates fall by about 3.25pp at the bottom of the income distribution, and gradually converge back to the reference tax schedule under homothetic utility, with a

[81] For example, Jaravel (2019) documents that, for consumer packaged goods in the United States, annual inflation is about 2.5pp lower in the top price decile (a proxy for quality), compared with the bottom price decile.

[82] Given our calibration for non-homotheticities, the induced change in the price index is 3.1 pp higher in the bottom decile of the income distribution, compared with the top decile. This corresponds to the level of inflation inequality reached over 8.5 years in U.S. data (Jaravel (2019)).

fall in marginal tax rates under 0.10pp for levels of income above $300,000 (panels (a) and (b)).

To understand the mechanism, it is instructive to examine how the exogenous shocks affect equilibrium prices. Without shocks, prices are identical to the baseline non-homothetic specification studied in Figure 5 (panel (c)). In partial equilibrium, the shocks would reduce the price of the high-quality good from 1.14 to 1.11 and would increase the price of the low-quality good from 0.9 to 0.92. Panel (c) of Figure 8 shows the amplification of the price shocks through consumer demand, additional redistribution and returns to scale: the equilibrium prices are 1.055 for the high-quality good and 0.96 for the low-quality goods. In general equilibrium, the convergence of relative prices is much larger than the partial equilibrium shocks. Consequently, there is a substantial increase in the value of transferring an additional dollar to high-income households, who have a higher marginal propensity to consume on the high-quality good (panels (d) and (e)).

Thus, it is desirable for the planner to redistribute toward high-income households, which can be done efficiently by reducing marginal tax rates at the bottom of the income distribution. The welfare effects are substantial, with an equivalent variation of -6% in the bottom decile and +8.5% in the top decile.

Sensitivity to increasing returns. Figure 9 shows the results with higher returns to scale ($\alpha = 0.4$), which magnifies the impact of the exogenous productivity change. The fall in marginal tax rates at the bottom of the income distribution is about 12pp (panel (b)). The GE amplification of price changes is much larger and flips the relative price of the high- and low-quality bundles (panel (c)). The distributional effects are large, with an equivalent variation ranging from -26% at the bottom to +24% at the top.[83]

Sensitivity to preferences for redistribution. Figure 10 documents the role of social preferences for redistribution, setting the Pareto weights to match the optimal schedule with constant returns to scale and a social welfare function with a CRRA coefficient of 0.5. The impact of exogenous price shocks is much larger than in the baseline specification, with a fall in marginal tax rates of 13pp at the bottom of the income distribution (panel (b)).

To understand the mechanism, panel (c) reports equilibrium prices. Before the exogenous shock, equilibrium prices are 0.965 for the low-quality product and 1.0375 for the high-quality product (identical to Figure 7). After the shock, the price of the low-quality product increases substantially, reaching 1.07, while the price of the high-quality good falls to 0.955 only. The amplification of price effects is sufficiently large to flip the relative price of the high- and low-quality bundles.

When social preferences for redistribution are low, the planner puts larger weight on the change in utility out of disposable income for high-skill agents. Therefore, the planner is more responsive to the initial fall in the relative price of the high-quality good and redistributes more toward the rich, which induces a feedback loop of changes in labor supply, spending, and prices, leading to further changes in redistribution, etc. Quantitatively, this mechanism is strong enough to flip equilibrium relative prices and increase high-skill agents' utility out of disposable income above 1 (panel (d)). As depicted on panel (f), the shock results in a large welfare loss at the bottom of the income distribution (-32%) and substantial welfare gains at the top (+10%).

The comparison of these results with those from Figure 7 are instructive to understand the mechanism driving the interplay between endogenous prices, increasing returns, and social preferences for redistribution. In Figure 7, weaker social preferences induced an optimal tax schedule with less redistribution toward the poor, implying smaller changes in relative market size, and hence smaller endogenous price changes. In that setting, absent exogenous shocks, weaker social preferences for redistribution reduce the importance of non-homotheticities for the optimal tax schedule, because prices change less. Introducing

[83] Conversely, Appendix Figure A6 shows the results with lower returns to scale ($\alpha = 0.2$), which reduce the impact of the exogenous productivity change. The fall in marginal tax rates at the bottom of the income distribution is about 1.70pp, the GE amplification of price changes is smaller, and the distributional effects are more modest, with an equivalent variation ranging from -2.4% at the bottom to +5% at the top.

exogenous price shocks, Figure 10 shows that the response to price shocks is magnified with weaker social preferences, which induce more redistribution toward those with high MPCs on the cheaper products, which amplifies the exogenous shocks and lead to larger changes in equilibrium prices.

Overall, the results show that exogenous price shocks can have a large impact on the optimal tax schedule, and that there are important amplification effects through increasing returns and the endogenous social value of redistribution. In all simulations, a unifying mechanism operates: changes in equilibrium prices and the distribution of marginal propensities to consume govern the change in the optimal tax schedule.

Conclusion

In this paper, we have shown that optimal commodity and income taxation is sensitive to exogenous price shocks, the elasticity of prices to market size, and non-homothetic preferences. We provided an explicit analytical characterization of the channels whereby prices and non-homotheticities affect optimal taxation in general equilibrium. Using simulations based on observed spending patterns and the empirical elasticity of prices to market size, we found that these novel channels have a sizable quantitative impact on optimal marginal tax rates and welfare across the skill distribution.

This analysis was motivated by the fact that observed price changes are heterogeneous across product categories and across the income distribution, and that empirically prices are endogenous to market size. Going forward, our framework could be used to study the response of optimal taxation to a variety of supply shocks that could affect prices, for example due to changes in technology, trade, immigration, or market concentration. Although we considered a closed economy, we conjecture that the mechanisms we highlighted might become even richer in a model with trade. Changes in domestic demand can be even more important in an open economy than in a closed economy (Matsuyama (2019)) because they have an impact on the equilibrium patterns of specialization, which in turn have an impact on the direction of productivity growth through market size effects. Analyzing optimal taxation in an open economy model with non-homothetic preferences and endogenous prices is thus a promising direction for future research.

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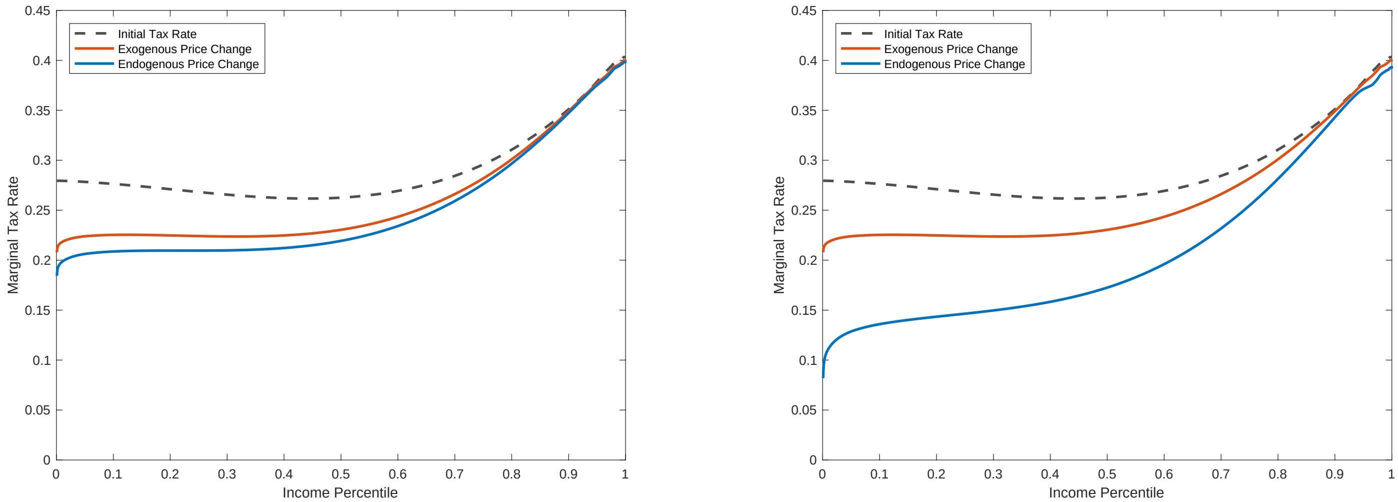
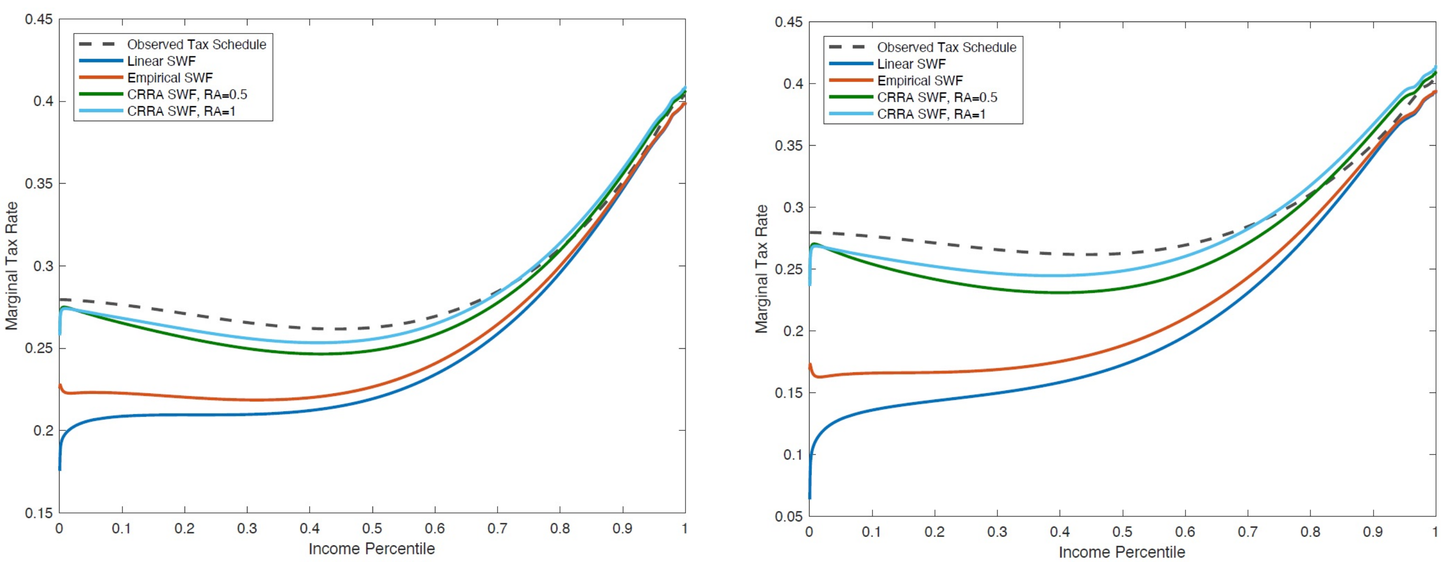
Figure 1: The Response of the Optimal Tax Schedule to Observed Price Shocks (2004-2015), CEX-CPI data



(i) $\sigma = 0.6$ (ii) $\sigma = 2$

Notes: the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.21$; the CEX-CPI dataset is used in both panels and the initial tax schedule is taken from Hendren (2020). See Section 5.2.1 for a description of the quantitative model and counterfactuals.

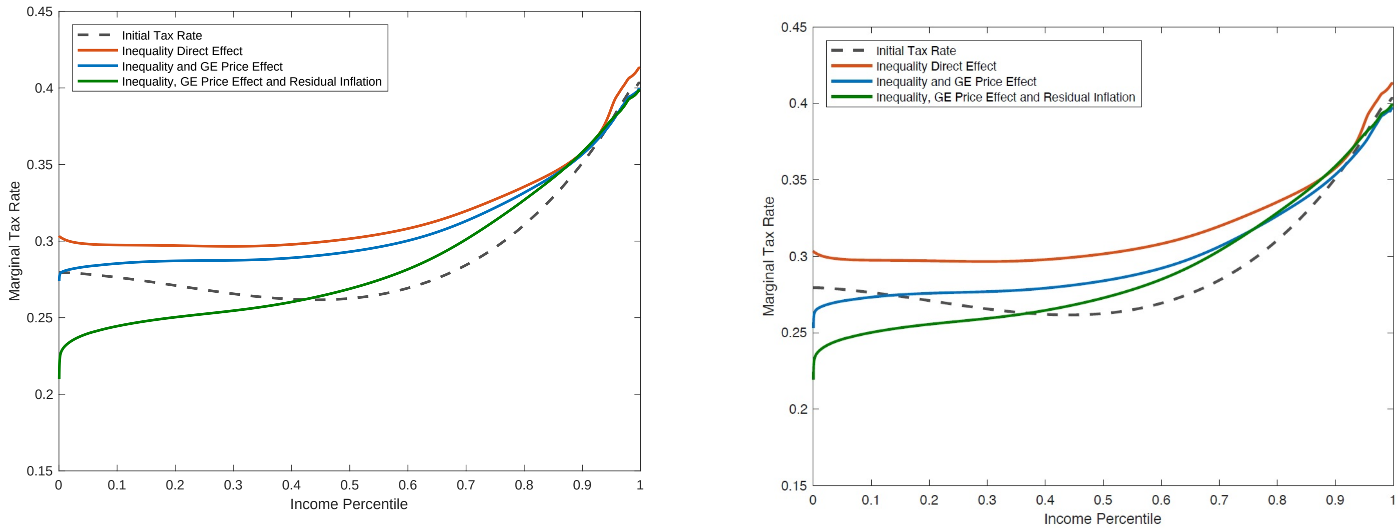
Figure 2: The Role of the Curvature of the Social Welfare Function



(i) $\sigma = 0.6$ (ii) $\sigma = 2$

Notes: the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.21$. The CEX-CPI dataset is used in both panels and the initial tax schedule is taken from Hendren (2020). See Section 5.2.1 for a description of the quantitative model and counterfactuals.

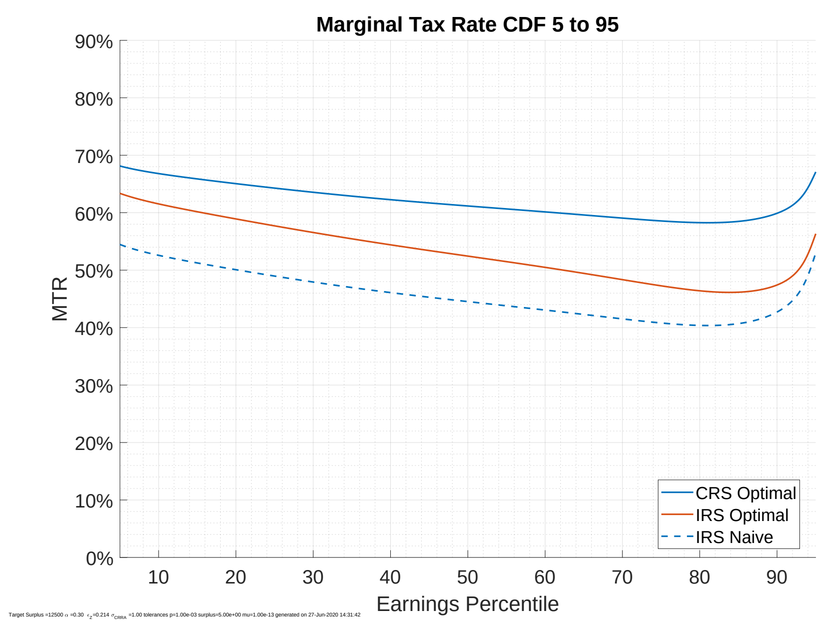
Figure 3: The Response of the Optimal Tax Schedule to Observed Shifts in the Skill Distribution (2004-2015)



(i) $\sigma = 0.6$ (ii) $\sigma = 2$

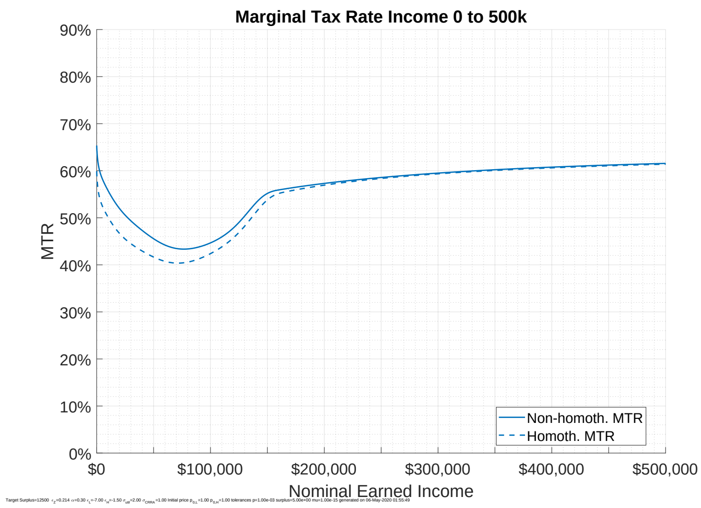
Notes: the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.21$; the U.S. Census and CEX-CPI datasets are used in both panels and the initial tax schedule is taken from Hendren (2020). See Section 5.2.2 for a description of the quantitative model and counterfactuals.

Figure 4: Returns to Scale and the Optimal Tax Schedule
($\alpha = 0.3$, $\varepsilon = 0.21$, SWF CRRA=1)

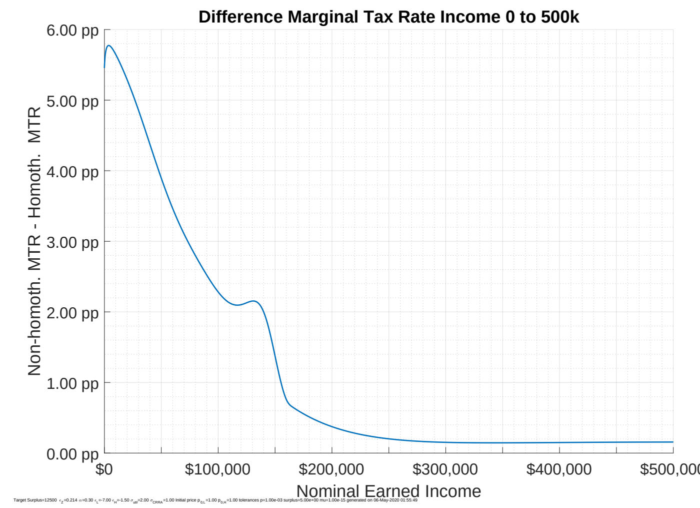


Notes: The "naive" correction uses the formula $1 - T'_{NAIVE}(\theta) = \frac{1}{1-\alpha}\left(1 - T'_{CRS}(\theta)\right)$.
See Section 5.3.1 for a description of the quantitative model and counterfactuals.

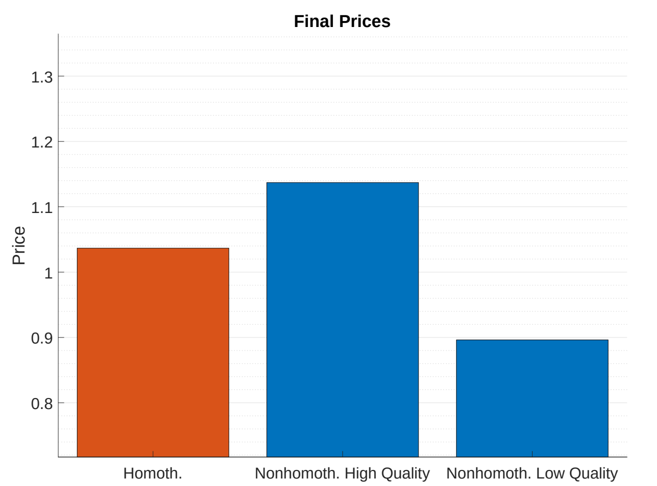
Figure 5: The Response of the Optimal Tax Schedule to Non-Homotheticities
($\alpha = 0.3$, $\varepsilon_z = 0.21$, Pareto weights from SWF CRRA=1)



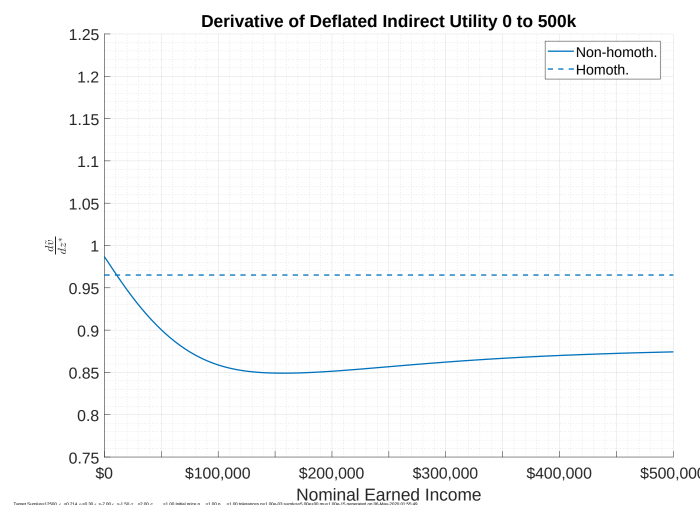
(a) Optimal Non-Homothetic vs. Homothetic MTRs



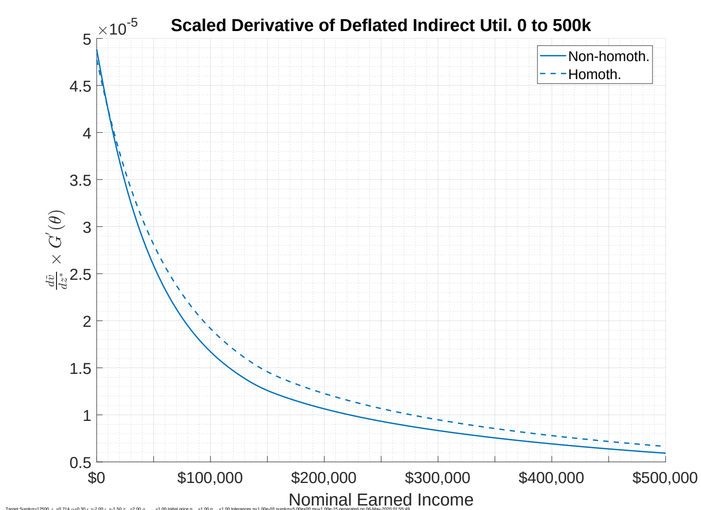
(b) Difference b/w Optimal Non-Homothetic and Homothetic MTRs



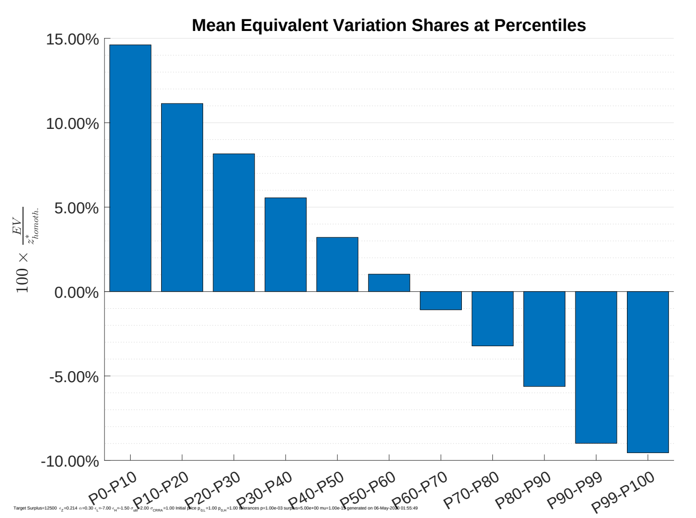
(c) Equilibrium Prices



(d) $\partial\widetilde{v}/\partial z^*$ by Earned Income



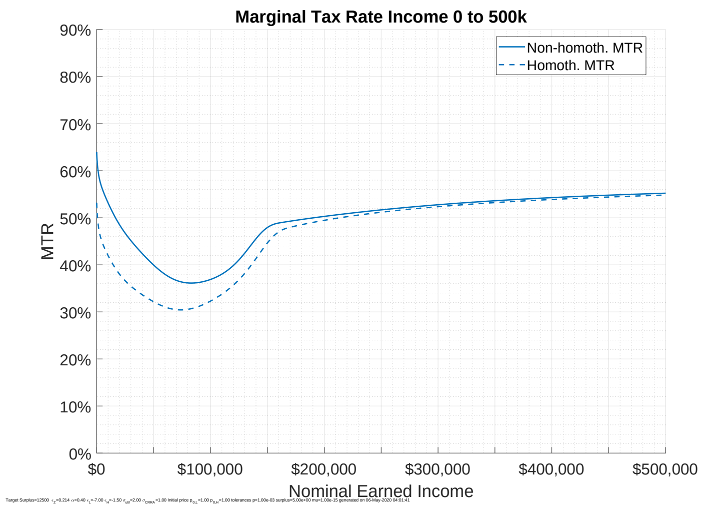
(e) $\partial\widetilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income



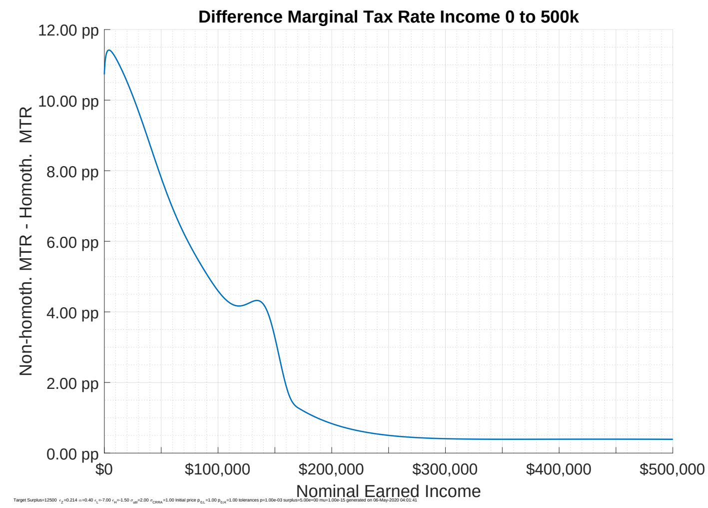
(f) EV relative to Optimal Homothetic Tax Schedule

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1, as described in Section 5.3.2.

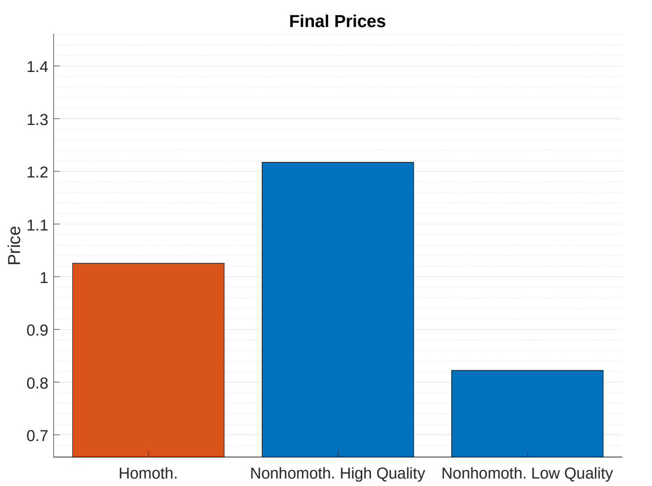
Figure 6: Higher Returns to Scale Magnify the Impact of Non-Homotheticicities
($\alpha = 0.4$, $\varepsilon_z = 0.21$, Pareto weights from SWF CRRA=1)



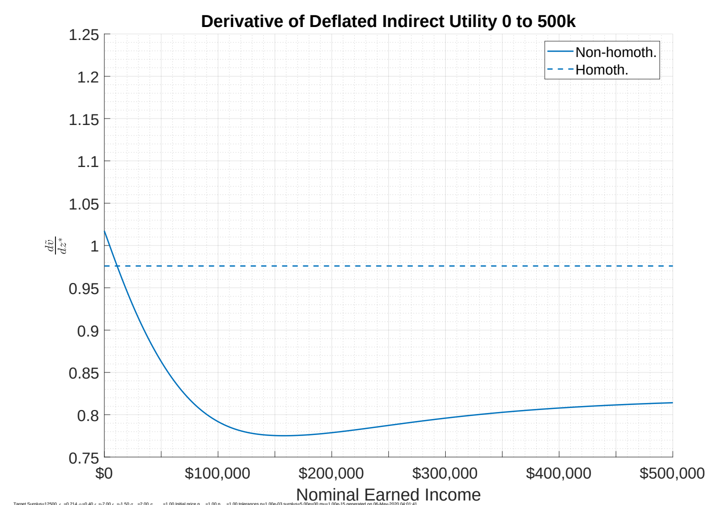
(a) Optimal Non-Homothetic vs. Homothetic MTRs



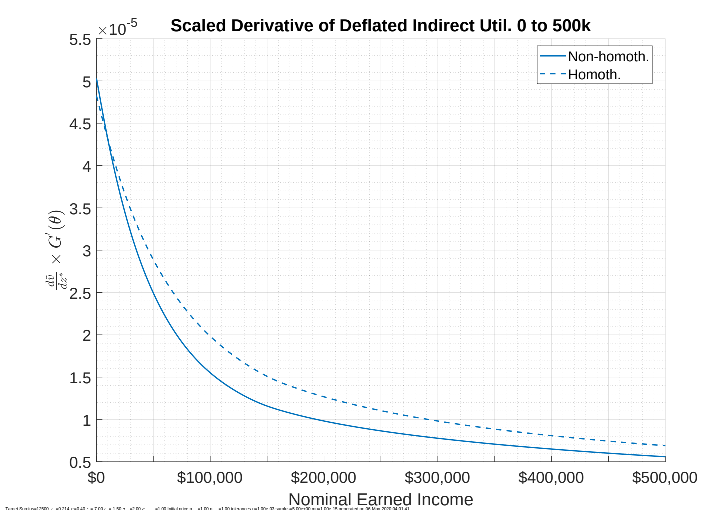
(b) Difference b/w Optimal Non-Homothetic and Homothetic MTRs



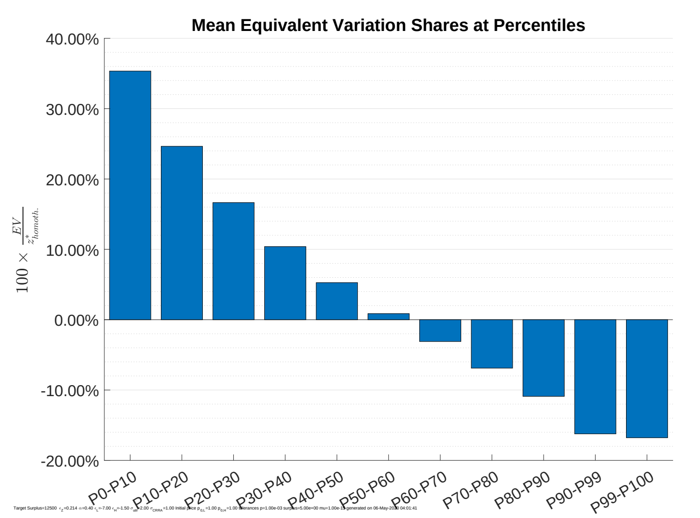
(c) Equilibrium Prices



(d) $\partial\widetilde{v}/\partial z^*$ by Earned Income



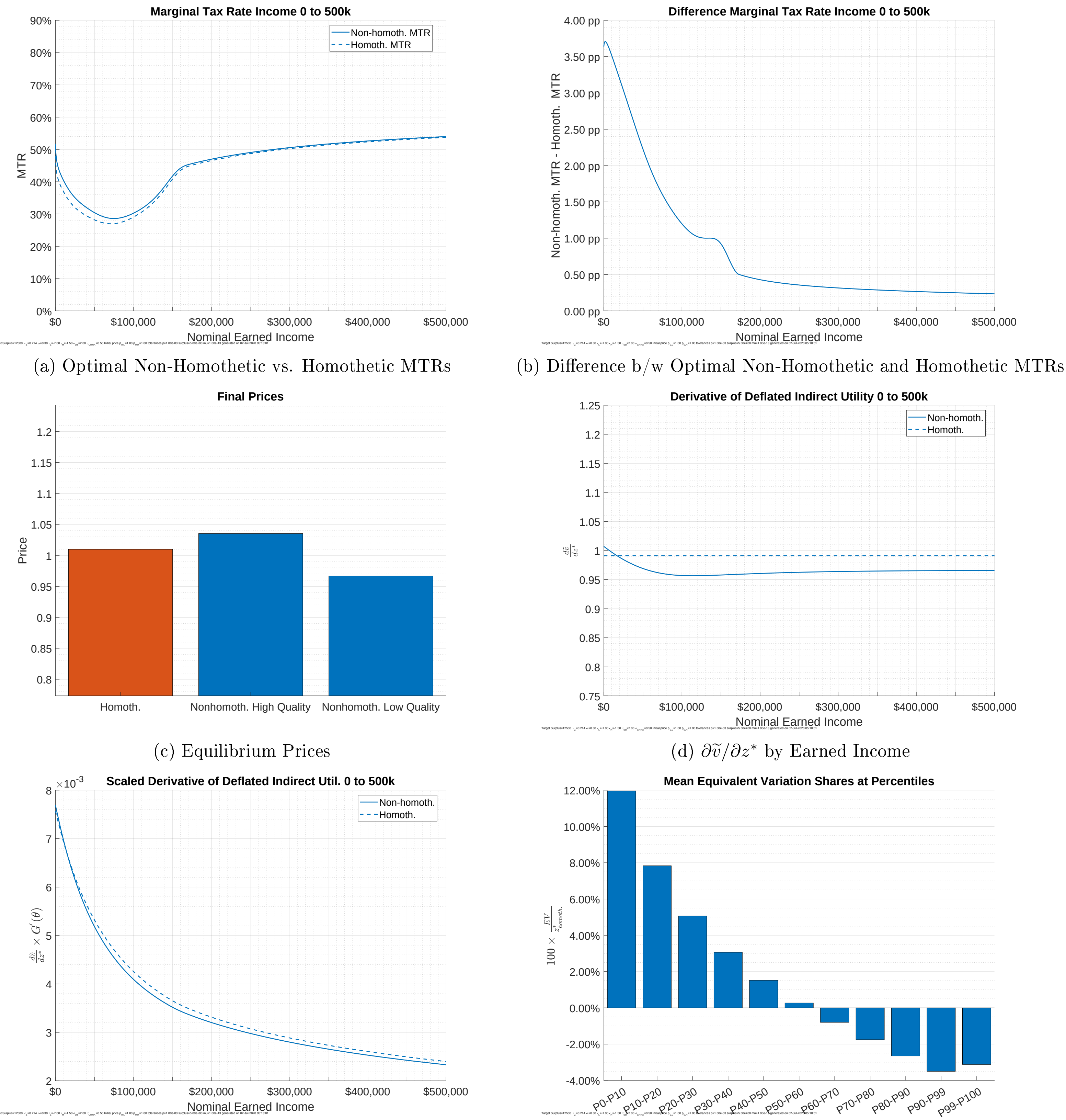
(c) $\partial\widetilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income



(f) EV relative to Optimal Homothetic Tax Schedule

Notes: The model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1, as described in Section 5.3.2.

Figure 7: Lower Social Preferences for Redistribution Reduce the Impact of Non-Homotheticicities
($\alpha = 0.3$, $\varepsilon_z = 0.21$, Pareto weights from SWF CRRA=0.5)



(a) Optimal Non-Homothetic vs. Homothetic MTRs

(b) Difference b/w Optimal Non-Homothetic and Homothetic MTRs

(c) Equilibrium Prices

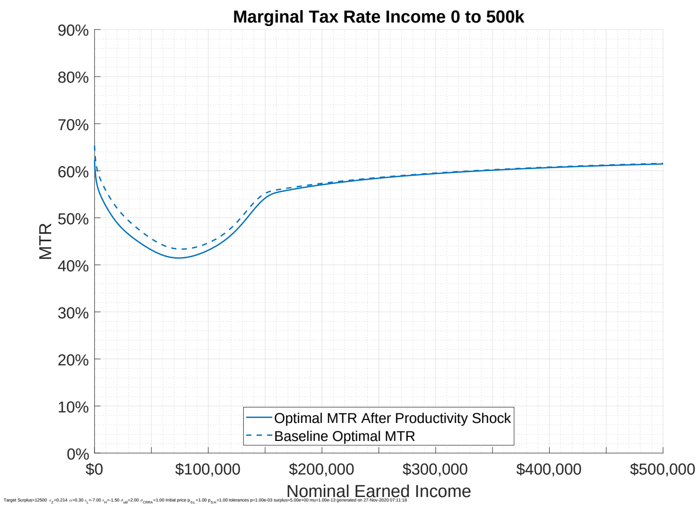
(d) $\partial\tilde{v}/\partial z^*$ by Earned Income

(e) $\partial\tilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income

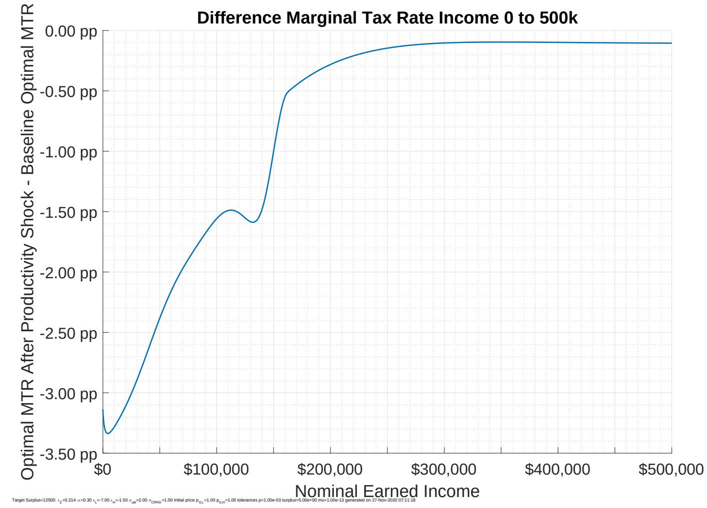
(f) EV relative to Optimal Homothetic Tax Schedule

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=0.5, as described in Section 5.3.2.

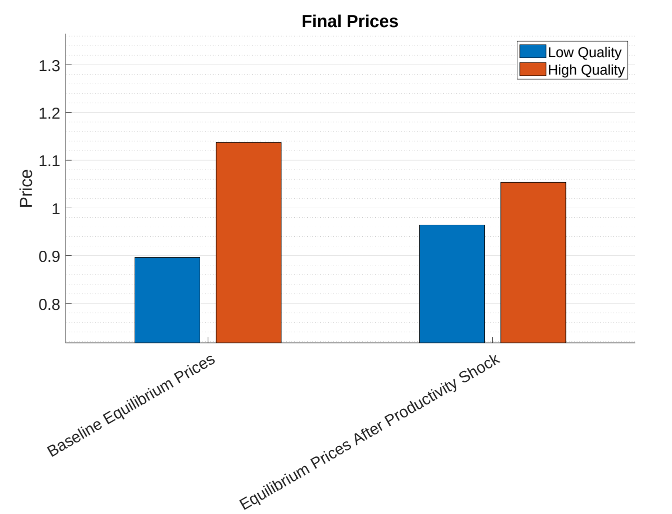
Figure 8: The Response of the Optimal Tax Schedule to Productivity Shocks
($\alpha = 0.3$, $\varepsilon_z = 0.21$, Pareto from SWF CRRA=1, PE price low-quality +2.5%, PE price high-quality -2.5%)



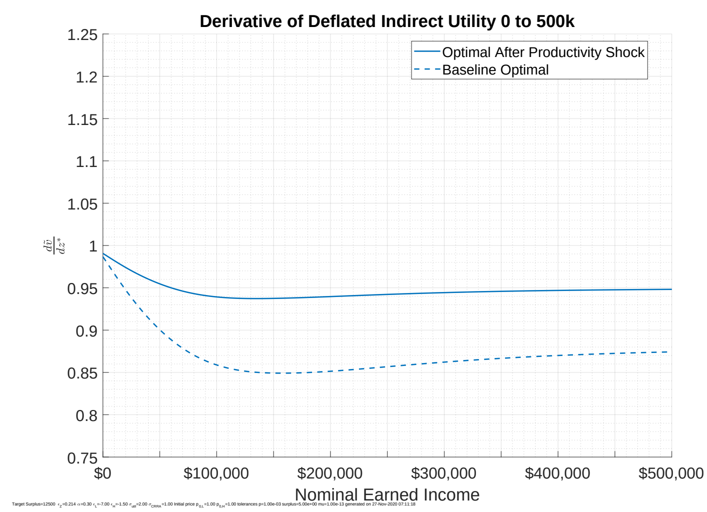
(a) Optimal MTRs Before vs. After Price Shocks



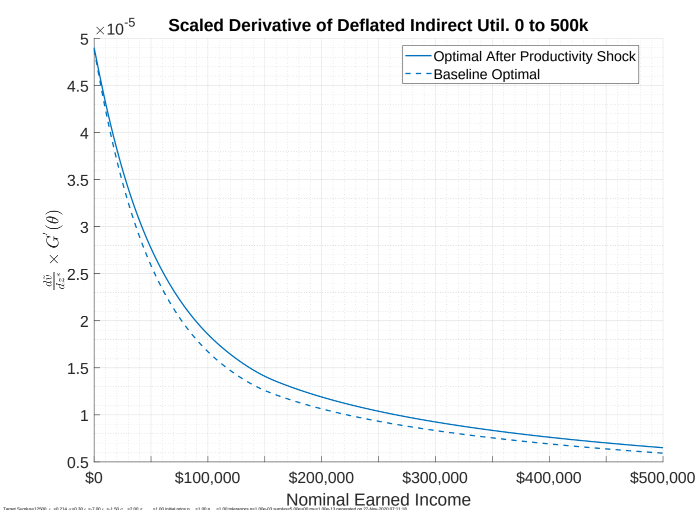
(b) Difference b/w MTRs



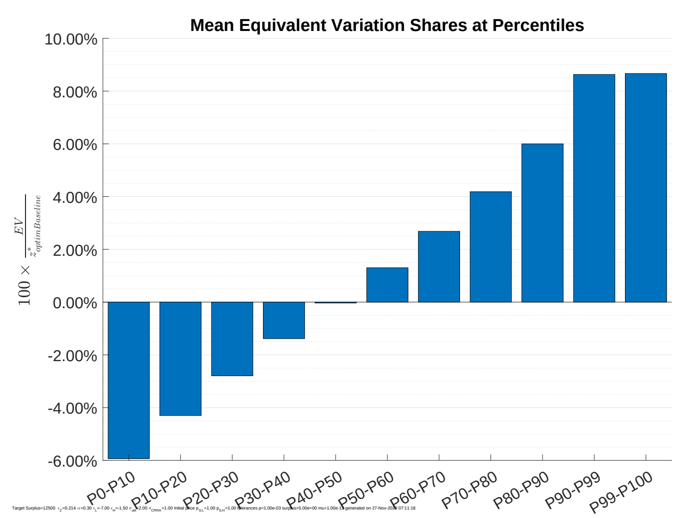
(c) Equilibrium Prices



(d) $\partial\tilde{v}/\partial z^*$ by Earned Income Before vs. After Price Shocks



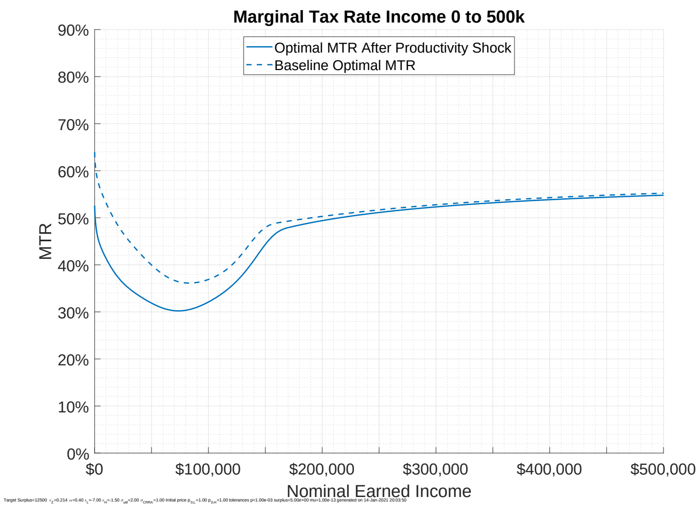
(e) $\partial\tilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income



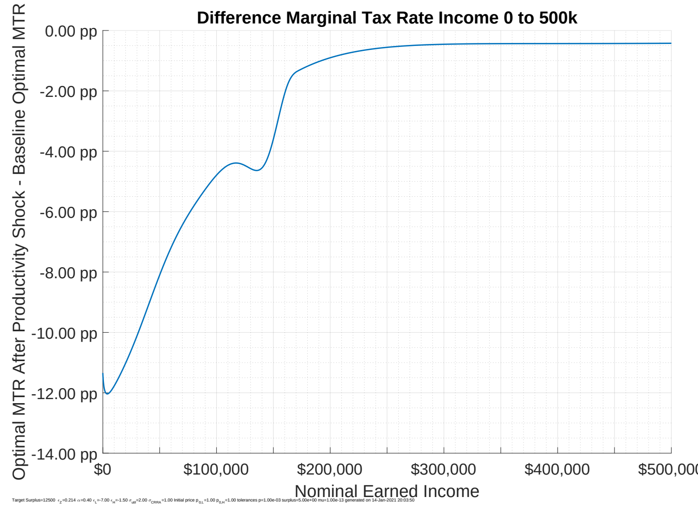
(d) EV, initial vs. new schedules

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1. The exogenous productivity changes are such that the partial equilibrium price of the low-quality bundle increases by 2.5% while the partial equilibrium price of the high-quality bundle decreases by 2.5%, as described in Section 5.3.3.

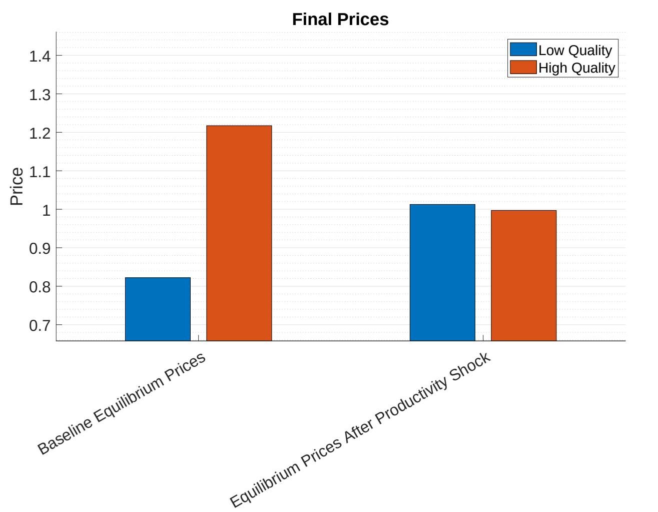
Figure 9: Higher Returns to Scale Magnify the Impact of Productivity Shocks
($\alpha = 0.4$, $\varepsilon_z = 0.21$, Pareto weights from SWF CRRA=1, PE price low-quality +2.5%, PE price high-quality -2.5%)



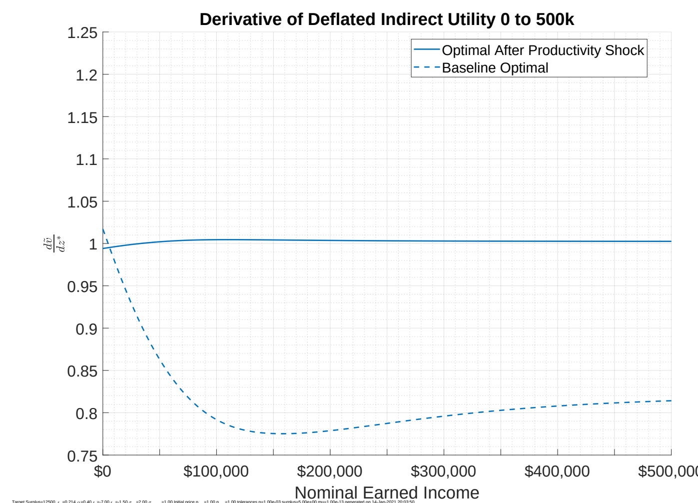
(a) Optimal MTRs Before vs. After Price Shocks



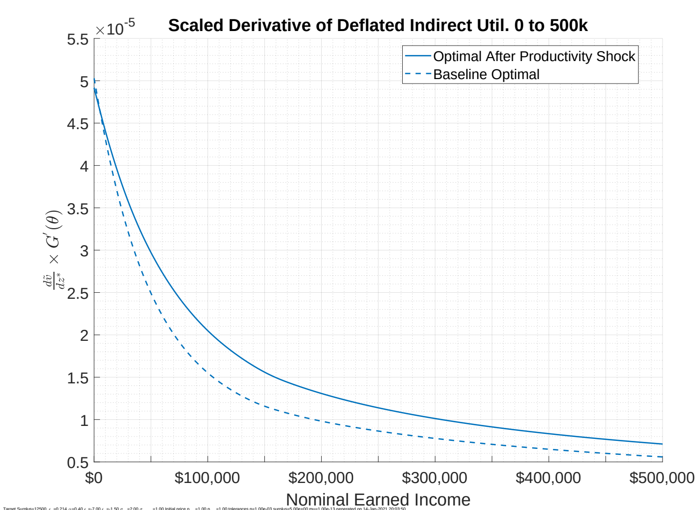
(b) Difference b/w MTRs



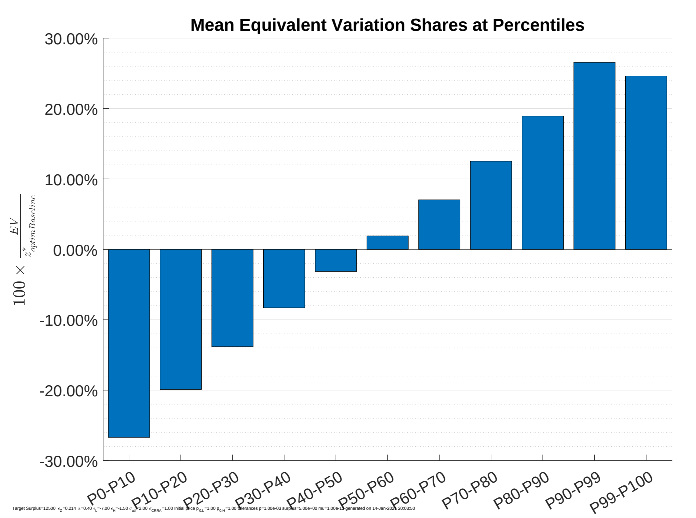
(c) Equilibrium Prices



(d) $\partial\widetilde{v}/\partial z^*$ by Earned Income Before vs. After Price Shocks



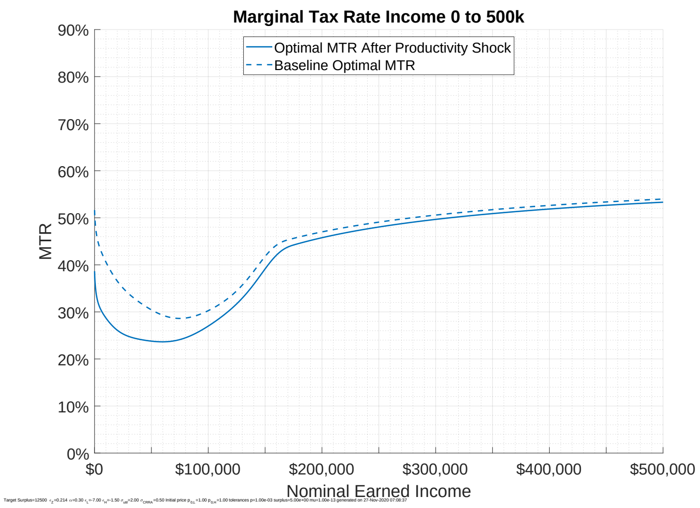
(e) $\partial\widetilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income



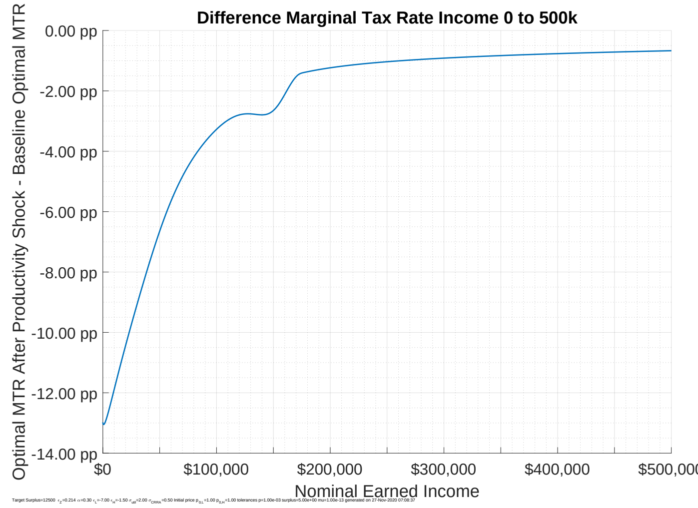
(f) EV, initial vs. new schedules

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1. The exogenous productivity changes are such that the partial equilibrium price of the low-quality bundle increases by 2.5% while the partial equilibrium price of the high-quality bundle decreases by 2.5%, as described in Section 5.3.3.

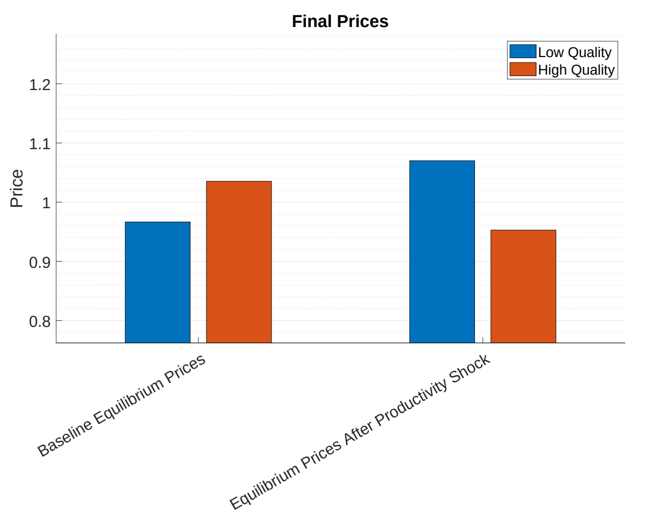
Figure 10: Lower Social Preferences for Redistribution Magnify the Impact of Productivity Shocks
($\alpha = 0.3$, $\varepsilon_z = 0.21$, Pareto weights from SFW CRRA=0.5, PE price low-quality +2.5%, PE price high-quality -2.5%)



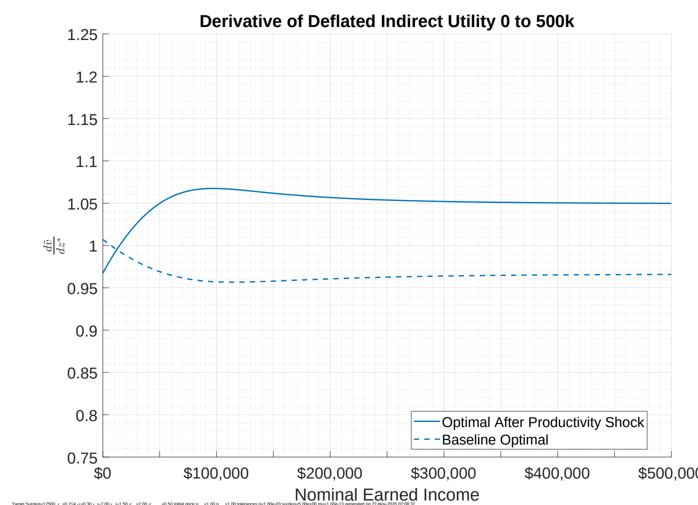
(a) Optimal MTRs Before vs. After Price Shocks



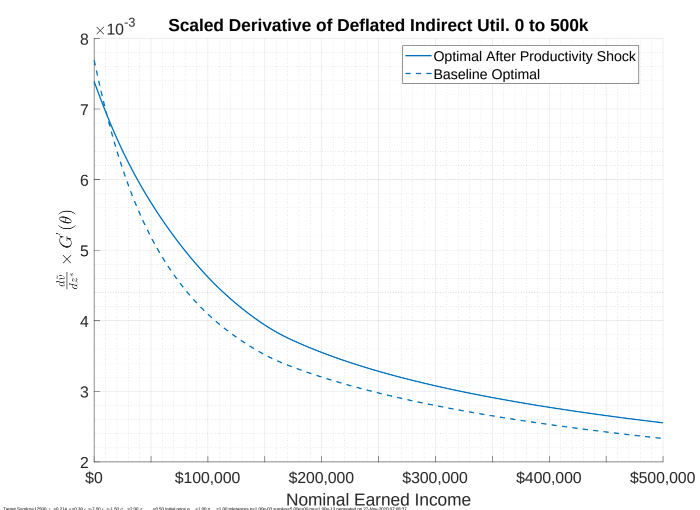
(b) Difference b/w MTRs



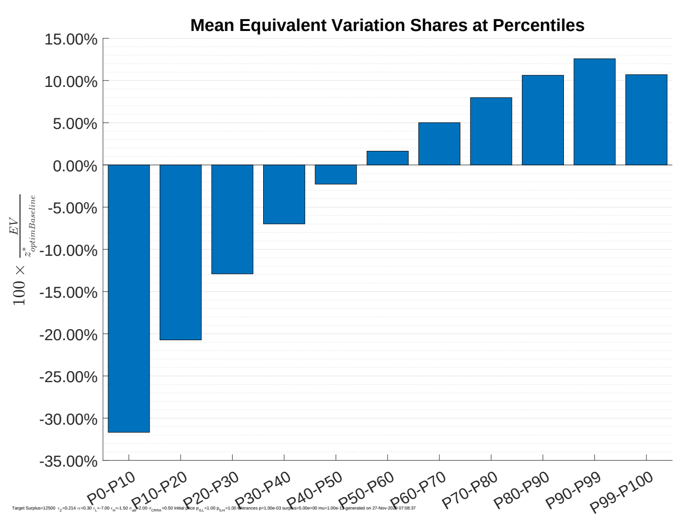
(c) Equilibrium Prices



(d) $\partial \widetilde{v}/\partial z^*$ by Earned Income Before vs. After Price Shocks



(e) $\partial \widetilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income



(f) EV, initial vs. new schedules

Notes: The model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=0.5. The exogenous productivity changes are such that the partial equilibrium price of the low-quality bundle increases by 2.5% while the partial equilibrium price of the high-quality bundle decreases by 2.5%, as described in Section 5.3.3.

Online Appendix

Prices, Non-homotheticities, and Optimal Taxation

Xavier Jaravel LSE

Alan Olivi UCL

August 2021

A Proofs

A.1 Proofs of Section 3

In this section we derive the optimal tax rates in our benchmark specification, and, for completeness, in the Diamoond-Mirrlees case. This will prove the results of Section 3.

A.1.1 Benchmark Model

We consider here the general planning problem.

$$\begin{aligned} &\sup_{V(\theta),z(\theta),q_i} \int G(V(\theta))\pi(\theta)d\theta \\ s.t.\quad & V'(\theta) = U_\theta(v(\theta),z(\theta),\theta) \quad and \int z^*(\theta) - z(\theta)\pi(\theta)d\theta - (q-p)\cdot C \leq 0 \\ with\quad & V(\theta) = U(v(\theta),z(\theta),\theta),\ \ v(\theta) = u(c_1(q,z^*(\theta)),...,c_n(q,z^*(\theta))),\ and\ z^*(\theta) = q\cdot c(q,z^*(\theta)) \\ & C_i = \int c_i(q,z^*(\theta))\pi(\theta)d\theta \quad and \quad p_i = \phi_i(C_1,...,C_n) \end{aligned}$$

Where v is the indirect sub-utility for consumption c_i denotes demand for i and z^* post-tax income.

After integration by parts, the corresponding lagrangian is:

$$\begin{aligned} \mathcal{L} = \int G(V(\theta))\pi(\theta)d\theta - \int (\mu'(\theta)V(\theta) + \mu(\theta)U_\theta(v(\theta),z(\theta),\theta))d\theta \\ -\lambda\left(\int z^*(\theta) - z(\theta)\pi(\theta)d\theta - (q-p)\cdot C\right) \end{aligned}$$

where $\mu(\theta)$ are the multipliers on the incentive constraints and λ is the multiplier on the resource constraint.

First Order Conditions. We start with the FOC with respect to consumer prices q_i. Denoting $c^h(q,v)$ the hicksian demand function at prices q for a given sub-utility v, we have:

$$\left.\frac{dc_j}{dq_i}\right|_{z,V} = \left.\frac{dc_j}{dq_i}\right|_{v} = \frac{\partial c_j^h}{\partial q_i}$$
$$\left.\frac{dz^*}{dq_i}\right|_{z,V} = \left.\frac{dz^*}{dq_i}\right|_{v} = c_i$$

We therefore have, denoting $\partial_{q_i} C_j^h = \int \partial_{q_i} c_j^h \pi(\theta) d\theta$:

$$\frac{d\mathcal{L}}{dq_i} = \lambda \left(C_i + \sum_j \left(q_j - p_j - \sum_k \partial_{Q_j} \phi_k C_k \right) \partial_{q_i} C_j^h - C_i \right)$$

Which gives for all i:

$$\sum_j \left(q_j C_j - \left(p_j C_j - \sum_k A_{k,j} p_k C_k \right) \right) \mathcal{S}_{j,i} = 0$$

Where A and $\mathcal{S}$ are defined in the main text. We therefore have:

$$(1-\beta) qC = (Id - A') pC$$

Where β is a scaling constant. Denoting α the scaling such that $q \cdot C - p \cdot C = 0$ gives:

$$q \cdot C - p \cdot C = \frac{1}{1-\beta} \left(\beta \sum_i p_i C_i - \sum_i p_i C_i \sum_j A_{i,j} \right)$$
$$\Rightarrow \quad \alpha = \frac{\sum_i (\sum_j A_{i,j}) p_i C_i}{\sum_i p_i C_i}$$

With this scaling, the ad valorem commodity taxes are:

$$1 + t_i = \frac{1 - \sum_j A_{j,i} p_j C_j / p_i C_i}{1 - \alpha}$$

Next, we derive the FOC associated with V. $V(\theta)$ impacts consumption and producer prices through

$z^*(\theta)$ with $dz^*(\theta)/dV(\theta) = (U_v v_{z^*})^{-1}$. We thus have:

$$
\begin{aligned}
0 &= G'(V(\theta))\pi(\theta) - \mu'(\theta) - \mu\frac{U_{\theta,v}}{U_v} - \frac{\lambda\pi(\theta)}{U_v v_{z^*}}\left[1 - \sum_i \left(q_i - p_i - \sum_j C_j \partial_{Q_i}\phi_j\right)\partial_{z^*} c_i(\theta)\right] \\
&= G'(V(\theta))\pi(\theta) - \mu'(\theta) - \mu\frac{U_{\theta,v}}{U_v} - \frac{\lambda\pi(\theta)}{U_v v_{z^*}}\left[1 - \sum_i \left(q_i - p_i\left(1 - \sum_j \frac{p_j C_j}{p_i C_i} A_{j,i}\right)\right)\partial_{z^*} c_i(\theta)\right] \\
&= G'(V(\theta))\pi(\theta) - \mu'(\theta) - \mu\frac{U_{\theta,v}}{U_v} - \frac{\lambda\pi(\theta)}{U_v v_{z^*}}\left[1 - \alpha\sum_i q_i\,\partial_{z^*} c_i(\theta)\right] \\
\Rightarrow& \mu'(\theta)\frac{U_v v_{z^*}}{\lambda} + \mu\frac{U_{\theta,v} v_{z^*}}{\lambda} = -\left(1 - \alpha - \frac{G'(V(\theta))U_v v_{z^*}}{\lambda}\right)\pi(\theta)
\end{aligned}
$$

Finally, defining $\tilde{\mu} = \mu\, U_v v_{z^*}/\lambda$, we have:

$$
\tilde{\mu}'(\theta) + \tilde{\mu}\,\partial_{z^*} MRS\, z'(\theta) = -\left(1 - \alpha - \frac{G'(V(\theta))U_v v_{z^*}}{\lambda}\right)\pi(\theta)
$$

With $MRS = -U_z/U_v v_{z^*}$ the marginal rate of substitution.

Finally, the FOC associated to z, using the same steps as above to derive the response of consumption and prices, are:

$$
\begin{aligned}
0 &= \mu(-U_{\theta,z} - U_{\theta,z^*} MRS) - \lambda\pi(\theta)\,(MRS - 1 - \alpha MRS) \\
\Rightarrow \quad & \tilde{\mu}\,\partial_\theta MRS = \pi(\theta)((1-\alpha)MRS - 1)
\end{aligned}
$$

Since $MRS = 1 - T'(z(\theta))$, and $z\tilde{\zeta}\partial_\theta MRS = -z'(\theta)(1 - T'(z(\theta)))$ where $\tilde{\zeta}$ is defined in the main text we therefore have, denoting $f(z(\theta)) = \pi(\theta)/z'(\theta)$

$$
\tilde{\mu}(\theta) = f(z)z\tilde{\zeta}\left(\frac{T'}{1-T'} + \alpha\right)
$$

Finally, using $-z\tilde{\zeta}\,\partial_{z^*} MRS = \tilde{\eta}$ we get:

$$
f(z)z\tilde{\zeta}\left(\frac{T'}{1-T'} + \alpha\right) + \int_{z(\theta)}^{z(\bar{\theta})} \tilde{\eta}\left(\frac{T'}{1-T'} + \alpha\right) f(z)dz = \int_{z(\theta)}^{z(\bar{\theta})}\left(1 - \alpha - \frac{G' U_v v_{z^*}}{\lambda}\right) f(z)dz
$$

Using $g = G' U_v v_{z^*}/((1-\alpha)\lambda)$ we obtain the formula of Proposition 2.

Optimal Taxation without Commodity Taxes The planner's problem is now:

$$\sup_{V(\theta),z(\theta)} \int G(V(\theta))\pi(\theta)d\theta$$

$$s.t. \quad V'(\theta) = U_\theta(v(\theta), z(\theta), \theta) \quad and \int z^*(\theta) - z(\theta)\pi(\theta)d\theta \leq 0$$

$$with \quad V(\theta) = U(v(\theta), z(\theta), \theta), \;\; v(\theta) = u(c_1(p, z^*(\theta)), ..., c_n(p, z^*(\theta))), \; and \; z^*(\theta) = p \cdot c(q, z^*(\theta))$$

$$C_i = \int c_i(p, z^*(\theta))\pi(\theta)d\theta \quad and \quad p_i = \phi_i(C_1, ..., C_n)$$

After integration by parts, the corresponding lagrangian is:

$$\mathcal{L} = \int G(V(\theta))\pi(\theta)d\theta - \int (\mu'(\theta)V(\theta) + \mu(\theta)U_\theta(v(\theta), z(\theta), \theta))d\theta$$
$$-\lambda \left(\int z^*(\theta) - z(\theta)\pi(\theta)d\theta \right)$$

where $\mu(\theta)$ are the multipliers on the incentive constraints and λ is the multiplier on the resource constraint.

First Order Conditions. We derive the FOC associated with V. Increasing the welfare of θ agents will endogenously move prices which requires compensating all other agents. We thus have: $dz^*(\theta')/dV(\theta) = \mathbb{1}_{\theta=\theta'}(U_v v_{z^*})^{-1} + \sum c_i(p, z^*(\theta)\frac{dp_i}{dV(\theta)}$. The change in aggregate consumption is given by:

$$\frac{dC_i}{dV(\theta)} = \sum \frac{\partial C_i^h}{\partial p_j}\frac{dp_j}{dV(\theta)} + \partial_{z^*}c_i(p, z^*(\theta))\frac{\pi(\theta)}{U_v v_{z^*}}$$

With $\partial C_i^h/\partial p_j = \mathbb{E}(\partial_{p_j} c^h(p, z^*(\theta))$ the average price derivative of Hicksian demand. We therefore have:

$$\frac{1}{p_j}\frac{dp_j}{dV(\theta)} = -\sum A_{jk}\mathcal{S}_{kl}\frac{1}{p_l}\frac{dp_l}{dV(\theta)} - \sum A_{jk}\frac{\partial_{z^*}c_k(p, z^*(\theta))\pi(\theta)}{C_k}\frac{1}{U_v v_{z^*}}$$

Which gives:

$$\left[\frac{1}{p_j}\frac{dp_j}{dV(\theta)}\right] = (Id + A\mathcal{S})^{-1}A\left[\frac{\partial_{z^*}c_k(p, z^*(\theta))\pi(\theta)}{C_k}\right]\frac{1}{U_v v_{z^*}}$$

We define the market elasticity $\tilde{\alpha}_i$ as:

$$\tilde{\alpha}_i = [s_1..s_n](Id + A\mathcal{S})^{-1}A[0..1/s_i..0]'$$

And $\tilde{\alpha} = \sum \tilde{\alpha}_i \mathbb{E}(\partial_{z^*} e_i)$. The first order conditions are therefore

$$0 = G'(V(\theta))\pi(\theta) - \mu'(\theta) - \mu \frac{U_{\theta,v}}{U_v} - \frac{\lambda \pi(\theta)}{U_v v_{z^*}} \left[1 - \sum_i \tilde{\alpha}_i \, \partial_{z*} e_i(p, z^*(\theta)) \right]$$

Defining $\tilde{\mu} = \mu \, U_v v_{z^*} / \lambda$, we have:

$$\tilde{\mu}'(\theta) + \tilde{\mu} \, \partial_{z^*} MRS \, z'(\theta) = - \left(1 - \frac{G'(V(\theta)) U_v v_{z^*}}{\lambda} - \sum_i \tilde{\alpha}_i \, \partial_{z*} e_i(p, z^*(\theta)) \right) \pi(\theta)$$

The FOC associated to z, following the same steps givse:

$$\tilde{\mu} \, \partial_\theta MRS = \pi(\theta)((MRS - 1 - MRS \sum_i \tilde{\alpha}_i \, \partial_{z*} e_i(p, z^*(\theta)))$$

Putting everything together, we therefore have:

$$\begin{aligned} f(z) z \tilde{\zeta} \frac{T'}{1-T'} + \int_{z(\theta)}^{z(\bar{\theta})} \tilde{\eta} \frac{T'}{1-T'} f(z) dz = \int_{z(\theta)}^{z(\bar{\theta})} \left(1 - \frac{G' U_v v_{z^*}}{\lambda} \right) f(z) dz \\ - \sum_i \tilde{\alpha}_i \left(z \tilde{\zeta} f(z) \partial_{z*} e_i + \mathbb{E}_{z'>z}((1-\tilde{\eta}) \partial_{z*} e_i) \right) \end{aligned}$$

Finally, noting that $\sum \tilde{\alpha} \partial_{z^*} e_i = \tilde{\alpha}$ and using $g = G' U_v v_{z^*} / ((1-\tilde{\alpha})\lambda)$ (Note that without income effects we have $\lambda = \mathbb{E}(G' U_v v_{z^*})/(1-\tilde{\alpha})$). We have:

$$\begin{aligned} f(z) z \tilde{\zeta} \left(\frac{T'}{1-T'} + \tilde{\alpha} \right) + \int_{z(\theta)}^{z(\bar{\theta})} \tilde{\eta} \left(\frac{T'}{1-T'} + \tilde{\alpha} \right) f(z) dz = (1-\tilde{\alpha}) \int_{z(\theta)}^{z(\bar{\theta})} \left(1 - \frac{G' U_v v_{z^*}}{\lambda} \right) f(z) dz \\ - \sum_i (\tilde{\alpha}_i - \tilde{\alpha}) \left(z \tilde{\zeta} f(z) \partial_{z*} e_i + \mathbb{E}_{z'>z}((1-\tilde{\eta}) \partial_{z*} e_i) \right) \end{aligned}$$

With $\tilde{\eta} = 0$ we obtain the formulas in the main text. With a constant elasticity of substitution and a diagonal A we obtain the formula of the main text for $\tilde{\alpha}_i$ with a simple application of the Sherman-Morrison formula.

A.1.2 Diamond-Mirrlees Specification

We consider the specification described in the main text: the cost of producing $Q_1, ..., Q_n$ is given by χ and all profits are taxed.

$$\sup_{V(\theta),z(\theta),q_i} \int G(V(\theta))\pi(\theta)d\theta$$

$$\begin{aligned}
s.t. \quad & V'(\theta) = U_\theta(v(\theta), z(\theta), \theta) \quad and \quad \chi(C_1, ..., C_n) - \int z(\theta)\pi(\theta)d\theta \leq 0 \\
with \quad & V(\theta) = U(v(\theta), z(\theta), \theta), \;\; v(\theta) = u(c_1(q, z^*(\theta)), ..., c_n(q, z^*(\theta))), \; and \, z^*(\theta) = q \cdot c(q, z^*(\theta)) \\
& C_i = \int c_i(q, z^*(\theta))\pi(\theta)d\theta
\end{aligned}$$

The lagrangian is:

$$\begin{aligned}
\mathcal{L} = \int G(V(\theta))\pi(\theta)d\theta - \int (\mu'(\theta)V(\theta) + \mu(\theta)U_\theta(v(\theta), z(\theta), \theta))d\theta \\
-\lambda \left(\chi(C_1, ..., C_n) - \int z(\theta)\pi(\theta)d\theta \right)
\end{aligned}$$

First Order Conditions. We start with the FOC with respect to consumer prices q_i. With the same notation as above, we have:

$$\sum_j \partial_{Q_j}\chi \partial_{q_i} C_j^h = 0$$

Therefore, we have $q_i = \partial_{Q_j}$.

Next, the first order condition for V are:

$$\begin{aligned}
& 0 = G'(V(\theta))\pi(\theta) - \mu'(\theta) - \mu \frac{U_{\theta,v}}{U_v} - \frac{\lambda\pi(\theta)}{U_v v_{z^*}} \sum_i \partial_{Q_i}\chi \, \partial_{z^*} c_i(\theta) \\
\Rightarrow \quad & \mu'(\theta)\frac{U_v v_{z^*}}{\lambda} + \mu \frac{U_{\theta,v} v_{z^*}}{\lambda} = -\pi(\theta) \left(1 - \frac{G'(V(\theta))U_v v_{z^*}}{\lambda} \right) \\
\Rightarrow \quad & \tilde{\mu}'(\theta) + \tilde{\mu} \, \partial_{z^*} MRS \, z'(\theta) = -\left(1 - \frac{G'(V(\theta))U_v v_{z^*}}{\lambda} \right) \pi(\theta)
\end{aligned}$$

Finally, the FOC for z

$$\begin{aligned}
& \tilde{\mu} \, \partial_\theta MRS = \pi(\theta)(\sum_j \partial_{Q_j}\chi \, \partial_{z^*} c_j MRS - 1) \\
\Rightarrow \quad & \tilde{\mu} \, \partial_\theta MRS = \pi(\theta)(MRS - 1)
\end{aligned}$$

So we have $q_j = \partial_{Q_j}\chi$ and:

$$f(z)z\tilde{\zeta}\frac{T'}{1-T'} + \int_{z(\theta)}^{z(\bar{\theta})} \tilde{\eta}\frac{T'}{1-T'} f(z)dz = \int_{z(\theta)}^{z(\bar{\theta})} \left(1 - \frac{G'U_v v_{z^*}}{\lambda} \right) f(z)dz$$

A.2 Proofs of Section 4

We prove here the comparative statics formulas in the case where $U(v(q,z^*),z,\theta) = v(q,z^*) - \chi(z/\theta)^{1+\frac{1}{\epsilon}}/(1+\frac{1}{\epsilon})$ and $v(q^0,z^*) = z^*$ at initial consumer prices q^0.

Proof of Proposition 4. Under the assumption of proposition 4, we have $G(V(\theta)) = \lambda(\theta)V(\theta)$, we consider here the derivative of the tax rate $T'(z(\theta))$ with respect to q_i keeping α (and the prices p) fixed at their initial values.

We have:

$$\begin{aligned} &\tilde{\mu}(\theta)\partial_\theta ln(MRS) = \pi(\theta)(1-\alpha-MRS^{-1}) \\ \Rightarrow \quad &\frac{d\tilde{\mu}(\theta)}{dq_i}\partial_\theta ln(MRS) + \tilde{\mu}(\theta)\frac{d\partial_\theta ln(MRS)}{dq_i} = -\pi(\theta)\frac{dMRS^{-1}}{dq_i} \end{aligned}$$

Since $\partial_\theta ln(MRS) = \partial_\theta\{-(1+\frac{1}{\epsilon})ln(\theta)\} = -(1+\frac{1}{\epsilon})\theta^{-1}$, we have $\frac{d\partial_\theta ln(MRS)}{dq_i} = 0$ which gives:

$$\begin{aligned} \frac{d\tilde{\mu}(\theta)}{dq_i} &= z\tilde{\zeta}\, f(z)\, \frac{dMRS^{-1}}{dq_i} \\ &= z\tilde{\zeta}\, f(z)\, \left.\frac{d}{dq_i}\frac{T'(z(\theta))}{1-T'(z(\theta))}\right|_\theta \end{aligned}$$

We now turn to the FOC associated to V. Note that since $v(q^0,z^*) = z^*$, we have $MRS_{z^*} = 0$ therefore:

$$\begin{aligned} &\tilde{\mu}'(\theta) + \tilde{\mu}\,\partial_{z^*}MRS\, z'(\theta) = -\left(1-\alpha-\frac{\lambda(\theta)v_{z^*}}{\lambda}\right)\pi(\theta) \\ \Rightarrow \quad &\frac{d}{d\theta}\frac{d\tilde{\mu}}{dq_i} + \tilde{\mu}\, z'(\theta)\,\frac{d\partial_{z^*}MRS}{dq_i} = \pi(\theta)\left(\frac{\lambda(\theta)}{\lambda}\frac{dv_{z^*}}{dq_i} - \frac{\lambda(\theta)v_{z^*}}{\lambda^2}\frac{d\lambda}{dq_i}\right) \end{aligned}$$

We first need to compute the derivatives of v_{z^*} and v_{z^*,z^*}. We have, using $v_{z^*z^*} = 0$:

$$\frac{dv_{z^*}(z(\theta)-T(z(\theta),q)}{dq_i} = v_{z^*z^*}\left((1-T')\frac{dz}{dq_i} + \frac{dT}{dq_i}\right) + \frac{\partial v_{z^*}}{\partial q_i} = \frac{\partial v_{z^*}}{\partial q_i}$$

Next using Roy's Identity for the sub-problem of choosing consumption conditional on z^*, we have $\partial_{q_i}v = -v_{z^*}c_i$, so:

$$\frac{\partial v_{z^*}}{\partial q_i} = \frac{\partial}{\partial z^*}\frac{\partial v}{\partial q_i} = -v_{z^*z^*}c_i - v_{z^*}\partial_{z^*}c_i = -v_{z^*}\partial_{z^*}c_i$$

Similarly, we have:

$$\frac{dv_{z^*z^*}}{dq_i} = \frac{\partial v_{z^*z^*}}{\partial q_i} = -v_{z^*}\partial_{z^*z^*}c_i$$

Using these we have:

$$\frac{dMRS_{z^*}}{dq_i} = \frac{d}{dq_i}\left\{\left(-\frac{U_z}{v_{z^*}}\right)\left(-\frac{v_{z^*z^*}}{v_{z^*}}\right)\right\} = (1-T')\partial_{z^*z^*}c_i$$

Plugging these in the FOC for V and integrating gives:

$$\begin{aligned} z\tilde{\zeta} f(z) \ \frac{d}{dq_i}\frac{T'(z(\theta))}{1-T'(z(\theta))}\bigg|_\theta &= \int_{z>z(\theta)} \frac{\lambda(\theta)}{\lambda}\left(\partial_{z^*}c_i - \frac{1}{\lambda}\frac{d\lambda}{dq_i}\right) f(z)dz \\ &+ \int_{z>z(\theta)} z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)(1-T')\partial_{z^*z^*}c_i\, f(z)dz \end{aligned}$$

Finally, using $\int 1-\alpha-\lambda(\theta)/\lambda f(z)dz = 0$ or $(1-\alpha)\lambda = \mathbb{E}(\lambda(\theta))$ and:

$$\begin{aligned} 0 &= \int \frac{\lambda(\theta)}{\lambda}\left(\partial_{z^*}c_i - \frac{1}{\lambda}\frac{d\lambda}{dq_i}\right) f(z)dz + \int z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)(1-T')\partial_{z^*z^*}c_i\, f(z)dz \\ \frac{1}{\lambda}\frac{d\lambda}{dq_i} &= \int \frac{\lambda(\theta)}{\mathbb{E}(\lambda(\theta))}\partial_{z^*}c_i f(z)dz + \frac{1}{1-\alpha}\int z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)(1-T')\partial_{z^*z^*}c_i\, f(z)dz \end{aligned}$$

Finally, using $-\int_{z>z(\theta)} \lambda(\theta)/\lambda f(z)dz = z\tilde{\zeta}f(z)(T'/(1-T')+\alpha) - \int_{z>z(\theta)} 1-\alpha\, f(z)dz$, and denoting $g(\theta) = \lambda(\theta)/\mathbb{E}(\lambda(\theta))$, we have:

$$\begin{aligned} z\tilde{\zeta} f(z) \ \frac{d}{dq_i}\frac{T'(z(\theta))}{1-T'(z(\theta))}\bigg|_\theta &= (1-\alpha)\, \mathbb{E}^g_{z>z(\theta)}(\partial_{z^*}c_i - \mathbb{E}^g(\partial_{z^*}c_i)) \\ +\mathbb{E}_{z>z(\theta)}&\left(z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)(1-T')\partial_{z^*z^*}c_i - \mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)(1-T')\partial_{z^*z^*}c_i\right)\right) \\ &-\frac{1}{1-\alpha}z\tilde{\zeta}f(z)(T'/(1-T')+\alpha)\mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)(1-T')\partial_{z^*z^*}c_i\right) \end{aligned}$$

Multiplying by q_i gives the formula of proposition 4.

Proof of Proposition 5. To derive the formula of proposition 5, we need to rewrite the second and third line of proposition 4 using the the FOC characterizing the optimal tax schedule. For any deviation $dT(z)$ of the tax schedule for $z > z(\theta)$, we have:

$$\begin{aligned} \int_{z>z(\theta)} z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)dT'(z)f(z)dz + z\tilde{\zeta}f(z)\left(\frac{T'}{1-T'}+\alpha\right)dT(z(\theta)) \\ = (1-\alpha)\int_{z>z(\theta)}(1-g)dT(z)f(z)dz \end{aligned}$$

In particular, for $dT(z) = \partial_{z^*} c_i$ since $dT'(z) = (1-T')\partial_{z^*z^*} c_i$, we have:

$$
\begin{aligned}
(1-\alpha)\, \mathbb{E}^g_{z>z(\theta)}(\partial_{z^*} c_i) =& (1-\alpha)\mathbb{E}_{z>z(\theta)}(\partial_{z^*} c_i) - z\tilde{\zeta} f(z) \left(\frac{T'}{1-T'} + \alpha\right) \partial_{z^*} c_i \\
& - \mathbb{E}_{z>z(\theta)} \left(z\tilde{\zeta} \left(\frac{T'}{1-T'} + \alpha\right) (1-T')\partial_{z^*z^*} c_i \right)
\end{aligned}
$$

Similarly, since $\tilde{\mu}(\underline{\theta}) = 0$, we have:

$$
\mathbb{E}^g(\partial_{z^*} c_i) = \mathbb{E}(\partial_{z^*} c_i) - \frac{1}{1-\alpha} \mathbb{E} \left(z\tilde{\zeta} \left(\frac{T'}{1-T'} + \alpha\right) (1-T')\partial_{z^*z^*} c_i \right)
$$

Therefore:

$$
\begin{aligned}
-(1-\alpha)\mathbb{E}^g_{z>z(\theta)} \left(\mathbb{E}^g(\partial_{z^*} c_i)\right) = & -(1-\alpha)\mathbb{E}_{z>z(\theta)} \left(\mathbb{E}(\partial_{z^*} c_i) - \frac{1}{1-\alpha} \mathbb{E} \left(z\tilde{\zeta} \left(\frac{T'}{1-T'} + \alpha\right) (1-T')\partial_{z^*z^*} c_i \right) \right) \\
& + z\tilde{\zeta} f(z) \left(\frac{T'}{1-T'} + \alpha\right) \left(\mathbb{E}(\partial_{z^*} c_i) - \frac{1}{1-\alpha} \mathbb{E} \left(z\tilde{\zeta} \left(\frac{T'}{1-T'} + \alpha\right) (1-T')\partial_{z^*z^*} c_i \right) \right)
\end{aligned}
$$

Putting everything together, we get:

$$
\begin{aligned}
z\tilde{\zeta}\, f(z) \left. \frac{d}{dq_i} \frac{T'(z(\theta))}{1-T'(z(\theta))} \right|_\theta = & (1-\alpha)\, \mathbb{E}_{z>z(\theta)}(\partial_{z^*} c_i - \mathbb{E}(\partial_{z^*} c_i)) \\
& - z\tilde{\zeta} f(z) \left(\frac{T'}{1-T'} + \alpha\right) (\partial_{z^*} c_i - \mathbb{E}(\partial_{z^*} c_i))
\end{aligned}
$$

Next we determine the sign of $d_{q_i} T'$. First note that its sign is the same as $z\tilde{\zeta} f(z) d_{q_i}(T'/(1-T')$ which can be rewritten:

$$
(1-\alpha)\, \mathbb{E}_{z>z(\theta)}(\partial_{z^*} c_i - \mathbb{E}(\partial_{z^*} c_i)) - (1-\alpha)\mathbb{E}_{z>z(\theta)}(1-g) \left(\partial_{z^*} c_i - \mathbb{E}(\partial_{z^*} c_i)\right)
$$

First note that the first term is negative (positive) when $\partial^*_z c_i$ decreases (increases) with $z(\theta)$ (its derivative is $f(z)(\mathbb{E}(\partial_{z^*} c_i) - (\partial_{z^*} c_i)$ so it is U-shaped in the first case and inverse U-shaped in the second and its value is 0 at $\underline{\theta}$ and $\bar{\theta}$). When $\partial_{z^*} c_i$ is decreasing, the second term is negative for $z < z^m$ where z^m is such that $\partial_{z^*} c_i(z^m - T(z^m), q) = \mathbb{E}(\partial_{z^*} c_i)$ and positive for $z > z^m$. Therefore $d_{q_i} T'$ is negative for $z < z^m$ when $\partial_{z^*} c_i$ is decreasing. Similarly $d_{q_i} T'$ is positive for $z < z^m$ when $\partial_{z^*} c_i$ is increasing. To determine the sign for $z > z^m$, we simply examine the derivative of the full expression which is:

$$
(1-\alpha) \left(-f(z(\theta)) g(z(\theta)) (\partial_{z^*} c_i - \mathbb{E}(\partial_{z^*} c_i)) - \mathbb{E}_{z>z(\theta)}(1-g)(1-T')\partial_{z^*z^*} c_i \right)
$$

The derivative is positive (negative) for $z > z^m$ when $\partial_{z^*} c_i$ is decreasing (increasing). Since the expression converges to 0 at $z(\bar{\theta})$ (possibly infinite), this proves the claim.

Finally if $fg(z(\underline{\theta})) > 0$ inspecting the derivative directly shows that it is negative (positive) at the bottom of the distribution when $\partial_{z^*} c_i$ is decreasing (increasing).

Proofs of Section 4.2 To derive the response of consumption to achange in prices – including the response of tax rates in partial equilibrium – it is easier to consider the change in taxes at fixed z rather than a fixed θ. Propositions 6 characterizes explicitly how the tax rate responds to a change in prices, at a given *percentile* of the income distribution. This characterization is not equivalent to the change in the tax schedule across the income distribution, since it omits the labor supply response of agents.

Lemma 1. *The change in tax rate at income z in response to an increase in the consumer price of good i is given by:*

$$\frac{\tilde{\zeta}}{\zeta}\frac{q_i}{1-T'}\frac{\partial T'}{\partial q_i}\bigg|_z = \frac{z\tilde{\zeta}T''}{1-T'}\partial_{z^*}e_i + \frac{q_i}{1-T'}\frac{\partial T'}{\partial q_i}\bigg|_\theta$$

Where θ is such that $z(\theta) = z$ before the price change.

The tax schedule is altered so that agents face exactly the tax rates derived in Proposition 6. Let us ignore for a moment the effect of prices. When the tax rate at z is increased by dT', the agent decreases her labor supply by $z\tilde{\zeta}dT'/(1-T')$ and is therefore subject to a new tax rate $dT'(1-\tilde{\zeta}T''/(1-T')) = \tilde{\zeta}/\zeta dT'$. The coefficient $\tilde{\zeta}/\zeta$ in the corollary therefore simply accounts for the change in labor supply, and the wedge on agents' incomes remains the same as in Proposition 5. In addition, when the price of i changes, the real wage of the agent decreases and the agent works less. The impact of a price change on labor supply is given by $-z\tilde{\zeta}(1-T')\partial_{z^*}e_i$. The term $z\tilde{\zeta}T''/(1-T')\partial_{z^*}e_i$ in the corollary corrects for the change in tax rate induced by the effect of prices on labor supply. Note that when the tax rate is initially constant, we then have:

$$\frac{\partial T'}{\partial q_i}\bigg|_z = \frac{\partial T'}{\partial q_i}\bigg|_\theta,$$

i.e. the correction of the tax schedule simply account for the interaction between labor supply and the non-linearity of the tax rate.

Finally, note that, in the homothetic case, even if the tax rate is unchanged at a given percentile of the income distribution, the tax schedule is modified. Indeed, we then have:

$$\frac{\partial T'}{\partial q_i}\bigg|_z = T''z\zeta\partial_{z^*}c_i.$$

This is a wave equation, meaning that the tax rate at $q_i' > q_i$ is such that $T'(z)[q_i'] = T'(a(q_i', q_i)z)[q_i]$, where $a(q_i', q_i)$ is a constant independent of z. Even with homothetic utility, the tax schedule is altered to correct for changes in labor supply.

Proof of Lemma 1. The proof is direct using the result of Saez (2002). A marginal change in the price q_i is equivalent to a change in tax $dT(z) = -c_i dq_i$ for both welfare and labor supply. Therefore we have:

$$\frac{dz(\theta)}{dq_i} = -z\tilde{\zeta}\left(\frac{1}{1-T'}\frac{dT'}{dq_i}\bigg|_z - \partial_{z^*}c_i\right)$$

Which gives:

$$\left.\frac{dT'}{dq_i}\right|_\theta = \left.\frac{dT'}{dq_i}\right|_z - T''z\tilde{\zeta}\left(\frac{1}{1-T'}\left.\frac{dT'}{dq_i}\right|_z - \partial_{z^*}c_i\right)$$
$$\Rightarrow \frac{\tilde{\zeta}}{\zeta}\left.\frac{dT'}{dq_i}\right|_z = \left.\frac{dT'}{dq_i}\right|_\theta + z\tilde{\zeta}T''\partial_{z^*}c_i$$

To streamline notations, it is useful to introduce the notion of "change in real wedge" to summarize both effects and derive the response of aggregate consumption and labor supply to price changes. Conceptually, the change in real wedge captures the change in the marginal return to labor supply after a price change, accounting for both the price change itself and the induced tax response.

Definition A.1. *The change in real wedge $\tilde{T}'$ at z in response to an increase in the price of i is defined as:*

$$\frac{q_i}{1-T'}\left.\frac{\partial\tilde{T}'}{\partial q_i}\right|_z := \partial_{z^*}e_i + \frac{q_i}{1-T'}\left.\frac{\partial T'}{\partial q_i}\right|_z$$

The change in real wedge can be expressed as:

$$\frac{\tilde{\zeta}}{\zeta}\frac{q_i}{1-T'}\left.\frac{\partial\tilde{T}'}{\partial q_i}\right|_z = (1-\alpha)(1-T')\left(\frac{1}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(\partial_{z^*}e_i - \mathbb{E}\left(\partial_{z^*}e_i\right)\right) + \partial_{z^*}e_i - \mathbb{E}\left(\partial_{z^*}e_i\right)\right) + \mathbb{E}\left(\partial_{z^*}e_i\right)$$

Finally, note that we have:

$$\frac{\tilde{\zeta}}{\zeta}\frac{q_i}{1-T'}\left.\frac{\partial\tilde{T}'}{\partial q_i}\right|_z = \partial_{z^*}e_i + \frac{q_i}{1-T'}\left.\frac{\partial T'}{\partial q_i}\right|_\theta$$

Proof of Proposition 6. The joint effect of the tax and price change on individual consumption (noting that $d_z c_j dz = (1-T')\partial_{z^*}c_j dz$ since preferences are weakly separable) is given by:

$$\frac{dc_j}{dq_i} = -\partial_{z^*}c_j\left(z\tilde{\zeta}\left\{(1-T')\partial_{z^*}c_i + \left.\frac{dT'}{dq_i}\right|_z\right\} + c_i + \frac{dT}{dq_i}\right) + \frac{\partial c_j^h}{\partial q_i}$$

Where the last term corresponds to the standard price derivative of Hicksian demand. The aggregate response is then given by:

$$\frac{dC_j}{dq_i} = -\mathbb{E}\left(\partial_{z^*}c_j\left(z\tilde{\zeta}\left\{(1-T')\partial_{z^*}c_i + \left.\frac{dT'}{dq_i}\right|_z\right\} + c_i + \frac{dT}{dq_i}\right)\right) + \frac{\partial C_j^h}{\partial q_i}$$

We have:

$$
\begin{aligned}
\mathbb{E}\left(\partial_{z^*}c_j\left(c_i+\frac{dT}{dq_i}\right)\right) &= \mathbb{E}\left(\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)\right)\left(c_i+\frac{dT}{dq_i}\right)\right)+\mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)\\
&= \mathbb{E}\left(\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)\right)\int_{z(\underline{\theta})}^{z}\left((1-T')\partial_{z^*}c_i+\left.\frac{dT'}{dq_i}\right|_z\right)dz\right)+\mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)\\
&= \int\left((1-T')\partial_{z^*}c_i+\left.\frac{dT'}{dq_i}\right|_z\right)\mathbb{E}_{\tilde{z}>z}\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)\right)\,dz+\mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)
\end{aligned}
$$

Next to express the second term in the equation above in terms of dT' we use the government's budget constraint:

$$
0=\mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)-\mathbb{E}\left(z\tilde{\zeta}\frac{T'}{1-T'}\left((1-T')\partial_{z^*}c_i+\left.\frac{dT'}{dq_i}\right|_z\right)\right)+\sum_j\left(q_j-p_j\left(1-\sum_k\frac{p_kC_k}{p_jC_j}A_{k,j}\right)\right)\frac{dC_j}{dq_i}
$$

Where we used the the fact that the elasticity of p_k with respect to C_j is $-A_{k,j}$. Using the definition of optimal commodity taxes, this gives:

$$
\begin{aligned}
0 &= \mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)-\mathbb{E}\left(z\tilde{\zeta}\frac{T'}{1-T'}\left((1-T')\partial_{z^*}c_i+\left.\frac{dT'}{dq_i}\right|_z\right)\right)+\sum_j\alpha q_j\frac{dC_j}{dq_i}\\
&= \mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)-\mathbb{E}\left(z\tilde{\zeta}\frac{T'}{1-T'}\left((1-T')\partial_{z^*}c_i+\left.\frac{dT'}{dq_i}\right|_z\right)\right)\\
&\quad -\alpha\mathbb{E}\left(z\tilde{\zeta}\,(1-T')\left\{\partial_{z^*}c_i+\frac{1}{1-T'}\left.\frac{dT'}{dq_i}\right|_z\right\}+c_i+\frac{dT}{dq_i}\right)
\end{aligned}
$$

Where the second line simply uses $q\cdot\partial_{z^*}c=1$ and $q\cdot\partial_{q_i}c^h=0$. Therefore, we have:

$$
\mathbb{E}\left(c_i+\frac{dT}{dq_i}\right)=\frac{1}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}(T'+\alpha(1-T'))\left(\partial_{z^*}c_i+\frac{1}{1-T'}\left.\frac{dT'}{dq_i}\right|_z\right)\right)
$$

Putting everything together, we can rewrite dC_j/dq_i using the definition of $\partial\tilde{T}'/\partial q_i$:

$$
\begin{aligned}
\frac{dC_j}{dq_i}-\frac{\partial C_j^h}{\partial q_i} &= -\mathbb{E}\left(z\tilde{\zeta}\frac{1}{1-T'}\frac{\partial\tilde{T}'}{\partial q_i}(1-T')\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)+\frac{1}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)\right)\right)\right)\\
&\quad -\mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(z\tilde{\zeta}\frac{1}{1-T'}\frac{\partial\tilde{T}'}{\partial q_i}\left(1-T'+\frac{T'+\alpha(1-T')}{1-\alpha}\right)\right)\\
&= -\mathbb{E}\left(z\tilde{\zeta}\frac{1}{1-T'}\frac{\partial\tilde{T}'}{\partial q_i}(1-T')\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)+\frac{1}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(\partial_{z^*}c_j-\mathbb{E}(\partial_{z^*}c_j)\right)\right)\right)\\
&\quad -\frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(z\tilde{\zeta}\frac{1}{1-T'}\frac{\partial\tilde{T}'}{\partial q_i}\right)
\end{aligned}
$$

Finally, using the definition of $\partial_{q_j}\tilde{T}'$ we have:

$$\frac{dC_j}{dq_i} = -\frac{1}{1-\alpha}\mathbb{E}\left(\frac{\tilde{\zeta}}{\zeta}\frac{\partial_{q_i}\tilde{T}}{1-T'}\frac{\partial_{q_j}\tilde{T}}{1-T'}z\tilde{\zeta}\right) + \frac{\partial C_j^h}{\partial q_i}$$

Using the definition of the real wedge $\frac{q_i}{1-T'}\frac{\partial \tilde{T}'}{\partial q_i}\Big|_z := \partial_{z^*}e_i + \frac{q_i}{1-T'}\frac{\partial T'}{\partial q_i}\Big|_z$ and of τ_i^{nh} we obtain the formula of proposition 6.

Proof of Proposition 7 and Lemma 2. To distinguish endogenous and exogenous changes in prices, we denote dp_i^* the exogenous change in p_i. Since we have already determined the impact of consumer prices on taxation, we first need to determine the effect of the endogenous change in α. Going back to the FOC associated with V we have:

$$\begin{aligned} f(z)z\tilde{\zeta}\frac{\partial}{\partial\alpha}\left\{\frac{T'}{1-T'}\right\} + f(z)z\tilde{\zeta} &= -\int_{z'>z}\left(1-(1-\alpha)\frac{\lambda(\theta)}{\mathbb{E}(\lambda(\theta))}\left\{\frac{1}{\lambda}\frac{\partial\lambda}{\partial\alpha}\right\}\right)f(z)dz \\ &= -\int_{\theta'>\theta}\left(1-\frac{\lambda(\theta)}{\mathbb{E}(\lambda(\theta))}\right)\pi(\theta)d\theta \end{aligned}$$

Where the second line directly use the definition of λ to derive $1/\lambda\partial_\alpha\lambda = 1/(1-\alpha)$. So we directly have:

$$\frac{\partial}{\partial\alpha}\left\{\frac{T'}{1-T'}\right\} = -\frac{1}{(1-\alpha)(1-T')}$$

We therefore have (where all derivatives are taken at fixed θ):

$$\frac{d}{dp_i^*}\left\{\frac{T'}{1-T'}\right\} = \sum_j \frac{d}{dq_j}\left\{\frac{T'}{1-T'}\right\}\frac{dq_j}{dp_i^*} + \frac{d}{d\alpha}\left\{\frac{T'}{1-T'}\right\}\frac{d\alpha}{dp_i^*}$$

To conclude we need dq_j/dp_i^* and $d\alpha/dp_i^*$.

First, we derive the response of aggregate consumption to a change in α holding all consumer prices fixed. We have:

$$\frac{1}{1-T'}\left.\frac{\partial T'}{\partial\alpha}\right|_z = \frac{1}{1-T'}\left.\frac{\partial T'}{\partial\alpha}\right|_z = \frac{\zeta}{\tilde{\zeta}}\frac{1}{1-T'}\left.\frac{\partial T'}{\partial\alpha}\right|_\theta = -\frac{\zeta}{\tilde{\zeta}}\frac{1}{1-\alpha}$$

As before, the response of consumption can be expressed in terms of $d\tilde{T}$:

$$\begin{aligned} \frac{\partial C_i}{\partial\alpha} &= -\mathbb{E}\left(\partial_{z^*}c_i z\tilde{\zeta}\frac{\partial T}{\partial\alpha}\right) - \mathbb{E}\left(\partial_{z^*}c_i\frac{\partial T}{\partial\alpha}\right) \\ &= -\mathbb{E}\left(\partial_{z^*}c_i\, z\tilde{\zeta}\frac{\partial T}{\partial\alpha}\right) - \int\left(\frac{\partial T'}{\partial\alpha}\mathbb{E}_{z'>z}(\partial_{z^*}c_i - \mathbb{E}(\partial_{z^*}c_i)\right)dz - \mathbb{E}(\partial_{z^*}c_i)\mathbb{E}\left(\frac{\partial T}{\partial\alpha}\right) \end{aligned}$$

Using again the government budget constraint gives:

$$\begin{aligned} &\mathbb{E}\left(\frac{\partial T}{\partial \alpha}\right) - \mathbb{E}\left(z\tilde{\zeta}\frac{T'}{1-T'}\frac{\partial T'}{\partial \alpha}\right) - \alpha\mathbb{E}\left(z\tilde{\zeta}\frac{\partial T'}{\partial \alpha} + \frac{\partial T}{\partial \alpha}\right) = 0 \\ \Rightarrow \quad &\mathbb{E}\left(\frac{\partial T}{\partial \alpha}\right) = \frac{1}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'} + \alpha\right)\frac{\partial T'}{\partial \alpha}\right) \end{aligned}$$

So the response of aggregate consumption is given by:

$$\begin{aligned} \frac{\partial C_i}{\partial \alpha} &= -\frac{1}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\,\frac{\tilde{\zeta}}{\zeta}\frac{1}{1-T'}\frac{\partial \tilde{T}'}{\partial q_i}\,\frac{1}{1-T'}\frac{\partial T'}{\partial \alpha}\right) \\ &= \frac{1}{(1-\alpha)^2}\mathbb{E}\left(\frac{z\tilde{\zeta}}{1-T'}\frac{\partial \tilde{T}'}{\partial q_i}\right) \end{aligned}$$

Finally, the exogenous change in p_i, keeping α and the consumer prices fixed only has an effect on consumption through the change in the intercept of the tax schedule (keeping government budget constant). It is simply given by $dT_0 = 1/(1-\alpha)C_i dp_i^*$ so we have:

$$\frac{\partial C_j}{\partial p_i^*} = -\frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)C_i$$

Now we determine the changes in α and q. We use the specification of the proposition, with a constant and diagonal matrix Δ_α. We start with consumer prices, the equilibrium definition of q_i and the supply side relationship between p_i and C_i give:

$$\frac{1}{q_j}\frac{dq_j}{dp_i^*} = \frac{1}{p_j}\frac{dp_j}{dp_i^*} + \frac{1}{1-\alpha}\frac{d\alpha}{dp_i^*} \qquad \textit{and} \qquad \frac{1}{p_j}\frac{dp_j}{dp_i^*} = -\alpha_j\frac{1}{C_j}\frac{dC_j}{dp_i^*} + \mathbb{1}_{i=j}\frac{1}{p_i}$$

To determine prices, we use these 2 relationship and our expression for the response of aggregate demand to q_j, p_i and α:

$$\frac{1}{q_j}\frac{dq_j}{dp_i^*} = -\alpha_j\left(\sum_k \frac{q_k}{C_j}\frac{dC_j}{dq_k}\frac{1}{q_k}\frac{dq_k}{dp_i^*} + \frac{1}{C_j}\frac{\partial C_j}{\partial \alpha}\frac{d\alpha}{dp_i^*} - \frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)\frac{C_i}{C_j}\right) + \frac{1}{1-\alpha}\frac{d\alpha}{dp_i^*} + \mathbb{1}_{i=j}\frac{1-\alpha_i}{1-\alpha}\frac{1}{q_i}$$

This expression can be simplified and made independent of $d\alpha$, indeed, we have:

$$\begin{aligned} \sum_k \frac{q_k}{C_j}\frac{dC_j}{dq_k} &= \frac{-1}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\frac{\partial_{q_j}\tilde{T}}{1-T'}\sum_k \frac{\tilde{\zeta}}{\zeta}\frac{q_k\partial_{q_k}\tilde{T}}{1-T'}\right) \\ &= \frac{-1}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\frac{\partial_{q_j}\tilde{T}}{1-T'}\right) = -(1-\alpha)\frac{\partial C_j}{\partial \alpha} \end{aligned}$$

Therefore, defining $1/q_j d\tilde{q}_j/dp_i^* = 1/q_j dq_j/dp_i^* - (1-\alpha)^{-1}d\alpha/dp_i^*$, we have:

$$\frac{1}{q_j}\frac{d\tilde{q}_j}{dp_i^*} = -\alpha_j\left(\sum_k \frac{q_k}{C_j}\frac{dC_j}{dq_k}\frac{1}{q_k}\frac{d\tilde{q}_k}{dp_i^*} - \frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)\frac{C_i}{C_j}\right) + \mathbb{1}_{i=j}\frac{1-\alpha_i}{1-\alpha}\frac{1}{q_i}$$

So we have (abusing notations):

$$\left[\frac{1}{q_j}\frac{d\tilde{q}_j}{dp_i^*}\right] = (Id + \Delta_\alpha \mathcal{C})^{-1}\left[\frac{\alpha_j}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)\frac{C_i}{C_j} + \mathbb{1}_{i=j}\frac{1-\alpha_i}{1-\alpha}\frac{1}{q_i}\right]$$

Since in addition $\sum_j \frac{q_j \partial}{\partial q_j}\left\{\frac{T'}{1-T'}\right\} = 0$, we have $\sum_j \frac{q_j \partial}{\partial q_j}\left\{\frac{T'}{1-T'}\right\}\frac{1}{q_j}\frac{d\tilde{q}_j}{dp_i^*} = \sum_j \frac{q_j \partial}{\partial q_j}\left\{\frac{T'}{1-T'}\right\}\frac{1}{q_j}\frac{dq_j}{dp_i^*}$. Redefining the endogenous change in consumer prices , $1/q_j dq_j^e/dp_i^* = 1/q_j d\tilde{q}_j/dp_i^* - \mathbb{1}_{i=j}\frac{1-\alpha_i}{1-\alpha}\frac{1}{q_i}$ gives the set of prices defined in Proposition 7.

To conclude, we derive the response of α in terms of dq_j^e/dp_i^*. By definition we have:

$$\alpha = \frac{\sum \alpha_i p_i C_i}{\sum p_i C_i}$$

Therefore:

$$\frac{d\alpha}{dp_i^*} = \frac{\sum_j (\alpha_j - \alpha) p_j C_j \left(\frac{1}{p_j}\frac{dp_j}{dp_i^*} + \frac{1}{C_j}\frac{dC_j}{dp_i^*}\right)}{\sum_j p_j C_j}$$

Using the definition of dq_j^e/dp_i^*, we have:

$$\frac{1}{q_j}\frac{dq_j^e}{dp_i^*} = \frac{1}{p_j}\frac{dp_j}{dp_i^*} - \mathbb{1}_{i=j}\frac{1}{p_i} = -\alpha_i \frac{1}{C_j}\frac{dC_j}{dp_i^*}$$

Therefore:

$$\frac{d\alpha}{dp_i^*} = \frac{\sum_j (\alpha_j - \alpha)(1 - \frac{1}{\alpha_j}) p_j C_j \frac{1}{q_j}\frac{dq_j^e}{dp_i^*}}{\sum_j p_j C_j} + \frac{C_i(\alpha_i - \alpha)}{\sum_j p_j C_j}$$

$$\frac{1}{1-\alpha}\frac{d\alpha}{dp_i^*} = \frac{\sum_j q_j C_j (\frac{\alpha}{\alpha_j} - 1)\frac{1}{q_j}\frac{dq_j^e}{dp_i^*}}{\sum_j q_j C_j} + \frac{\alpha_i - \alpha}{1-\alpha}\frac{C_i}{\sum_j q_j C_j}$$

This proves the first part of Proposition 7. To express the response of the tax rate at z, we simply use Lemma 1:

$$\frac{\tilde{\zeta}}{\zeta}\frac{q_i}{1-T'}\left.\frac{\partial T'}{\partial q_i}\right|_z = \frac{z\tilde{\zeta}T''}{1-T'}\left(\partial_{z^*}e_i\frac{1-\alpha_i}{1-\alpha} + \sum_j \partial_{z^*}e_i\frac{dq_j^e}{dp_i^*} + \frac{1}{1-\alpha}\frac{d\alpha}{dp_i^*}\right) + \frac{q_i}{1-T'}\left.\frac{\partial T'}{\partial q_i}\right|_\theta$$

The proof of Lemma 2 follows exactly the same steps.

Finally, we consider the general case where $p_i = \phi_i(C_1, ..., C_n)$. Denoting the exogenous change in

p_i, dp_i^*, we have:

$$\frac{dp_j}{dp_i^*} = \mathbb{1}(i=j) - p_j \sum A_{jk} \frac{1}{C_k} \frac{dC_k}{dp_i^*}$$

and the change in aggregate demand for goods is given as before by:

$$\frac{1}{C_j}\frac{dC_j}{dp_i^*} = \sum_k \frac{q_k}{C_j}\frac{dC_j}{dq_k}\frac{1}{q_k}\frac{dq_k}{dp_i^*} + \frac{\partial C_j}{\partial \alpha}\frac{d\alpha}{dp_i^*} - \frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*} c_j)\frac{C_i}{C_j}$$

with dC_j/dq_k and $\partial C_j/\partial\alpha$ defined as before. The equilibrium consumer prices are determined by:

$$q_i = \frac{p_i}{1-\alpha}\left(1 + \sum_j \frac{\partial_{Q_i}\phi_j C_i}{\phi_j}\frac{p_j C_j}{p_i C_i}\right)$$

Since $\frac{\partial_{Q_i}\phi_j C_i}{\phi_j}$ only depends on $C_1, ..., C_n$, we have:

$$\begin{aligned}\frac{dq_j}{dp_i^*} &= \frac{1}{1-\alpha}\frac{d\alpha}{dp_i^*}q_j + \frac{1}{p_j}\frac{dp_j}{dp_i^*}q_j \\ &- \frac{p_j}{1-\alpha}\left(\sum_k A_{kj}\frac{p_k C_k}{p_j C_j}\left(\frac{1}{p_k}\frac{dp_k}{dp_i^*} + \frac{1}{C_k}\frac{dC_k}{dp_i^*} - \frac{1}{p_j}\frac{dp_j}{dp_i^*} + \frac{1}{C_j}\frac{dC_j}{dp_i^*}\right) + \sum_k A_{kj}\frac{p_k C_k}{p_j C_j}\sum_l a^l_{kj}\frac{1}{C_l}\frac{dC_l}{dp_i^*}\right)\end{aligned}$$

with $a^l_{kj} = Q_l \partial_{Q_l} A_{kj}/A_{kj}$ is the elasticity of the price elasticity of product k with respect to market size j. As before, defining $1/q_j d\tilde{q}_j/dp_i^* = 1/q_j dq_j/dp_i^* - (1-\alpha)^{-1} d\alpha/dp_i^*$, we have:

$$\begin{aligned}\frac{1}{C_j}\frac{dC_j}{dp_i^*} &= \sum_k \frac{q_k}{C_j}\frac{dC_j}{dq_k}\frac{1}{q_k}\frac{d\tilde{q}_k}{dp_i^*} - \frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*} c_j)\frac{C_i}{C_j} \\ \frac{1}{q_j}\frac{d\tilde{q}_j}{dp_i^*} - \frac{1}{p_j}\frac{dp_j}{dp_i^*} &= -\frac{p_j/q_j}{1-\alpha}\left(\sum_k A_{kj}\frac{p_k C_k}{p_j C_j}\left(\frac{1}{p_k}\frac{dp_k}{dp_i^*} + \frac{1}{C_k}\frac{dC_k}{dp_i^*} - \frac{1}{p_j}\frac{dp_j}{dp_i^*} + \frac{1}{C_j}\frac{dC_j}{dp_i^*}\right)\right. \\ &\left. + \sum_k A_{kj}\frac{p_k C_k}{p_j C_j}\sum_l a^l_{kj}\frac{1}{C_l}\frac{dC_l}{dp_i^*}\right)\end{aligned}$$

Therefore, denoting $\tilde{A}^0$ and $\tilde{A}^1$ the matrix with entry $\tilde{A}^0_{ij} = ((1-\alpha)(1+t_i))^{-1}(A_{ji}\frac{p_j C_j}{p_i C_i} - \mathbb{1}(i=j)\sum_k A_{ki}\frac{p_k C_k}{p_i C_i})$ and $\tilde{A}^1_{ij} = ((1-\alpha)(1+t_i))^{-1}\sum_k A_{ki}\frac{p_k C_k}{p_i C_i}a^j_{ki})$, we have:

$$\left[\frac{1}{p}\frac{dp}{dp_i^*}\right] = \mathbb{1}(i=j)(1+t_i)\frac{1}{q_i} - A\mathcal{C}\left[\frac{1}{q}\frac{d\tilde{q}}{dp_i^*}\right] + \frac{1}{1-\alpha}A\left[\mathbb{E}(\partial_{z^*}c)\frac{C}{C_i}\right]$$

And the consumer price changes are given by:

$$\left[\frac{1}{q}\frac{d\tilde{q}}{dp_i^*}\right] = (Id + \tilde{A}\mathcal{C})^{-1}\left((Id + \tilde{A}^0)\left[\mathbb{1}(i=j)(1+t_i)\frac{1}{q_i}\right] + \frac{1}{1-\alpha}\tilde{A}\left[\mathbb{E}(\partial_{z^*}c)\frac{C}{C_i}\right]\right)$$

With $\tilde{A} = A + \tilde{A}^0(Id - A) + \tilde{A}^1$. The change in tax is given by:

$$\frac{d}{dp_i^*}\left\{\frac{T'}{1-T'}\right\} = \sum_j \frac{d}{dq_j}\left\{\frac{T'}{1-T'}\right\}\frac{d\tilde{q}_j}{dp_i^*} + \frac{d}{d\alpha}\left\{\frac{T'}{1-T'}\right\}\frac{d\alpha}{dp_i^*}$$

with

$$\frac{d\alpha}{dp_i^*} = \frac{\sum_j(\sum_k A_{jk} - \alpha)p_jC_j(dln(p_j)/dp_i^* + dln((C_j)/dp_i^*)}{\sum p_jC_j} + \frac{\sum_j(\sum_k A_{jk}\sum_l(a_{jk}^l)dln((C_l)/dp_i^*)p_jC_j}{\sum p_jC_j}$$

Proof of Corollary 1. The proof of (1) in Corollary 1 follows exactly the Proof of Proposition 4 and 5. To derive aggregate consumption, we follow Proposition 6, we have:

$$\frac{dC_j}{dq_i} = -\mathbb{E}\left(\partial_{z^*}c_j\left(z\tilde{\zeta}\left\{(1-T')\partial_{z^*}c_i + \left.\frac{dT'}{dq_i}\right|_z\right\} + c_i + \frac{dT}{dq_i}\right)\right) + \frac{\partial C_j^h}{\partial q_i}$$

and

$$\begin{aligned}
&\mathbb{E}\left(\partial_{z^*}c_j\left(c_i + \frac{dT}{dq_i}\right)\right) = \mathbb{E}\left(\left(\partial_{z^*}c_j - \mathbb{E}(\partial_{z^*}c_j)\right)\left(c_i + \frac{dT}{dq_i}\right)\right) + \mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(c_i + \frac{dT}{dq_i}\right)\\
&= \int\left((1-T')\partial_{z^*}c_i + \left.\frac{dT'}{dq_i}\right|_z\right)\mathbb{E}_{\tilde{z}>z}\left(\partial_{z^*}c_j - \mathbb{E}(\partial_{z^*}c_j)\right)\,dz + \mathbb{E}(\partial_{z^*}c_j)\mathbb{E}\left(c_i + \frac{dT}{dq_i}\right)
\end{aligned}$$

Next to express the second term in the equation above in terms of dT' we use the feasibility constraint:

$$\begin{aligned}
0 &= \sum_j \partial_{Q_j}\chi\frac{dC_j}{dq_i} + \mathbb{E}\left(z\tilde{\zeta}\left(\frac{1}{1-T'}\frac{dT'}{dq_i} + \partial_{z^*}c_i\right)\right)\\
&= -\mathbb{E}\left(z\tilde{\zeta}\left((1-T')\partial_{z^*}c_i + \frac{dT'}{dq_i}\right) + c_i + \frac{dT}{dq_i}\right) + \mathbb{E}\left(z\tilde{\zeta}\left(\frac{1}{1-T'}\frac{dT'}{dq_i} + \partial_{z^*}c_i\right)\right)\\
\Rightarrow\quad \mathbb{E}\left(c_i + \frac{dT}{dq_i}\right) &= -\mathbb{E}\left(z\tilde{\zeta}T'\left(\frac{1}{1-T'}\frac{dT'}{dq_i} + \partial_{z^*}c_i\right)\right)
\end{aligned}$$

Therefore as before, we have:

$$\frac{dC_j}{dq_i} = -\mathbb{E}\left(\frac{\tilde{\zeta}}{\zeta}\frac{\partial_{q_i}\tilde{T}}{1-T'}\frac{\partial_{q_j}\tilde{T}}{1-T'}z\tilde{\zeta}\right) + \frac{\partial C_j^h}{\partial q_i}$$

The response of aggregate consumption to a change in total cost is defined by the change in the tax schedule intercept:

$$\begin{aligned}
\frac{d\chi}{dp_i^*} + \sum_j \partial_{Q_j}\chi\,\mathbb{E}(\partial_{z^*}c_j\frac{dT_0}{dp_i^*}) &= 0\\
\Rightarrow\quad \frac{\partial C_j}{\partial\chi} &= -\mathbb{E}(\partial_{z^*}c_j)\frac{d\chi}{dp_i^*}
\end{aligned}$$

To determine equilibrium prices, we then simply use:

$$\left[\frac{1}{q_j}\frac{dq_j}{dp_i^*}\right] = A\mathcal{C}\left[\frac{1}{q_j}\frac{dq_j}{dp_i^*}\right] - A\left[\frac{\mathbb{E}(\partial_{z^*}c_j)}{C_j}\frac{d\chi}{dp_i^*}\right] - [0..1/q_i..0]'$$

Which gives the prices in the main text.

A.3 Extensions

As a preliminary, we introduce a tool that we use extensively in this section: the tax rate derivative with respect to exogenous consumer price changes. Exogenous consumer price changes are abstract objects that can be interpreted as decisions, from the social planner, to arbitrarily increase or decrease the tax on good i while keeping all other commodity taxes fixed.

Lemma 2. *Consider an economy without spillovers ($p_i = \phi_i(C_i)$). The response of the optimal tax rate to an exogenous increase in the consumer price of i, that we denote dq_i^* is:*

$$\frac{d}{dq_i^*}\left\{\frac{T'}{1-T'}\right\}_\theta = \frac{\partial}{\partial q_i}\left\{\frac{T'}{1-T'}\right\}_\theta + \sum_j \frac{q_j\partial}{\partial q_j}\left\{\frac{T'}{1-T'}\right\}_\theta \frac{1}{q_j}\frac{dq_j^e}{dq_i^*} - \frac{1}{(1-\alpha)(1-T')}\frac{d\alpha}{dq_i^*}$$

Where the endogenous price response $1/q_j dq_j^e/dq_i^*$ solves:

$$\left[\frac{1}{q_i}\frac{dq_j^e}{dq_i^*} + \mathbb{1}_{i=j}\frac{1}{q_i}\right] = (Id + \Delta_\alpha \mathcal{C})^{-1}\left[\frac{1}{q_i}\right]$$

and the endogenous response to the average market size effect $d\alpha/dq_i^*$ is given by:

$$\frac{1}{1-\alpha}\frac{d\alpha}{dq_i^*} = \frac{\sum q_j C_j (\frac{\alpha}{\alpha_i} - 1)\frac{1}{q_j}\frac{dq_j}{dq_i^*}}{\sum q_j C_j}$$

The tax response at z, is given by:

$$\frac{\tilde{\zeta}}{\zeta}\frac{q_i}{1-T'}\frac{dT'}{dq_i^*}\bigg|_z = \frac{z\tilde{\zeta}T''}{1-T'}\left(\partial_{z^*}e_i + \sum \partial_{z^*}e_j \frac{dq_j}{dq_i^*}\right) + \frac{q_i}{1-T'}\frac{dT'}{dq_i^*}\bigg|_\theta$$

The tax rate responses to consumer price changes are essentially the same as the response to producer price changes. The main difference is that they do not generate amplification through their impact on government revenue.

Market Size Effects and Redistribution With the theoretical tools developed in section 4, we can now reexamine the impact of of market size effects on the tax schedule. To streamline exposition, we consider an economy where the price elasticity with respect to market size is constant across sectors ($\alpha_i = \alpha$).[1]

[1]For example, changes in the price elasticity with respect to market size could stem from technology shocks (e.g. with the rise of IT and intangible capital, as in **?**, **?**, and **?**), from competition policy (e.g., **?**), or from trade policy (e.g., **?**).

Proposition A.1. *Consider an economy with constant α_i across markets. The tax rate response to an increase in the average market size elasticity α is:*

$$\left.\frac{dT'}{d\alpha}\right|_{\theta} = -\frac{1-T'}{1-\alpha} + \sum \left.\frac{dT'}{dq_i^*}\right|_{\theta} \frac{dq_i^0}{d\alpha} \tag{1}$$

where dT'/dq_i^* is defined in Lemma 2 and the initial price reactions $dq_i^0/d\alpha$ are given by:

$$\frac{dq_i^0}{d\alpha} = -\frac{\alpha\zeta}{(1-\alpha)^2}\left(1 + \frac{1}{s_i}\mathbb{E}_z\left(\partial_{z^*}E_i - s_i + q_i\tau_i^{nh}\right)\right)$$

As discussed in section 3, a naive interpretation of the optimal tax formula of Proposition 3 is that the correction is simply a wage subsidy applied to the optimal tax rate at fixed prices. If this were the case, the tax rate at $\alpha > 0$ would be $1 - T' = (1 - T'_{fp})/(1-\alpha)$, where T'_{fp} is the tax rate at fixed prices ($\alpha = 0$), i.e. the social planner uniformly subsidize nominal net-of-tax wages $1 - T'$ by $1/(1-\alpha)$. The derivative of the of the tax rate at θ with respect to α would then be given by $dT'/d\alpha = -(1-T')/(1-\alpha)$, which is exactly the first term in equation 1. Keeping endogenous variables fixed (in particular the pareto weights g), the mechanical effect of an in increase in α is to increase the wage subsidy $1/(1-\alpha)$. However, this interpretation is incomplete: implementing the subsidy shifts consumption and leads to price changes in general equilibrium. The advantage of our comparative statics approach is to make these effects explicit.

The non-trivial interaction between the corrective tax $-(1-T')/(1-\alpha)$ and redistributive operates through prices. First, implementing the corrective tax modifies agents' disposable income, shifts consumption and, through market size effects, has a direct impact on prices, given by $dq_i^0/d\alpha$ in Proposition A.1. Second, this direct price effect leads to a new tax change, which further shifts consumption through income and substitution effects, creating new price changes, and so on. This indirect effect is summarized by the derivative of the tax rate with respect to consumer prices, which we derived in Lemma 2. Since we already know how the optimal tax rate responds to changes in consumer prices, we only need to determine the direct effect of α on prices to obtain the full response of the tax rate to a change in the market size elasticity of prices.

Since we have discussed the indirect effect in detail in subsection 4.2, we focus here on the direct effect of α on prices. Decreasing the tax rate at z by $(1-T')/(1-\alpha)$ mechanically increases post-tax income at $z' > z$ by $(z'-z) \times (1-T')/(1-\alpha)$ and, through labor supply, increases income at z by $z\tilde{\zeta}(1-T')/(1-\alpha)$. The lower taxes reduces government revenue: the intercept of the tax schedule is higher and the post-tax income of households below z is lower. Thus, aggregate income rises and the impact of the corrective tax is regressive: it increases disposable income at the top of the distribution and makes households at the bottom poorer. The initial price change dq^0 captures both effects. As aggregate income increases the price of all goods decreases by $-\alpha\zeta/(1-\alpha)^2$. However, as households earns more and that the tax rates becomes more regressive, income increases more at the top than at the bottom of the distribution: the share of luxuries increases in the economy ($\mathbb{E}_z\left(\partial_{z^*}E_i - s_i + q_i\tau_i^{nh}\right) > 0$ for luxuries, assuming $\partial_{z^*}E_i > s_i$ – which is the case when the aggregate share of luxuries does not decrease with aggregate income) while the share of necessities declines. Through this reallocation of income from luxuries to necessities, the relative price of luxuries decreases.

When producer prices become more sensitive to demand, the tax schedule therefore becomes more regressive. First, a higher α calls for a larger wage subsidy, second this larger wage subsidy induces a shift of demand away from necessities and towards luxuries through income effects. As a result the relative price of luxuries decreases and it becomes more valuable to redistribute income to higher ability households. Through the general equilibrium effects discussed in section 4.2, the increase in the relative price of luxuries is then amplified which leads to more redistribution towards higher income households. The corrective tax therefore directly affects the value of redistribution in a non trivial way through prices: the tax schedule becomes more progressive when prices are less sensitive to demand and more regressive when they are more sensitive.

The lessons for optimal tax design are more general. Corrective policies are often considered independently from redistributive ones, as they can be compensated for. In partial equilibrium, a tax on sin goods or polluting goods can be compensated for (when preferences are separable between consumption and labor) and therefore do not affect redistribution. We show that this is no longer true if prices endogenously respond to corrective policies: as prices directly impact the value of redistribution, even when the direct impact of corrective policies is compensated for, they would alter optimal redistributive policies through their effect on prices.

In the homothetic case, the price effects are nil. Indeed, as discussed in the previous section, we have $dT'/dq_i^* = 0$: there is no interaction between the corrective tax and redistribution. While this is true when considering the tax rate at a given *percentile* of the income distribution, we find that the tax schedule as a function of *income* responds in a non-trivial way to the change in the average market size elasticity α:

$$\left.\frac{dT'}{d\alpha}\right|_z = T''z\zeta\left(\partial_{z^*}c_i \cdot \frac{dq_i}{d\alpha} - \frac{1}{1-\alpha}\right) - \frac{1-T'}{1-\alpha}$$

In this wave equation, the tax rate at α is such that $1 - T'_\alpha(z) = (1 - T'_{fp}(a(\alpha)z))/(1-\alpha)$, where $a(\alpha)$ is a constant independent of z, and T'_{fp} is the tax rate at fixed prices ($\alpha = 0$). Thus, the tax schedule is adjusted to correct for changes in labor supply.

Proof of Proposition A1. We denote by $d\alpha^*$ the exogenous change in elasticity across markets. Since the change in elasticity is common and that we initially have $\alpha_i = \alpha$ for all i, then the total change in elasticity is $d\alpha = d\alpha^*$. Then using the derivation of Proposition 7, we have:

$$\left.\frac{dT'}{d\alpha^*}\right|_\theta = \sum_j \left.\frac{\partial T'}{\partial q_j}\right|_\theta \frac{dq_j}{d\alpha^*} - \frac{1-T'}{1-\alpha}$$

Next we have:

$$\frac{1}{q_j}\frac{dq_j}{d\alpha^*} = -\alpha\left(\sum_k \frac{q_k}{C_j}\frac{dC_j}{dq_k}\frac{1}{q_k}\frac{dq_k}{d\alpha^*} + \frac{1}{C_j}\frac{\partial C_j}{\partial \alpha}\right)$$

Therefore, we have:

$$\left[\frac{1}{q_j}\frac{dq_j}{d\alpha^*}\right] = (Id + \alpha\mathcal{C})^{-1}\left[\frac{1}{q_j}\frac{dq_j^0}{d\alpha^*}\right]$$

With $C_j dq_j^0/d\alpha^* = -\alpha/(1-\alpha)^2\mathbb{E}(z\tilde{\zeta}q_j\partial_{q_j}\tilde{T}'/1-T')$. Using the definition of Corollary 3 then gives the expression of the main text.

Inequality and Prices We examine how exogenous shifts in the distribution of income alter the design of optimal taxes. This allows us to evaluate the direct impact of income inequality on redistribution policies and its indirect impact, through prices. We continue to assume a linear social welfare function.[2]

For a given distribution of income f, we consider a small deviation $f + \epsilon \tilde{f}$. We denote the Gateaux derivative of a function F of f in the direction $\tilde{f}$ as $dF[\tilde{f}, f]$.

Proposition A.2. *Consider an economy with constant α_i across markets. The partial equilibrium response of the tax rate at θ to a change $\tilde{f}$ in the income distribution is:*

$$d\left\{\frac{T'}{1-T'}\right\}_{PE}[\tilde{f}, f] = \underbrace{\frac{1-\alpha}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}\left(\frac{\tilde{f}}{f} - g\left(\frac{\tilde{f}}{f} - \mathbb{E}^g\left(\frac{\tilde{f}}{f}\right)\right)\right)}_{\text{Change In the Value of Redistribution}} - \underbrace{\frac{\tilde{f}}{f}\left(\frac{T'}{1-T'} + \alpha\right)}_{\text{Change in the Efficiency Cost of Taxation}} \tag{2}$$

The general equilibrium response is given by:

$$dT'_{GE}[\tilde{f}, f] = dT'_{PE}[\tilde{f}, f] + \sum \left.\frac{dT'}{dq_i^*}\right|_{\theta} dq_i^0[\tilde{f}, f] \tag{3}$$

where $dq_i^0[\tilde{f}, f]$ is the direct effect of $\tilde{f}$ and $dT'_{PE}[\tilde{f}, f]$ on prices and is given by:

$$C_i dq_i^0[\tilde{f}, f] = \frac{\alpha}{(1-\alpha)}\left(\mathbb{E}\left(\frac{q_i \partial_{q_i}\tilde{T}'}{1-T'} z\tilde{\zeta}\frac{dT'_{PE}[\tilde{f}, f]}{1-T'}\right) - \mathbb{E}(\partial_{z^*}e_i)\mathbb{E}\left(\frac{\tilde{f}}{f}((1-\alpha)T + \alpha z)\right)\right) - \alpha\mathbb{E}\left(\frac{\tilde{f}}{f}e_i\right)$$

Proposition A.2 shows that a change in the distribution of income has two effects on the tax rate in partial equilibrium. First, it affects the value of redistribution. For example, the value of redistribution increases everywhere in response to a spread of the income distribution that keeps the cost of public fund constant ($\mathbb{E}^g(\tilde{f}/f) = 0$), where the bottom and the top are thicker while the middle of the distribution is thinner. When income at the bottom of the distribution is stagnant and increases at an increasing rate with higher income, the value of redistribution will increase at the bottom and at the top. Indeed, the marginal cost of public funds decreases ($\mathbb{E}^g(\tilde{f}/f) < 0$), so it is more valuable to tax at the bottom, and a larger tax rate near the top collects more income to redistribute to lower income households. Second, distributional changes affects the cost of taxation. When $\tilde{f}$ is positive, the mass at $z(\theta)$ increases and a larger tax rate creates a larger reduction in labor supply.

Although the two effects can work in opposite direction, so that the impact of a change in the income distribution may be ambiguous, we derive a simpler formula for the top and bottom tax rate when $\tilde{f}/f$ is bounded:

$$d\left\{\frac{T'}{1-T'}\right\}_{PE}[\tilde{f}, f] = \mathbb{E}^g(\tilde{f}/f)\frac{g}{1-g}\left(\frac{T'}{1-T'} + \alpha\right)$$

The tax response at the top and bottom only depends on the cost of public funds $\mathbb{E}^g(\tilde{f}/f)$. If the cost does not change, the tax rate remains the same at the tails. For a spread in the income distribution (with $\mathbb{E}^g(\tilde{f}/f) = 0$), the tax rate increases in the middle of the distribution (since the value of redistribution increases and the cost decreases). Indeed, as the cost of taxation decreases in the middle of the distribution and in-

[2]The tax rate can be corrected exactly as in the previous subsection when social preferences are non linear.

creases at the top, it becomes more efficient to increase the tax rate in the middle to redistribute to poorer households.

As in Proposition A.1, the general equilibrium effects of the change in distribution works through prices. A shift in income inequality has a direct effect on prices summarized by $dq^0[\tilde{f}, f]$, which will then be amplified through income and substitution effects and further changes in taxes. The direct effect, $dq^0[\tilde{f}, f]$ is the sum of three terms. The first term captures the change in income due to the partial equilibrium adjustment in taxes $dT'_{PE}[\tilde{f}, f]$. As discussed in Proposition A.1, since $dT'_{PE}[\tilde{f}, f]$ is positive, the share of luxury goods decrease relatively more than the share of necessity goods. This first effect is less important quantitatively than the second and especially the third term of $dq^0[\tilde{f}, f]$. The second term captures the increase in tax revenue (as there are more higher income households, tax receipts are larger) and its effect on consumption. The additional revenue is distributed lump sum and all prices decrease proportionally to the aggregate marginal propensity to spend across goods. The last and most important term is the direct effect of the spread in the income distribution on aggregate demand for goods. A spread in the income distribution increases the mass of households at the tails. This mechanically affects aggregate demand for goods and, since households at the top hold a larger share of aggregate income, demand for luxury goods increases relatively more.

Therefore, the price of luxury goods decreases relatively more and the overall price effect mutes the incentive to redistribute to lower-income households. The magnitude of the price effect depends on the degree of heterogeneity in consumption and, implicitly, on preferences for redistribution: the order of magnitude of the price derivative of the tax rate is $(1 - T')^2$.[3] This is also the order of magnitude of the tax rate derivative with respect to the distribution. Thus, the direct impact of inequality on redistributive policies and its indirect impact through prices are of the same order, and it is not meaningful to consider one without the other.[4].

Proof of Proposition A2. First note that for a given tax schedule, we have that for any distribution of ability $\pi(\theta)$ the associated distribution of income is given by $f(z(\theta)) = \pi(\theta)/z'(\theta)$ where $z'(\theta)$ is independent of $\pi(\theta)$ and we have $d\pi[\tilde{f}, f](\theta) = z'(\theta)\tilde{f}(z(\theta))$. Given that $\partial_\theta ln(MRS)$ is independent of the tax system in the additively separable/isoelastic case, we then have at constant q_i:

$$z\tilde{\zeta}f(z)d\left\{\frac{T'}{1-T'}\right\}[\tilde{f}, f](\theta) + \partial_\theta ln(MRS)d\pi[\tilde{f}, f](\theta)\left(\frac{T'}{1-T'} + \alpha\right)$$
$$= (1-\alpha)\int_{\theta'>\theta} d\pi[\tilde{f}, f](\theta) - g\left(d\pi[\tilde{f}, f](\theta) - \int g d\pi[\tilde{f}, f](\theta)d\theta\right)d\theta$$

Plugging $d\pi[\tilde{f}, f](\theta) = z'(\theta)\tilde{f}(z(\theta))$ and changing variable in the integral then gives the formula of the change in tax rate in partial equilibrium.

The change in aggregate consumption is given by:

$$dC_j[\tilde{f}, f] = \sum_k \frac{dC_j}{dq_k} dq_k[\tilde{f}, f] - \mathbb{E}\left(\partial_{z^*} c_j\left(z\tilde{\zeta}dT'[\tilde{f}, f](z) + dT[\tilde{f}, f](z)\right)\right) + \int c_i \tilde{f} dz$$

[3] Moreover the direct change in consumption generates a price change $dq_i = -\alpha\mathbb{E}\left(\tilde{f}/fe_i\right)$, which is of the same order as the change in distribution.

[4] We illustrate this point quantitatively in subsection 5.2.

where the last term is the mechanical change in C_j due to the shift in f and $dT'_{PE}[\tilde{f}, f](z) = \zeta/\tilde{\zeta} dT'_{PE}[\tilde{f}, f](\theta)$. The impact, at fixed consumer prices, of the shift in distribution on government budget constraint is given by:

$$\int (1-\alpha)T + \alpha z \tilde{f} dz + (1-\alpha)\mathbb{E}\left(dT[\tilde{f}, f](z)\right) - \mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'} + \alpha\right) dT'[\tilde{f}, f](z)\right) = 0$$

Using the same steps as in proposition 7 then gives the formula of the main text.

Comparative Statics with a Non-Linear Social Welfare Function Next, we allow the social welfare function to be non-linear. We continue to assume that agents' utility is separable in consumption and labor, that there is no income effect on labor supply at initial prices, and that the market size elasticities are constant and equal to α across markets. As before, g are the pareto weights (i.e., $g = G'(v)v_{z^*}/\lambda$ with v the indirect utility of the agent and λ the social cost of public funds), and g' denote their derivatives with respect to disposable income. We denote by $d\hat{T}'/dp_i^* = dT'/dp_i^*|_z + c \cdot dq/dp_i^*$ the response to price changes of the real wedge with a non-linear social welfare function; it is distinct from the linear case, which we denote by $d\tilde{T}'_\ell/dp_i^*$.

Proposition A.3. *Consider an economy with constant α_i across markets. The response of the real wedge at z to an exogenous increase in the producer price of i is given by:*

$$\left.\frac{d\hat{T}'}{dp_i^*}\right|_z = \underbrace{\left.\frac{d\tilde{T}'_\ell}{dp_i^*}\right|_z}_{\substack{\text{Price Effect} \\ \text{Linear Pareto Weights}}} + \overbrace{\underbrace{\left.\frac{d\hat{T}'_g}{dp_i^*}\right|_z}_{\text{Change in Pareto Weights}} + \underbrace{\sum \left.\frac{d\tilde{T}'_\ell}{dq_j^*}\right|_z \frac{d\hat{q}_j^0}{dp_i^*}}_{\text{Price Correction}}}^{\text{Non Linear Correction}}$$

where $d\tilde{T}'_\ell/dp_i^*$ is defined in Proposition 7 and $d\hat{T}'_g/dp_i^*$ captures the change in the tax rate induced by change in the non linear pareto weights:

$$\frac{\tilde{\zeta}}{\zeta}\frac{z\tilde{\zeta}f(z)}{(1-T')^2}\left.\frac{d\hat{T}'_g}{dp_i^*}\right|_z = (1-\alpha)\mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}}{dp_i^*} - \mathbb{E}^g\left(\frac{g'}{g}\frac{d\hat{T}}{dp_i^*}\right)\right) \tag{4}$$

$d\hat{T}'_g/dp_i^*$ induces a direct change in prices $d\hat{q}_i^0/dp_i^*$ given by:

$$C_i\frac{d\hat{q}_i^0}{dp_i^*} = \frac{\alpha}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\frac{q_i\partial_{q_i}\tilde{T}'_\ell}{1-T'}\frac{\tilde{\zeta}/\zeta}{1-T'}\left.\frac{d\hat{T}'_g}{dp_i^*}\right|_z\right) \tag{5}$$

Finally, $d\hat{T}/dp_i^*(z_{min})$ is determined by the budget constraint:

$$(1-\alpha)\mathbb{E}\left(\frac{d\hat{T}}{dp_i^*}\right) = \mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'} + \alpha\right)\frac{d\hat{T}'}{dp_i^*}\right) + C_i \tag{6}$$

Similarly, the tax rate derivative with respect to the market size elasticity α is:

$$\left.\frac{d\hat{T}'}{d\alpha}\right|_z = \left.\frac{d\tilde{T}'_\ell}{d\alpha}\right|_z + \left.\frac{d\hat{T}'_g}{d\alpha}\right|_z + \sum \left.\frac{d\tilde{T}'_\ell}{dq_j^*}\right|_z \frac{d\hat{q}_j^0}{d\alpha}$$

where $d\hat{T}'_g/d\alpha$ and $d\hat{q}^0_j/d\alpha$ are defined as in equations 4, 5.

In Proposition A.3, the response of optimal tax rates to prices and market size elasticity is given implicitly as the solution to an integro-differential equation, which can be solved efficiently by iterations. The iterative procedure conveys the intuition for the non-linear tax correction. For example, consider an increase in the price of necessity goods. As seen in Proposition 7, with linear pareto weights, the tax schedule becomes more regressive: the tax burden increases at the bottom of the distribution, and decreases at the top. When the social welfare function is concave, this means that the value of a dollar transfer increases at the bottom and decreases at the top: there is an incentive for the social planner to partially compensate lower-income households for their higher burden.

In general, if the change in tax and price burden $d\hat{T}/dp^*_i$ is decreasing and if the coefficient of inequality aversion g'/g is increasing, then $d\hat{T}'_g/dp^*_i$ is positive: the planner levies funds to reduce the income loss at the bottom of the distribution. This change in taxes leads to an increase in the consumption of necessity goods, which lowers their prices. This direct effect on prices is captured by $d\hat{q}^0/dp^*_i$. As in Lemma 2, the change in prices generates a further change in tax rates and prices through income and substitution effects.

Overall, the non-linearities dampen both the regressive change in the tax schedule and the price increases for necessities induced by general equilibrium effects. These channels may be difficult to see directly in Proposition A.3, since both prices and taxes are a fixed point of the equation. However, when the derivatives of the pareto weights are small, the term $d\hat{T}'_g/dp^*_i$ is well approximated by:

$$\frac{\tilde{\zeta}}{\zeta}\frac{z\tilde{\zeta}f(z)}{(1-T')^2}\frac{d\hat{T}'_g}{dp^*_i} = (1-\alpha)\mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\tilde{T}_\ell}{dp^*_i} - \mathbb{E}^g\left(\frac{g'}{g}\frac{d\tilde{T}_\ell}{dp^*_i}\right)\right)$$

In words, the planner corrects for the increase in tax burden due to the regressive tax change (and price changes) of Proposition 7: the next round change in tax burden induced by the correction is of second order. This formula is useful to characterize the first-order impact of the tax correction. If the tax schedule of Proposition 7 is regressive (progressive), the correction makes it more (less) progressive provided that g'/g is increasing.

When the derivatives of the pareto weights are larger, then the changes in tax burden induced by the correction become first order and more iteration are needed. However, regardless of the number of iterations, the correction will generally not overturn the regressivity (or progressivity) of the tax response derived in Proposition **??**. If the price changes impose a larger burden at the bottom of the distribution with a linear social welfare function, the non-linear correction will merely dampen this increase. We formalize these statements in the following corollary, making additional assumptions to obtain unequivocal results:

Proposition A.4. *Suppose that $\alpha = 0$, that the exogenous price changes dp^* are such that $C \cdot dp^* = 0$ (average inflation is zero) and that $\tilde{\zeta}/\zeta\, z\tilde{\zeta}f((z)/(1-T')^2\, d\tilde{T}'_\ell/dp^*$ is negative and inverse U-shaped (implying that the price changes benefit higher-income households[5]). Then:*

- *If $g'(z)/g(z)$ is constant and negative, there exists an income level z_0 such that $d\hat{T}/dp^* > 0$ if $z < z_0$ and $d\hat{T}/dp^* < 0$ if $z > z_0$*

[5]Indeed we have $1/(1-T')d\tilde{T}'_\ell/dp^* = \sum_i(\partial_{z^*}E_i - s_i + \mathbb{E}_z(p_i\tau^{nh}_i))dp^*_i/p_i$ which is negative if the price of luxuries decreases while the price of necessities decreases

- *If $g'(z)/g(z)$ is increasing and negative, there exists an income level z_0 such that $d\hat{T}/dp^* > 0$ if $z < z_0$ and $\mathbb{E}^g_{z>z_0}(d\hat{T}/dp^*) < 0$.*

In both cases we have $d\hat{T}'/dp^* \geq d\tilde{T}'_\ell/dp^*$. The opposite is true when $\tilde{\zeta}/\zeta\, z\tilde{\zeta}f((z)/(1-T')^2\, d\tilde{T}'_\ell/dp^*_i$ is positive and U-shaped.

Let us briefly discuss the assumptions of the corollary. First, imposing $\alpha = 0$ allows us to ignore the endogenous response of prices, which depends on substitution patterns across products.[6] Second, to capture a price change that benefits higher-income households, we assume that $\tilde{\zeta}/\zeta\, z\tilde{\zeta}f((z)/(1-T')^2\, d\tilde{T}'_\ell/dp^*_i$ is negative, imposing in addition that it is inverse U-shaped is mild technical assumption.[7] Finally, assuming that average inflation is zero allows us to focus on price dispersion rather than changes in aggregate real income.[8]

Although it is feasible to fully compensate agents for the price changes when average inflation is zero (as shown by Equation 6, with $d\hat{T}/dp^* = 0$), Proposition A.4 states that it is not optimal to do so. When the price change benefits higher-income households, we have $d\hat{T}/dp^* > 0$ at lower income levels, so the welfare of lower-income households decreases at the optimum. This is true even if welfare losses at the bottom of the distribution are socially more costly, which is the case when g'/g increases, which covers, in particular, $CRRA$ social welfare functions. In the next subsection, we discuss how these results extend to non-isoelastic disutilities of labor supply.

When preferences for consumption are homothetic and the social welfare function is linear, we have seen that the impact of prices and the market size elaticity α on taxes rate is rather limited. This is not the case when the social welfare function is non linear. Indeed, we have shown that when markets become more elastic, the planner imposes a corrective tax and the tax rate at a given percentile of the distribution decreases by $-(1-T')/(1-\alpha)$: there is no interaction between the corrective tax and the redistribution motive. But imposing the corrective tax reduces government revenue and increases the tax burden at the bottom of the distribution. With non linear social preferences, this increases the value of a dollar transfer at the bottom. The tax rate is adjusted accordingly and decreases by less than $-(1-T')/(1-\alpha)$. There is therefore a non trivial interaction between corrective and redistributive taxation. Similarly, even if price changes do not affect directly the tax rate in the homothetic case, they indirectly have an impact through the non-linear correction.

[6] In some simple cases, for example when there only a luxury good and a necessity good that are substitutes, the result can be generalized with $\alpha \neq 0$. In general, as long as the total price change – incorporating endogenous responses – benefits higher income households, the result remains true.

[7] Indeed, when the prices of luxury goods decrease, the real wedge decreases with a linear social welfare function. Assuming that $\tilde{\zeta}/\zeta\, z\tilde{\zeta}f((z)/(1-T')^2\, d\tilde{T}'_\ell/dp^*_i$ is inverse U-shaped is not very restrictive because (i) the term $\tilde{\zeta}/\zeta\,/(1-T')\, d\tilde{T}'_\ell/dp^*_i$ is decreasing at the bottom of the distribution, and increasing a higher income levels; (ii) empirically, we observe that $zf(z)\tilde{\zeta}/(1-T')$ is U-shaped. Finally, the fact that $\tilde{\zeta}/\zeta\, z\tilde{\zeta}f((z)/(1-T')^2\, d\tilde{T}'_\ell/dp^*_i$ is inverse U-shaped is verified in our empirical analysis.

[8] When average inflation is instead positive, the planner would need to levy more funds and the tax burden would increase even more at the bottom of the distribution. When g'/g is constant, we can in addition show that when inflation is not zero, the additional fund is simply levied via a lump sum tax.

Proof of Proposition A3. With a non-linear social welfare function, we have that the derivative of the pareto weights g is given by:

$$\frac{dg}{dp_i^*} = \frac{d}{dp_i^*}\left\{\frac{G'(V)v_{z^*}}{\mathbb{E}(G'(V)v_{z^*})}\right\} = -\left(g'\left(\frac{dT}{dp_i^*}+c\cdot\frac{dq}{dp_i^*}\right) - g\mathbb{E}\left(g'\left(\frac{dT}{dp_i^*}+c\cdot\frac{dq}{dp_i^*}\right)\right)\right)$$
$$- g\left(\partial_{z^*}c\cdot\frac{dq}{dp_i^*} - \mathbb{E}\left(g\partial_{z^*}c\cdot\frac{dq}{dp_i^*}\right)\right)$$

Where the first line simply uses Roy's identity and $g' = G''(V)/\mathbb{E}(G'(V))$ (recall that v_{z^*} is a constant at initial prices. Denoting with the subscript ℓ the tax derivative with respect to price in the linear case – defined in Proposition 5 and 7, we have:

$$\left.\frac{dT'}{dp_i^*}\right|_z = \sum_j \left.\frac{\partial T_\ell'}{\partial q_j}\right|_z \frac{dq_j}{dp_i^*} + \frac{\zeta}{\tilde{\zeta}}\frac{(1-T')^2}{z\tilde{\zeta}f(z)}(1-\alpha)\mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\left(\frac{dT}{dp_i^*}+c\cdot\frac{dq}{dp_i^*}\right) - \mathbb{E}^g\left(\frac{g'}{g}\left(\frac{dT}{dp_i^*}+c\cdot\frac{dq}{dp_i^*}\right)\right)\right)$$

Defining $d\hat{T}'/dp_i^* = dT'/dp_i^* + \partial_{z^*}c\cdot dq/dp_i^*$ the change in real wedge, we have:

$$\begin{aligned}\left.\frac{d\hat{T}'}{dp_i^*}\right|_z &= \sum_j \left.\frac{\partial \tilde{T}_\ell'}{\partial q_j}\right|_z \frac{dq_j}{dp_i^*} + \frac{\zeta}{\tilde{\zeta}}\frac{(1-T')^2}{z\tilde{\zeta}f(z)}(1-\alpha)\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp_i^*} - \mathbb{E}^g\left(\frac{g'}{g}\frac{d\hat{T}}{dp_i^*}\right)\right)\\ &= \sum_j \left.\frac{\partial \tilde{T}_\ell'}{\partial q_j}\right|_z \frac{dq_j}{dp_i^*} + \left.\frac{dT_g'}{dp_i^*}\right|_z\end{aligned}$$

Where the second line defines dT_g'/dp_i^* as a function of $d\hat{T}'/dp_i^*$

Next, the response of consumption to the joint change in price and taxes is:

$$\frac{dC_j}{dp_i^*} = \sum_k \frac{dC_j}{dq_k}\frac{dq_k}{dp_i^*} - \frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)C_i - \mathbb{E}\left(\partial_{z^*}c_j z\tilde{\zeta}\frac{dT_g'}{dp_i^*}\right) - \mathbb{E}\left(\partial_{z^*}c_i\frac{dT_g}{dp_i^*}\right)$$

Using the same steps as in proposition 7 we can rewrite this:

$$\frac{dC_j}{dp_i^*} = \sum_k \frac{dC_j}{dq_k}\frac{dq_k}{dp_i^*} - \frac{1}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)C_i - \frac{1}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\frac{\partial\tilde{T}'}{\partial q_i}\frac{\tilde{\zeta}}{\zeta}\frac{1}{1-T'}\frac{dT_g'}{dp_i^*}\right)$$

Given that $1/q_j dq_j/dp_i^* = -\alpha 1/C_j dC_j/dp_i^* + \mathbb{1}_{j=i}1/p_i$, we have:

$$\left[\frac{1}{q_j}\frac{dq_j}{dp_i^*}\right] = (Id + \Delta_\alpha\mathcal{C})^{-1}\left[\frac{\alpha_j}{1-\alpha}\mathbb{E}(\partial_{z^*}c_j)\frac{C_i}{C_j} + \mathbb{1}_{i=j}\frac{1-\alpha_i}{1-\alpha}\frac{1}{q_i} + \frac{1}{q_j^0}\frac{dq_j}{dp_i^*}\right]$$

With $C_j\frac{1}{q_j}\frac{dq_j^0}{dp_i^*} = \frac{\alpha}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\frac{q_j\partial\tilde{T}'}{\partial q_i}\frac{\tilde{\zeta}}{\zeta}\frac{1}{1-T'}\frac{dT_g'}{dp_i^*}\right)$.

Therefore, the tax schedule is determined by the 4 following equations:

$$\left.\frac{d\hat{T}'}{dp_i^*}\right|_z = \left.\frac{d\tilde{T}'_\ell}{dp_i^*}\right|_z + \left.\frac{dT'_g}{dp_i^*}\right|_z + \sum_j \left.\frac{d\tilde{T}'_\ell}{dq_j}\right|_z \frac{dq_j^0}{dp_i^*}$$

$$\frac{\tilde{\zeta}}{\zeta}\frac{z\tilde{\zeta}f(z)}{(1-T')^2}\left.\frac{dT'_g}{dp_i^*}\right|_z = (1-\alpha)\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp_i^*} - \mathbb{E}^g\left(\frac{g'}{g}\frac{d\hat{T}}{dp_i^*}\right)\right)$$

$$C_j\frac{1}{q_j}\frac{dq_j^0}{dp_i^*} = \frac{\alpha}{1-\alpha}\mathbb{E}\left(z\tilde{\zeta}\frac{q_j\partial\tilde{T}'}{\partial q_i}\frac{\tilde{\zeta}}{\zeta}\frac{1}{1-T'}\frac{dT'_g}{dp_i^*}\right)$$

$$(1-\alpha)\mathbb{E}\left(\left.\frac{d\hat{T}}{dp_i^*}\right|_z\right) = \mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)\left.\frac{d\hat{T}'}{dp_i^*}\right|_z\right) + C_i$$

Proof of Proposition A4. In all the proof, we assume that the marginal tax rates are well defined everywhere. As shown in Proposition 9, the change in government budget in response to a change of prices $[dp^*]$ is given by:

$$(1-\alpha)\mathbb{E}\left(\sum_i \left.\frac{d\hat{T}}{dp_i^*}\right|_z dp_i^*\right) = \mathbb{E}\left(z\tilde{\zeta}\left(\frac{T'}{1-T'}+\alpha\right)\sum_i \left.\frac{d\hat{T}'}{dp_i^*}\right|_z dp_i^*\right) + \sum_i C_i dp_i^*$$

When average inflation is 0 ($C \cdot dp^* = 0$) then the change in taxes is budget neutral at initial prices. Given that the initial schedule is optimal, this means that $\mathbb{E}(gd\hat{T}/dp^* \cdot dp^*) = 0$ (otherwise there would be a budget neutral deviation, at initial prices, improving welfare).

First consider the case where g'/g negative and constant. Since $\mathbb{E}(gd\hat{T}/dp^* \cdot dp^*) = 0$, we have $\mathbb{E}(g'd\hat{T}/dp^* \cdot dp^*) = 0$. and we have:

$$\frac{\tilde{\zeta}}{\zeta}\frac{z\tilde{\zeta}f(z)}{(1-T')^2}\left.\frac{d\hat{T}'}{dp^*}\right|_z \cdot dp^* = (1-\alpha)\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right) + \frac{\tilde{\zeta}}{\zeta}\frac{z\tilde{\zeta}f(z)}{(1-T')^2}\left.\frac{\partial\tilde{T}'}{\partial q}\right|_z \cdot dp^*$$

Since by assumption, the second term on the right hand side is negative (except at $z(\underline{\theta})$ and $z(\bar{\theta})$ where it is potentially 0), we necessarily have that $d\hat{T}/dp^* \cdot dp^*$ is strictly positive at $z(\underline{\theta})$. if not, $\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right)$ is non increasing at $z(\underline{\theta})$. Since the second term is decreasing at $z(\underline{\theta})$, this means that there is a small open interval $(z(\underline{\theta}), z^*)$ such that $d\hat{T}/dp^* \cdot dp^*$ is negative. But this implies that $d\hat{T}/dp^* \cdot dp^*$ is negative on $(z(\underline{\theta}), z(\bar{\theta}))$. Indeed, by contradiction call z_0 the first z such that $d\hat{T}/dp^* \cdot dp^*$ is 0. Then, since $\mathbb{E}(g'd\hat{T}/dp^* \cdot dp^*) = 0$, $\mathbb{E}^g_{z'>z_0}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right) < 0$, since in addition $\partial\tilde{T}'/\partial q \cdot dp^*$ is negative at z_0, we have $d\hat{T}'/dp^* \cdot dp^*$ strictly negative at z_0. Therefore we cannot have $d\hat{T}/dp^* \cdot dp^*$ negative on $(z(\underline{\theta}), z_0)$, which is a contradiction. Therefore, $d\hat{T}/dp^* \cdot dp^*$ is negative on $(z(\underline{\theta}), z(\bar{\theta}))$ which contradicts $\mathbb{E}(g'd\hat{T}/dp^* \cdot dp^*) = 0$ and $d\hat{T}/dp^* \cdot dp^*$ is strictly positive at $z(\underline{\theta})$. Using the same logic, $d\hat{T}/dp^* \cdot dp^*$ cannot be positive in a neighborhood of $z(\bar{\theta})$. In addition, $\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right) > 0$ on $(z(\underline{\theta}), z(\bar{\theta}))$. Indeed suppose not and denote again z_0 the smallest z_0 such that $\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right) = 0$. $d\hat{T}/dp^* \cdot dp^*$ cannot be positive at z_0 since then $\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right)$ would be increasing at z_0 (contradicting the fact that it is positive for $z < z_0$). therefore $d\hat{T}/dp^* \cdot dp^*$ is non negative at z_0, but using the same reasoning as before this would imply $d\hat{T}/dp^* \cdot dp^* < 0$ for $z > z_0$ so that $\mathbb{E}^g_{z'>z_0}\frac{g'}{g}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right) > 0$.

Since $d\hat{T}/dp^* \cdot dp^*$ is strictly positive at $z(\underline{\theta})$ is strictly positive and since we have $\mathbb{E}^g_{z'>z_0} \frac{g'}{g}\left(\frac{d\hat{T}}{dp^*} \cdot dp^*\right) \geq 0$ for all z, there exists z_0 such that $d\hat{T}/dp^* \cdot dp^*$ is 0. We now show that when the second term on the RHS is inverse U-shaped, $d\hat{T}/dp^* \cdot dp^* \leq 0$ for $z > z_0$. Suppose not, and call z_1 the smallest $z > z_0$ such that $d\hat{T}/dp^* \cdot dp^*$ is positive above z_1. Since $\mathbb{E}^g_{z'>z_0} \frac{g'}{g}\left(\frac{d\hat{T}}{dp^*} \cdot dp^*\right) geq0$, there exists a smallest $z_2 > z_1$ such that $d\hat{T}/dp^* \cdot dp^*$ is 0 at z_2 . Therefore we have:

$$
\begin{aligned}
\frac{\tilde{\zeta}}{\zeta} \frac{z_0\tilde{\zeta} f(z_0)}{(1-T')^2} \left.\frac{\partial \tilde{T}'}{\partial q}\right|_{z_0} \cdot dp^* - \frac{\tilde{\zeta}}{\zeta} \frac{z_1\tilde{\zeta} f(z_1)}{(1-T')^2} \left.\frac{\partial \tilde{T}'}{\partial q}\right|_{z_1} \cdot dp^* &= -(1-\alpha)\mathbb{E}^g_{z_1>z'>z_0} \frac{g'}{g}\left(\frac{d\hat{T}}{dp^*} \cdot dp^*\right) \\
&+ \frac{\tilde{\zeta}}{\zeta} \frac{z_0\tilde{\zeta} f(z_0)}{(1-T')^2} \left.\frac{d\hat{T}'}{dp^*}\right|_{z_0} \cdot dp^* - \frac{\tilde{\zeta}}{\zeta} \frac{z_1\tilde{\zeta} f(z_1)}{(1-T')^2} \left.\frac{d\hat{T}'}{dp^*}\right|_{z_1} \cdot dp^* \\
&< 0
\end{aligned}
$$

The first term on the RHS is negative since $d\hat{T}/dp^* \cdot dp^*$ is negative on $(z_), z_1)$, the second is also negative since $d\hat{T}/dp^* \cdot dp^*$ is non-increasing at z_0 and non-decreasing at z_1. In the same way, we have

$$
\begin{aligned}
\frac{\tilde{\zeta}}{\zeta} \frac{z_1\tilde{\zeta} f(z_1)}{(1-T')^2} \left.\frac{\partial \tilde{T}'}{\partial q}\right|_{z_1} \cdot dp^* - \frac{\tilde{\zeta}}{\zeta} \frac{z_2\tilde{\zeta} f(z_2)}{(1-T')^2} \left.\frac{\partial \tilde{T}'}{\partial q}\right|_{z_2} \cdot dp^* &= -(1-\alpha)\mathbb{E}^g_{z_2>z'>z_1} \frac{g'}{g}\left(\frac{d\hat{T}}{dp^*} \cdot dp^*\right) \\
&+ \frac{\tilde{\zeta}}{\zeta} \frac{z_1\tilde{\zeta} f(z_1)}{(1-T')^2} \left.\frac{d\hat{T}'}{dp^*}\right|_{z_1} \cdot dp^* - \frac{\tilde{\zeta}}{\zeta} \frac{z_2\tilde{\zeta} f(z_2)}{(1-T')^2} \left.\frac{d\hat{T}'}{dp^*}\right|_{z_2} \cdot dp^* \\
&> 0
\end{aligned}
$$

Which contradicts the fact that the linear term is inverse U shaped.

Next we consider the case where g'/g is negative and increasing. We can decompose the solution of our equation as $d\hat{T}/dp^* \cdot dp^* = d\hat{T}_0/dp^* \cdot dp^* + d\hat{T}_1/dp^* \cdot dp^*$ where the components solve respectively:

$$
\begin{aligned}
\frac{\tilde{\zeta}}{\zeta} \frac{z\tilde{\zeta} f(z)}{(1-T')^2} \left.\frac{d\hat{T}_0'}{dp^*}\right|_{z} \cdot dp^* &= (1-\alpha)\mathbb{E}^g_{z'>z} \frac{g'}{g}\left(\frac{d\hat{T}_0}{dp^*} \cdot dp^*\right) + \frac{\tilde{\zeta}}{\zeta} \frac{z\tilde{\zeta} f(z)}{(1-T')^2} \left.\frac{\partial \tilde{T}'}{\partial q}\right|_{z} \cdot dp^* \\
\frac{\tilde{\zeta}}{\zeta} \frac{z\tilde{\zeta} f(z)}{(1-T')^2} \left.\frac{d\hat{T}_1'}{dp^*}\right|_{z} \cdot dp^* &= (1-\alpha)\mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*} \cdot dp^* - \mathbb{E}^g \frac{g'}{g}\left(\frac{d\hat{T}_1}{dp^*} \cdot dp^*\right)\right)
\end{aligned}
$$

and $\mathbb{E}^g \frac{g'}{g}\left(\frac{d\hat{T}_0}{dp^*} \cdot dp^*\right) = 0$, $\mathbb{E}^g\left(\frac{d\hat{T}}{dp^*} \cdot dp^*\right) = 0$.

We first analyze $d\hat{T}_0/dp^* \cdot dp^*$. Using the same step as in the previous case, we necessarily have $d\hat{T}_0/dp^* \cdot dp^*$ positive at $z(\underline{\theta})$, and there exists z_0 such that $d\hat{T}_0/dp^* \cdot dp^*$ is positive for $z < z_0$ and non-positive for $z > z_0$. In addition, we have:

$$
\begin{aligned}
& \mathbb{E}^g_{z'>z} \frac{g'}{g}\left(\frac{d\hat{T}_0}{dp^*} \cdot dp^*\right) < \frac{g'}{g}(z_0)\mathbb{E}^g_{z'>z}\left(\frac{d\hat{T}_0}{dp^*} \cdot dp^*\right) \\
\Rightarrow \quad & 0 < -\mathbb{E}^g_{z'>z}\left(\frac{d\hat{T}_0}{dp^*} \cdot dp^*\right)
\end{aligned}
$$

This means that $d\hat{T}_0/dp^* \cdot dp^*$ generates a deficit (otherwise it would improve welfare at initial prices since it increases welfare) and that necessarily, $-\mathbb{E}^g_{z'>z}\left(\frac{d\hat{T}_1}{dp^*} \cdot dp^*\right) < 0$. Next, we analyze

$d\hat{T}_1/dp^* \cdot dp^*$. First suppose $\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right) < 0$. Then we necessarily have that $g'/gd\hat{T}_1/dp^* \cdot dp^* < \mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ in a neighborhood of $z(\underline{\theta})$. Indeed, using the same reasoning as above, if $g'/gd\hat{T}_1/dp^* \cdot dp^* > \mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ then we have $g'/gd\hat{T}_1/dp^* \cdot dp^* > \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ for all z which would be a contradiction (we can use the same reasoning as before, if not there is a smallest z_0 such that $g'/gd\hat{T}_1/dp^* \cdot dp^* = \mathbb{E}^g_{z'>z_1}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$, but at z_0 we necessarily have $\frac{d\hat{T}_1'}{dp^*} < 0$ at z_0 and since g'/g is negative increasing and $d\hat{T}_1/dp^* \cdot dp^* > 0$ at z_0 this means that $g'/gd\hat{T}_1/dp^* \cdot dp^*$ is increasing at z_0 which is a contradiction). If $g'/gd\hat{T}_1/dp^* \cdot dp^* > \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ in a neighborhood of $z(\underline{\theta})$, the same reasoning applies. Finally, if $g'/gd\hat{T}_1/dp^* \cdot dp^* = \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ in a neighborhood of $z(\underline{\theta})$ we would have that $d\hat{T}_1/dp^* \cdot dp^* = 0$ in the same neighborhood. Since, g'/g is increasing, this is a contradiction.

Next we have that $D(z) = \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^* - \mathbb{E}^g\frac{g'}{g}\left(\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)\right)$ is positive on the interval $(z(\underline{\theta}), z(\bar{\theta}))$. Again consider the smallest z_0 such that it is 0 at z_0 and negative in a neighborhood above. First we cannot have $g'/gd\hat{T}_1/dp^* \cdot dp^* < \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ at z_0 or in a neighborhood above since $D(z)$ would be locally increasing. This means that $g'/gd\hat{T}_1/dp^* \cdot dp^* \geq \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ in the neighborhood above z_0 which implies by the same reasoning as at $z(\underline{\theta})$ that $g'/gd\hat{T}_1/dp^* \cdot dp^* \geq \mathbb{E}^g_{z'>z}\left(\frac{g'}{g}\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right)$ for all $z > z_0$ and implies (since $g'/gd\hat{T}_1/dp^* \cdot dp^*$ cannot be constant) that $D(z)$ is positive everywhere above z_0.

Therefore, when $\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right) < 0$, we have that $d\hat{T}_1/dp^* \cdot dp^*$ is non negative and increasing. Since the equation determining $d\hat{T}_1/dp^* \cdot dp^*$ is linear, when it is positive instead, $d\hat{T}_1/dp^* \cdot dp^*$ would be non positive an decreasing. So to have $-\mathbb{E}^g_{z'>z}\left(\frac{d\hat{T}_1}{dp^*}\cdot dp^*\right) < 0$ we need $d\hat{T}_1/dp^* \cdot dp^*$ to be non negative and increasing which implies there exists a smallest z_0 such that that $d\hat{T}/dp^* \cdot dp^*$ is positive below z_0 and $-\mathbb{E}^g_{z'>z_0}\left(\frac{d\hat{T}}{dp^*}\cdot dp^*\right) > 0$, meaning that households with income larger than z_0 gain on average.

Non Constant Elasticity. Our comparative statics equation can easily be extended to the general quasilinear case with $U(c, z/\theta) = u(c_1, .., c_n) - \psi(z/\theta)$ and $v(q_1, .., q_n, z^*)$ linear at initial prices. The only difference is that now ζ depends on z.

Coming back to the proof of Proposition 4, we now have:

$$\begin{aligned}\partial_\theta ln(MRS) &= -\frac{1}{\theta}\left(1 + \frac{z/\theta\psi''(z/\theta)}{\psi'(z/\theta)}\right)\\ &= -\frac{1}{\theta}\left(1 + \frac{1}{\zeta(z/\theta)}\right)\end{aligned}$$

Therefore, denoting ϵ_ζ the curvature of ζ ($\epsilon_\zeta = (z/\theta)\zeta'/\zeta$) we have, defining as before $d\tilde{T}/dp_i^* = dT/dp_i^*|_z + \partial_{z^*}c \cdot dq/dp_i^*$:

$$\frac{d}{dp_i^*}\left\{\partial_\theta ln(MRS)\right\} = -\frac{1}{\theta}\epsilon_\zeta\frac{\tilde{\zeta}}{\zeta}\frac{1}{1-T'}\frac{d\tilde{T}'}{dp_i^*}$$

Note that ϵ_ζ can be measured in this case either by examining how the elasticity ζ responds to wage changes or how the elasticity varies across the income distribution. Using this it is then immediate to rederive the results of Section 3. For simplicity, we keep the assumption that the α_i are constant across markets.

Proposition A.5. *Consider a linear social welfare function. The change in real wedge in partial equilibrium is given by:*

$$(1-\kappa(z))\frac{\partial \tilde{T}'_\ell}{\partial q_i} = \frac{\partial \tilde{T}'_{iso}}{\partial q_i}$$

Where $\partial \tilde{T}'_{iso}/\partial q_i$ is the real wedge change in the isooelastic case defined in Definition 1 of the main text, and $\kappa(z) = \zeta/(1+\zeta)\epsilon_\zeta(T'(1-\alpha)+\alpha))$.

The response of aggregate consumption is given by:

$$\frac{dC_j}{dq_i} = \frac{-1}{1-\alpha}\mathbb{E}\left((1-\kappa(z))\frac{\tilde{\zeta}}{\zeta}\frac{\partial_{q_i}\tilde{T}'_\ell}{1-T'}\frac{\partial_{q_j}\tilde{T}'_\ell}{1-T'}z\tilde{\zeta}\right)$$

Finally, the response of the real wedge to a price change is given by:

$$\frac{d\tilde{T}_\ell}{dp_i^*} = \sum_j \frac{\partial \tilde{T}'_\ell}{\partial q_i}\frac{dq_j}{dp_i^*}$$

and the set of consumer prices is given by:

$$\left[\frac{1}{q_j}\frac{dq_j}{dp_i^*}\right] = (Id+\alpha\mathcal{C})^{-1}\left[\frac{\alpha}{1-\alpha}\mathbb{E}(\partial_{z^*}c)\frac{C}{C_i}+\frac{1}{q_i}\right]$$

When the social welfare function is non linear, the response to price change of the real wedge is given by:

$$\frac{d\tilde{T}}{dp_i^*} = \frac{d\tilde{T}_\ell}{dp_i^*} + \frac{dT_g}{dp_i^*} + \sum_j \frac{\partial \tilde{T}'_\ell}{\partial q_i}\frac{dq_j^0}{dp_i^*}$$

with:

$$(1-\kappa(z))\frac{\tilde{\zeta}}{\zeta}\frac{z\tilde{\zeta}f(z)}{(1-T')^2}\left.\frac{dT'_g}{dp_i^*}\right|_z = (1-\alpha)\mathbb{E}^g_{z'>z}\frac{g'}{g}\left(\frac{d\hat{T}}{dp_i^*} - \mathbb{E}^g\left(\frac{g'}{g}\frac{d\hat{T}}{dp_i^*}\right)\right)$$

$$C_j\frac{1}{q_j}\frac{dq_j^0}{dp_i^*} = \frac{\alpha}{1-\alpha}\mathbb{E}\left((1-\kappa(z))z\tilde{\zeta}\frac{q_j\partial\tilde{T}'_\ell}{\partial q_i}\frac{\tilde{\zeta}}{\zeta}\frac{1}{1-T'}\frac{dT'_g}{dp_i^*}\right)$$

Therefore, with non isoelastic preferences the results are the same as in the elastic case with a scaling given by $1-\kappa(z)$. This means in particular that the qualitative properties (in particular the ones of Proposition A.4) of the response of the marginal tax rate to prices are similar when $\kappa(z) < 1$.

A.4 Examples of Supply Side Specifications

We first describe an abstract entry model a la Melitz (2003) with general homothetic preferences across subvarieties satisfying our specification of the pricing function. We then provide more concrete examples.

In each sector k, a large mass $\tilde{M}_k$ of potential entrants which can produce subvarieties of the of the aggregate product k. Entry is done in two steps. First firms pay a fixed labor cost ξ_k^0. Upon paying the investment cost, they learn their productivity type i (ex-ante uniformly distributed) and decide to pay a second fixed labor investment ξ_k^p to produce. The subvariety $c_k(i)$ is then produced according to the variable cost function $\kappa_k(i)\psi(c)$, with $\kappa(i)$ non decreasing.

On the demand side, we assume first that preferences are separable between the sub-varieties produced across markets, second that preferences for the sub-varieties in market k are defined by an homothetic aggregator $C_k(\{c_k(i)\}_i, M_k)$.[9] Since preferences for subvarieties are homothetic, prices will only depend on aggregate spending in market k, E_k rather than the distribution of individual spending $\{e_k(\theta)\}_\theta$. In addition the agent's problem defines an ideal price index P_k with $P_k C_k = E_k$ and a demand function for subvarieties i $c_k(i) = d_k(p(i), P_k, \delta(\{p(i)\}_i, M_k), M_k)E_k$, with δ an aggregate demand shifter.

Then it is direct to see that P_k is only a function of E_k, and therefore of C_k which is consistent with our pricing function. Indeed, the firm first order condition

$$d_k((p(i), P_k, \delta, M_k) + p(i)\partial_p d_k((p(i), P_k, \delta, M_k) = \kappa(i)\psi'(d_k((p(i), P_k, \delta, M_k)E_k)\partial_p\, d_k(p(i), P_k, \delta, M_k),$$

determines the prices of producing firms $p(i)$ and $d_k((p(i), \delta, P_k, M_k)$ as functions of E_k,P_k M_k and δ. Since δ is itself a function of $p(i)$ and M_k, it is implicitly a function of E_k, P_k and M_k and so are the prices $p(i)$ and $d_k(i)$, which can be written $\tilde{d}_i(E_k, P_k, M_k)$ and $\tilde{p}_i(E_k, P_k, M_k)$.

Next, the entry condition determines the set $\mathcal{I}$ of producing firms through:

$$i \in \mathcal{I} \Leftrightarrow \tilde{p}_i(E_k, P_k, M_k)\tilde{d}_i(E_k, P_k, M_k)E_k - \kappa(i)\chi(\tilde{d}_i(E_k, P_k, M_k)E_k) \geq \xi_p$$

so $\mathcal{I}$ only depend on E_k, P_k and M_k. Finally the free entry condition:

$$\int \mathbb{1}(i \in \mathcal{I})(\tilde{p}_i\tilde{d}_i E_k - \kappa(i)\psi(\tilde{d}_i E_k) - \xi_p)g di = \xi_0 \tilde{M}_k$$

determines implicitly M_k as a function of E_k and P_k. Since we have $P_k C_k = E_k$ and C_k is only a function of E_k and P_k, $C_k = C_k(\{\tilde{d}_i(E_k, P_k, M_k(E_k, P_k))E_k, M_k(E_k, P_k)\})$, P_k itself is only a function of E_k or equivalently of C_k when a solution exists and is unique (some additional assumptions would be needed to ensure it is the case).

With Kimball preferences, for example, we have:

$$\int \mathcal{Y}(c_k(i)/C_k)di = 1$$

The first order conditions of the agent's problem give:

$$c_k(i) = \frac{E_k}{P_k}\mathcal{Y}'^{-1}\left(\lambda \int \frac{c_k(j)}{C_k}\mathcal{Y}'\left(\frac{c_k(j)}{C_k}\right) dj\, p(i)\right)$$

[9] The dependency of the aggregator on M_k can capture for example love for variety

We can then define $\delta = \lambda \int c_k(j)/C_k \mathcal{Y}'\left(c_k(j)/C_k\right)$ we have that δ only depends on the set of prices $\{p(i)\}_i$ since the function is homothetic so $c_k(j)/C_k$ and λ only depend on prices and not on E_k.

Second, with a continuous version of the homothetic translog, we have, denoting U_k the sub-utility of consumption of market k subvarieties:

$$ln(E_k) = ln(U_k) + \alpha_o + \frac{\tilde{M}_k - M_k}{2\gamma\tilde{M}_k M_k} + \int_{i\in\mathcal{I}} \frac{1}{M_k} ln(p(i))di + \frac{\gamma}{2M_k}\int_{i\in\mathcal{I}}\int_{j\in\mathcal{I}} ln(p(i))(ln(p(j)) - ln(p(i)))didj$$

So we directly have:

$$p(i)c_k(i) = E_k\left(\frac{1}{M_k} - \gamma(ln(p(i)) - \int_{j\in\mathcal{I}}\frac{ln(p(j))}{M_k}dj)\right)$$

the demand function takes the form above with $\delta(\{p(i)\}_i, M_k) = \int_{j\in\mathcal{I}} ln(p(j))/M_k dj$

B Quantitative Analysis of the Optimal Tax Schedule

This section describes the quantitative model. We first describe the economic environment. We then describe consumer preferences, contrasting the homothetic specification with non-homothetic preferences. Third, we describe the social planner's problem and the ordinary differential equations (ODEs) characterizing the solution. Finally, we present the solution algorithm for the ODEs.

B.1 Setting

B.1.1 Indirect Utility Function

The quantitative model uses a standard additively separable specification:

$$U\left(z^*, z, \boldsymbol{p}, \theta\right) = v\left(z^*, \boldsymbol{p}\right) - \psi\left(\frac{z}{\theta}\right) \tag{7}$$

$$\psi\left(\frac{z}{\theta}\right) = \frac{1}{1+\frac{1}{\varepsilon_z}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon_z}} \tag{8}$$

where $\psi\left(\frac{z}{\theta}\right)$ is the cost of earning z given ability θ, and $v\left(z^*, \mathbf{p}\right)$ is the indirect utility function given prices and disposable income.

B.1.2 Pricing Function

Denoting aggregate consumption by C_i, the quantitative model is based on an isoelastic pricing function:

$$p_i = \gamma_i C_i^{-\alpha} \quad \forall i \in \mathcal{I} \tag{9}$$

We calibrate γ_i to fit prices at the observed schedule, which are normalized to one without loss of generality, using the relationship:

$$\gamma_i \equiv p_{0,i} C_{0,i}^{\alpha} \tag{10}$$

where $C_{0,i}$ is aggregate quantity consumed in sector i at initial prices. To obtain observed consumption, we compute disposable income at the observed schedule as defined in appendix B.1.3, and then compute sectoral consumption given the expenditure shares described in appendix B.2.1 and appendix B.2.2.

B.1.3 Skill Distribution

The skill distribution $f(\theta)$ plays a key role in the shape of the optimal tax schedule. We use data from Hendren (2020) on the observed tax schedule to calibrate the skill distribution. As the data is only available for each percentile of the observed income distribution, we interpolate for marginal tax rates at income levels within the observed bounds using p-chip interpolation.

We then create a mapping from earned income at the observed tax schedule to skill type θ. Following Saez (2001), we obtain this mapping using the individual's utility function described in appendix B.1.1, which depends on the functional form of $v(z^*, \boldsymbol{p})$. We use two altenative forms of this indirect utility function - homothetic as described in appendix B.2.1 and non-homothetic as described in appendix B.2.2.

At the observed schedule in the homothetic case, we fit the skill distribution after setting: $p_{obs} = p_0 \equiv 1$. When computing income at the observed schedule in case of non-homothetic preferences, we use the "deflator" as defined in definition B.1. When we apply the deflator at any initial prices $\mathbf{p}_0$ the indirect utility of the agent will always be the same as in the homothetic case with $p_0 = 1$. This approach allows us to use the same skill distribution in the homothetic and in non-homothetic cases.

B.2 Consumer Preferences

This section describes the indirect utility function $v(z^*, \boldsymbol{p})$ from appendix B.1.1.

B.2.1 Homothetic Preferences

With homothetic preferences, the indirect utility function $v(z^*, \boldsymbol{p})$ described in appendix B.1.1 is given by:

$$v(z^*, \boldsymbol{p}) \equiv \frac{z^*}{p} \tag{11}$$

where p is the price in the economy. The individual's utility function, per equation (7), is:

$$U(z^*, z, \boldsymbol{p}, \theta) = \frac{z^*}{p} - \psi\left(\frac{z}{\theta}\right) = \frac{z^*}{p} - \frac{1}{1+\frac{1}{\varepsilon_z}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon_z}} \tag{12}$$

Plugging in the definition of disposable income, the optimal $z(\theta)$ satisfies the FOC:

$$\frac{dU(\theta)}{z(\theta)} = \frac{1-T'(z(\theta))}{p} - \left(\frac{z(\theta)}{\theta}\right)^{\frac{1}{\varepsilon_z}} \frac{1}{\theta} = 0$$

We can thus express income or skill parameters as functions of observables:

$$z(\theta) = \theta^{1+\varepsilon_z} \left(\frac{1-T'(z(\theta))}{p}\right)^{\varepsilon_z} \tag{13}$$

$$\theta = \left[z\left(\frac{p}{1-T'(z)}\right)^{\varepsilon_z}\right]^{\frac{1}{1+\varepsilon_z}} \tag{14}$$

With $\theta = 0$, we apply the limiting case described in appendix B.3.2.

B.2.2 Non-Homothetic CES Preferences

Definitions and Properties We use the *General Non-Homothetic CES Preferences* as defined in Appendix A.1 of Comin, Lashkari and Mestieri revision 3 (2019). The indirect utility function $v(z^*, \boldsymbol{p})$ described in appendix B.1.1 is given by $v \equiv v(z^*, \boldsymbol{p}) \equiv F(\mathbf{C})$, where $\mathbf{C}$ is the consumption vector of the agent.

Indirect utility v is implicitly defined by:

$$\sum_{i\in\mathcal{I}} \Omega_i^{\frac{1}{\sigma}} \left(\frac{C_i}{v^{\frac{\varepsilon_i}{1-\sigma}}}\right)^{\frac{\sigma-1}{\sigma}} = \sum_{i\in\mathcal{I}} (\Omega_i v^{\varepsilon_i})^{\frac{1}{\sigma}} C_i^{\frac{\sigma-1}{\sigma}} = 1, \tag{15}$$

where parameters ε_i denote the utility elasticities of each good, the elasticity of substitution between sectors is denoted σ , and taste parameters are denoted Ω_i, for an arbitrary set of sectors $i \in \mathcal{I}$. For the quantitative analysis, we consider two sectors, labelled "high quality" (H) and "low quality" (L).

Under this specification, Marshallian Demand (spending shares) and the price index are:

$$\omega_i(z^*) = \Omega_i \left(\frac{p_i}{P}\right)^{1-\sigma} \left(\frac{z^*}{P}\right)^{\varepsilon_i-(1-\sigma)} \tag{16}$$

$$P(\mathbf{p}, z^*) = \left[\sum_{i\in\mathcal{I}} \left(\Omega_i p_i^{1-\sigma}\right)^{\chi_i} \left(\omega_i\left(z^*\right)^{1-\sigma}\right)^{1-\chi_i}\right]^{\frac{1}{1-\sigma}} \tag{17}$$

$$\text{where } \chi_i \equiv \frac{1-\sigma}{\varepsilon_i}$$

Using this specification, we obtain quantity consumed as:

$$C_i(z^*) = \frac{\omega_i(z^*)z^*}{p_i} \tag{18}$$

Definition of Deflated Non-Homothetic Indirect Utility Function In the social planner's problem in appendix B.3.1 we use a deflated indirect utility function.

Definition B.1 (Deflated Indirect Utility Function). *Deflated indirect utility function $\widetilde{v}(z^*, \boldsymbol{p})$ is the inverse of the indirect utility function at initial prices, under constant returns to scale. It can be thought of as the level of "virtual disposable income" $\tilde{z}^*$ that satisfies $v(\tilde{z}^*, \boldsymbol{p}_0) = v(z^*, \boldsymbol{p})$. Formally,*

$$\widetilde{v}(z^*, \boldsymbol{p}) = v^{-1}(v(z^*, \boldsymbol{p}), \boldsymbol{p}_0) \tag{19}$$

Properties of the deflated indirect utility function are listed below. At $\boldsymbol{p}_0$, the non-homothetic indirect utility is equivalent to the homothetic case from appendix B.2.1.

$$\frac{d\widetilde{v}(z^*, \boldsymbol{p})}{dz^*} = \frac{dv(z^*, \boldsymbol{p})}{dz^*} \left(\frac{dv(\tilde{z}^*, \boldsymbol{p}_0)}{d\tilde{z}^*} \right)^{-1} \text{ where:} \tag{20}$$

$$\tilde{z}^* = v^{-1}(v(z^*, \boldsymbol{p}), p), \boldsymbol{p}_0)$$

$$\widetilde{v}(z^*, \boldsymbol{p}_0) = z^* \tag{21}$$

$$\frac{d\widetilde{v}(z^*, \boldsymbol{p}_0)}{dz^*} = 1 \tag{22}$$

B.3 ODEs from Social Planner's Problem

B.3.1 Social Planner's Problem

The social planner chooses the optimal tax schedule to maximize total utility over the distribution of types θ, subject to budget constraint, agents' FOC and market clearing, according to an arbitrary social welfare function $G\left(U(\theta, \mathbf{p})\right)$:

$$max_{z(\theta)} \int_{\underline{\theta}}^{\overline{\theta}} G\left(U(\theta, \mathbf{p})\right) f(\theta) d\theta \quad s.t: \tag{23}$$

$$G\left(U(\theta, \mathbf{p})\right) = G\left(\widetilde{v}\left(z^*(\theta), \boldsymbol{p}\right) - \psi\left(\frac{z(\theta)}{\theta}\right)\right) \tag{24}$$

$$G'\left(U(\theta, \mathbf{p})\right) = \frac{dG}{dU} \tag{25}$$

$$R \geq \int_{\underline{\theta}}^{\overline{\theta}} \left(z(\theta) - z^*(\theta)\right) f(\theta) d\theta \tag{26}$$

$$p_i = \gamma_i \left(\overline{C_i}\right)^{-\alpha}, \forall i \tag{27}$$

where:

i. $\overline{C_i} = \int_{\underline{\theta}}^{\overline{\theta}} C_i(\theta) dF(\theta)$ denotes aggregate consumption in sector i

ii. R denotes the government surplus (government revenue requirement)

iii. The state variable is $G(U(\theta, \mathbf{p}))$ and the control variable is $z(\theta)$

Using the envelope theorem and our functional form for U, we can write:

$$\dot{U}(\theta) = \frac{1}{\theta}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon_z}} \tag{28}$$

Call $\mu(\theta)$ the costate variable for the evolution of $G(\cdot)$. The equation for μ is

$$\dot{\mu}(\theta) = \left((1-\alpha)\frac{\lambda}{\frac{d\tilde{v}}{dz^*}} - G'\left(U(\theta)\right)\right) f(\theta), \tag{29}$$

where

- $\frac{d\tilde{v}}{dz^*}$ is the derivative of deflated indirect utility $\tilde{v}$ with respect to disposable income, evaluated at the level of disposable income z^* (which we express below as a function of θ, U and μ)
- λ is the multiplier on the government's budget constraint

The first-order condition for z gives:

$$\mu(\theta) \cdot \left(\frac{1+\frac{1}{\varepsilon_z}}{\theta^2}\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}\right) = -\lambda\left(1 - \frac{(1-\alpha)\cdot\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}}{\theta \cdot \frac{d\tilde{v}}{dz^*}}\right) f(\theta) \tag{30}$$

$$\mu(\theta) = \theta^2 \frac{\lambda\left((1-\alpha)\frac{\frac{1}{\theta}\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}}{\frac{d\tilde{v}}{dz^*}} - 1\right) f(\theta)}{\left(1+\frac{1}{\varepsilon_z}\right)\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}} \tag{31}$$

The **boundary conditions** are:

$$\mu(\underline{\theta}) = 0$$
$$\mu(\overline{\theta}) = 0$$

The government resource constraint in equation (26) results from the fact that government revenue is distributed among the agents in the economy through a lump sum transfer such that an amount R is not redistributed. With R denoting government surplus, we have

$$\overline{C} = \int C(\theta)\, dF(\theta) = \int C(z^*(\theta))\, dF(\theta) = \int \frac{z^*(\theta)}{p}\, dF(\theta) = \int \frac{z(\theta) - R}{p}\, dF(\theta) \tag{32}$$

B.3.2 Limiting Case

We need to address the case when $\theta = 0$ since several equations from appendix B.3.1 become indeterminate. We have

$$\zeta(\theta) \equiv \frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{\frac{1}{\varepsilon_z}} \tag{33}$$

$$\ell(\theta) \equiv \frac{z(\theta)}{\theta} = (\theta\zeta(\theta))^{\varepsilon_z} \tag{34}$$

These two functions are bounded functions of θ near 0. Using equation (30) and the definitions above we can express:

$$\zeta(\theta) = \left[\frac{1-\alpha}{\frac{d\tilde{v}}{dz^*}} - \frac{\mu(\theta)}{\theta}\frac{1+\frac{1}{\varepsilon_z}}{\lambda f(\theta)}\right]^{-1} \tag{35}$$

$$\dot{U}(\theta) = \zeta(\theta)\ell(\theta) \tag{36}$$

We still need to address the fact that $\frac{\mu(\theta)}{\theta}$ is undefined for $\theta = 0$. Given our specification we know that:

$$\begin{aligned} &f(0) > 0 \\ &\dot{\mu}(0) < 0 \\ &\mu(\theta) < 0 \quad \theta \to 0_+ \end{aligned}$$

Therefore we can use:

$$\lim_{\theta\to 0}\frac{\mu(\theta)}{\theta} = \dot{\mu}(\theta) = \left((1-\alpha)\frac{\lambda}{\frac{d\tilde{v}}{dz^*}} - G'\left(U(\theta)\right)\right) f(\theta) \tag{37}$$

Using these relationships we can express several of our key variables for $\theta = 0$:

$$\dot{U}(0) = 0 \tag{38}$$

$$z(0) = 0 \tag{39}$$

$$\psi(\frac{0}{0}) = 0 \tag{40}$$

$$\zeta(0) = \left[\frac{1-\alpha}{\frac{d\tilde{v}}{dz^*}} - \dot{\mu}(0)\frac{1+\frac{1}{\varepsilon_z}}{\lambda f(0)}\right]^{-1} \tag{41}$$

B.3.3 System of ODEs

The solution to the general case allowing for non-homotheticities in agents' utility function and an arbitrary social welfare function $G(U(\theta, \mathbf{p}))$ is given by the following system of ODEs and boundary conditions:

$$\dot{U}(\theta) = \frac{z}{\theta^2}\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}} \tag{42}$$

$$\dot{\mu}(\theta) = \left((1-\alpha)\frac{\lambda}{\frac{d\widetilde{v}}{dz^*}} - G'\left(\widetilde{v}-\psi\right)\right) f(\theta) \tag{43}$$

$$\mu(\theta) = \theta^2 \frac{\lambda\left((1-\alpha)\frac{\frac{1}{\theta}\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}}{\frac{d\widetilde{v}}{dz^*}} - 1\right) f(\theta)}{\left(1+\frac{1}{\varepsilon_z}\right)\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}} \tag{44}$$

with the boundary conditions:

$$\mu(\underline{\theta}) = 0$$
$$\mu(\overline{\theta}) = 0$$

Furthermore, we can express incomes as a function of other variables:

$$z = \theta \cdot \left(\frac{1-\alpha}{\theta \cdot \frac{d\widetilde{v}}{dz^*}} - \frac{\mu(\theta)}{\lambda f(\theta)} \cdot \frac{1+\frac{1}{\varepsilon_z}}{\theta^2}\right)^{-\varepsilon_z} \tag{45}$$

$$z^* = \widetilde{v}^{-1}\left(U(\theta) + \psi\left(\frac{z}{\theta}\right)\right) = \widetilde{v}^{-1}\left(U(\theta) + \frac{1}{1+\frac{1}{\varepsilon_z}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon_z}}\right) \tag{46}$$

This is a system of non-linear equations we can solve for to obtain z and z^* given θ, $U(\theta)$, $\mu(\theta)$, λ. We apply the limit case as per appendix B.3.2.

Homothetic Case With homothetic indirect utility, the system of ODEs can be expressed as:

$$\dot{U}(\theta) = \frac{z}{\theta^2} \cdot \left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}$$

$$\dot{\mu}(\theta) = \left((1-\alpha)\cdot\lambda\cdot p - \left(\frac{z^*}{p} - \psi\right)^{-\widetilde{\sigma}}\right) \cdot f(\theta)$$

$$\mu(\theta) = \theta^2 \frac{\lambda\left(p(1-\alpha)\frac{1}{\theta}\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}} - 1\right) f(\theta)}{\left(1+\frac{1}{\varepsilon_z}\right)\left(\frac{z}{\theta}\right)^{\frac{1}{\varepsilon_z}}}$$

$$z = \theta \cdot \left(\frac{-\theta^2 \lambda f(\theta)}{\mu(\theta)\left(1+\frac{1}{\varepsilon_z}\right) - \theta\lambda(1-\alpha) f(\theta) p}\right)^{\varepsilon_z}$$

$$z^* = p \cdot \left(U(\theta) + \frac{1}{1+\frac{1}{\varepsilon_z}} \cdot \left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon_z}}\right)$$

with boundary conditions:

$$\mu(\underline{\theta}) = 0$$
$$\mu(\overline{\theta}) = 0$$

B.3.4 Social Welfare Function

To study the role of non-linearities in the social welfare function, we consider a specification with constant relative risk aversion, where the CRRA risk parameter is denoted $\widetilde{\sigma}$. The functional form is:

$$G'\left(U(\theta, \mathbf{p})\right) = \frac{dG}{dU} = \left(U(\theta, \mathbf{p})\right)^{-\widetilde{\sigma}}, \quad \widetilde{\sigma} \geq 0 \tag{47}$$

$$G\left(U(\theta, \mathbf{p})\right) \equiv \begin{cases} \log\left(U(\theta, \mathbf{p})\right) & \text{if } \widetilde{\sigma} = 1 \\ \frac{(U(\theta, \mathbf{p}))^{1-\widetilde{\sigma}}}{1-\widetilde{\sigma}} & \text{if } \widetilde{\sigma} \geq 0 \wedge \widetilde{\sigma} \neq 1 \end{cases} \tag{48}$$

B.3.5 Pareto Analysis

We also perform analysis using Pareto weights, denoted $\lambda(\theta)$ and set to match the results obtained with the CRRA social welfare function $G(\cdot)$, with the CRRA risk parameter $\widetilde{\sigma}$. The Pareto weight are given by:

$$\lambda(\theta) \equiv \left(U_{optim}(\theta)\right)^{-\widetilde{\sigma}}, \tag{49}$$

where $U_{optim}(\theta)$ is the solution of the optimal taxation problem with homothetic indirect utility function, $\alpha = 0$, and the CRRA parameter $\widetilde{\sigma}$.

With Pareto weights, the social welfare function and its derivative become:

$$G\left(\theta\right) \equiv \lambda(\theta) U(\theta, \mathbf{p}), \tag{50}$$

$$G'\left(\theta\right) = \frac{dG}{dU} = \lambda(\theta). \tag{51}$$

B.3.6 Defining the Equivalent Variation

The equivalent variation (EV) is defined by:

$$\widetilde{v}\left(z^*_{ref}(\theta) + EV(\theta), \boldsymbol{p}_{ref}\right) - \psi\left(\frac{z_{ref}(\theta)}{\theta}\right) = u_{optim}(\theta),$$

where "*ref*" denotes the reference point and "*optim*" the new equilibrium. In our main specifications, the reference point is the outcome at the optimal tax schedule with homothetic preferences, to which we compare the outcome with non-homothetic preferences.

B.4 Solution Algorithm

This section describes the algorithm to solve the problem described in appendix B.3.1, using nested bisection in Matlab.

B.4.1 Convergence to Optimal Schedule

The algorithm relies on three nested loops. Each loop ensures that we satisfy one of the conditions in the social planner's problem (see appendix B.3.1); we guess the value of one parameter in each loop, and then solve for all inner loops.

The loops are structured as follows, from the outer loop to the inner loop:

i. *Price loop* - ensures prices in the economy converge such that equation (27) is satisfied by guessing $\boldsymbol{p}$. The convergence condition is base on the price change.

ii. *Surplus loop* - ensures government surplus equation (26) converges by guessing a value of λ. The convergence condition is the distance from the revenue requirement.

iii. *Utility or μ loop* - ensures that the boundary condition $\mu(\overline{\theta}) = 0$ is satisfied by guessing value of $U(\underline{\theta})$. The convergence condition is the distance between $\mu(\overline{\theta})$ and the coundary condition of 0.

We set a convergence condition (tolerance) for each loop, which determines whether the variable of interest has converged. In what follows, $\epsilon_{\mathbf{p}}, \epsilon_\lambda, \epsilon_\mu$ denote tolerance for price, surplus and utility loops, respectively.

In the description of the algorithm below, for any variable the indexes represent the iteration of price, surplus and utility loop, respectively. The optimal schedule is defined by $\mathbf{p}_{optim}$, λ_{optim} and $U_{optim}(\underline{\theta})$, denoting the values under which the variable determining convergence of each loop converged.

For illustration, assume the values of counters at convergence were 3 for price, 7 for surplus and 10 for utility. Then, the optimal value of utility is denoted $U_{3,7,10}(\underline{\theta})$, which is the value used to solve the ODE in the 10th utility loop, within the 7th surplus loop, within the 3rd price loop.

Thus, the optimal schedule can be defined as:

$$\mathbf{p}_{optim} = \mathbf{p}_{10} = \mathbf{p} \quad \textbf{such that} \tag{52}$$

$$\epsilon_{\mathbf{p}} \geq \frac{1}{\boldsymbol{p}} \left(\gamma \int_{\underline{\theta}}^{\overline{\theta}} \boldsymbol{C} \left(z^*(\theta; \boldsymbol{p}, \lambda_{optim}, U_{optim}(\underline{\theta})) \right) dF(\theta) - \boldsymbol{p} \right) \quad \textbf{where} \tag{53}$$

$$\lambda_{optim} = \lambda_{3,7} = \lambda \quad \textbf{such that} \tag{54}$$

$$\epsilon_\lambda > \left| \int_{\underline{\theta}}^{\overline{\theta}} z(\theta; \boldsymbol{p}, \lambda, U_{optim}(\underline{\theta})) - z^*(\theta; \boldsymbol{p}, \lambda, U_{optim}(\underline{\theta})) f(\theta) d\theta - R \right| \quad \textbf{where} \tag{55}$$

$$U_{optim}(\underline{\theta}) = U_{3,7,10}(\underline{\theta}) = U(\underline{\theta}) \quad \textbf{such that} \tag{56}$$

$$\epsilon_\mu > \left| \mu(\overline{\theta}; \boldsymbol{p}, \lambda, U(\underline{\theta})) \right| \tag{57}$$

B.4.2 Adjustment of Bounds

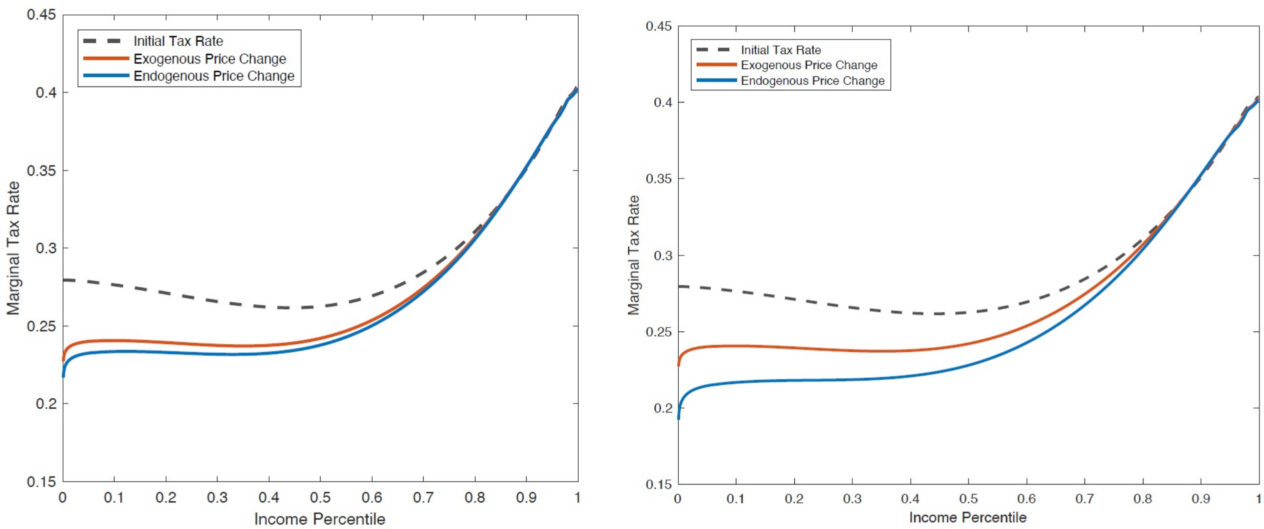
In the bisection algorithm, we update the bounds at the end of each iteration of loop based on the value of the variable of interest. In the case of the **utility or μ loop**, we change bounds on $U(\underline{\theta})$ according to the rule:

$$U_{UB}(\underline{\theta}) = U_{current}(\underline{\theta}) \text{ if } \mu(\overline{\theta}) < 0 \text{ or the solver failed}$$
$$U_{LB}(\underline{\theta}) = U_{current}(\underline{\theta}) \text{ if } \mu(\overline{\theta}) \geq 0$$

In the case of the **surplus loop**, we change bounds on λ according to the rule:

$$\lambda_{UB} = \lambda_{current} \text{ if } \int_{\underline{\theta}}^{\overline{\theta}} (z(\theta) - z^*(\theta)) f(\theta) d\theta \geq R$$
$$\lambda_{LB} = \lambda_{current} \text{ if } \int_{\underline{\theta}}^{\overline{\theta}} (z(\theta) - z^*(\theta)) f(\theta) d\theta < R$$

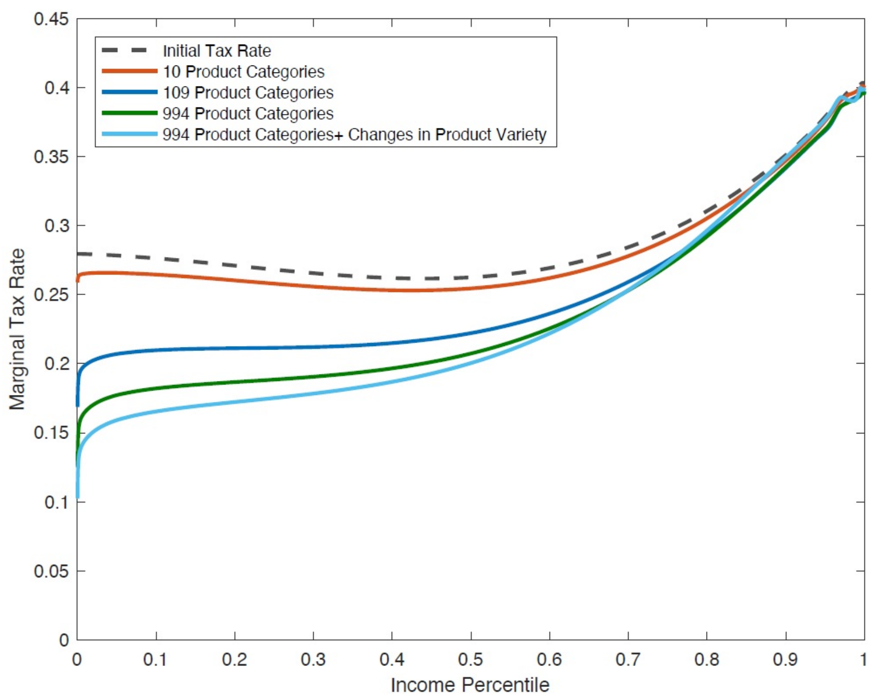
Figure A1: Sensitivity Analysis for the Response to Observed Price Shocks (2004-2015), CEX-CPI data: Results with Labor Supply Elasticity $\varepsilon = 0.33$



(i) $\sigma = 0.6$ (ii) $\sigma = 2$

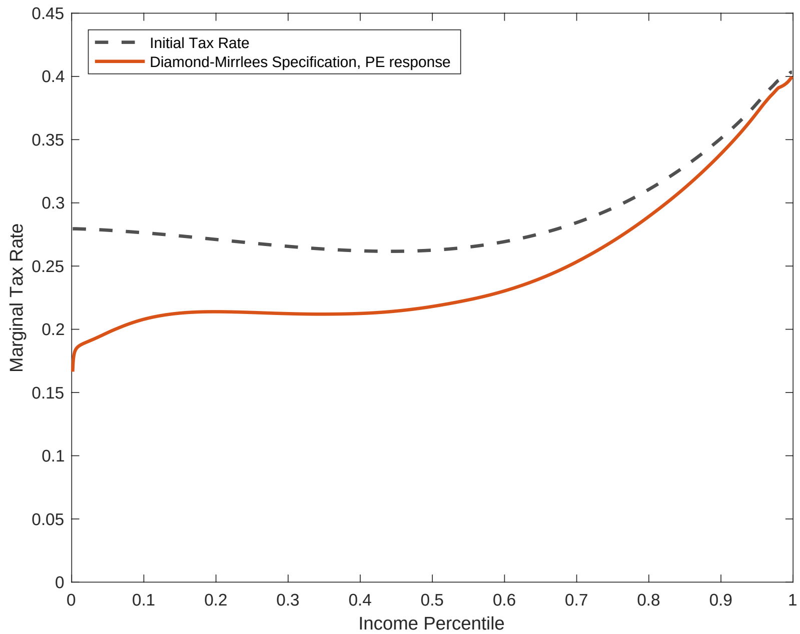
Notes: in all specifications, the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.33$. The CEX-CPI dataset is used in both panels and the initial tax schedule is taken from Hendren (2020). See Section 5.2.1 for a description of the quantitative model and counterfactuals.

Figure A2: The Response of the Optimal Tax Schedule to Observed Price Shocks (2004-2015), Nielsen data



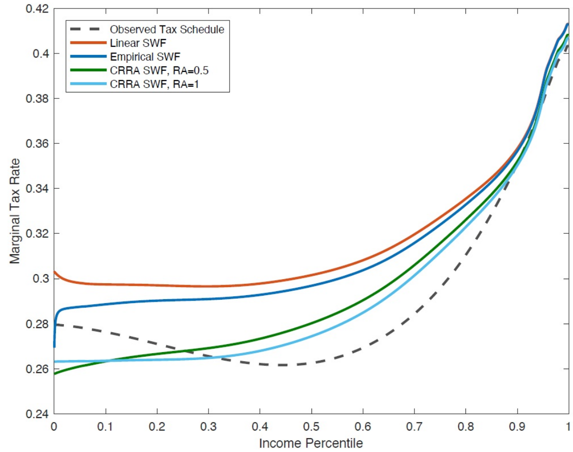
Notes: in this panel, we only consider the exogenous shock to prices, without taking into account the endogenous response of prices. The Nielsen data is used to measure price changes, and the initial tax schedule is taken from Hendren (2020). Section 5.2.2 describes the quantitative model and counterfactuals.

Figure A3: The Response of the Optimal Tax Schedule to Observed Price Shocks (2004-2015) in the Diamond-Mirrlees Supply-Side Model

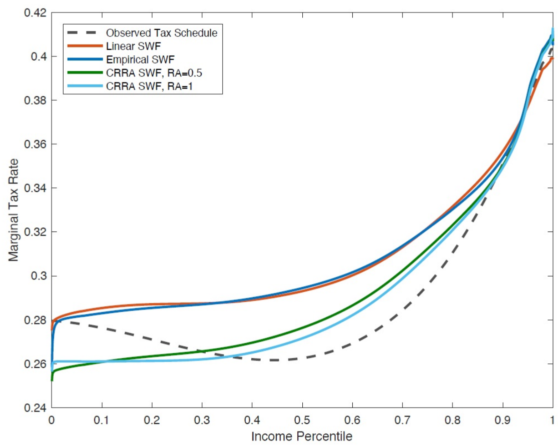


Notes: in this panel, we only consider the exogenous shock to prices, without taking into account the endogenous response of prices. The CEX-CPI linked dataset is used to measure price changes, and the initial tax schedule is taken from Hendren (2020). Section 5.2.2 describes the Diamond-Mirrlees model of the supply side, where goods are produced competitively and all profits are taxed.

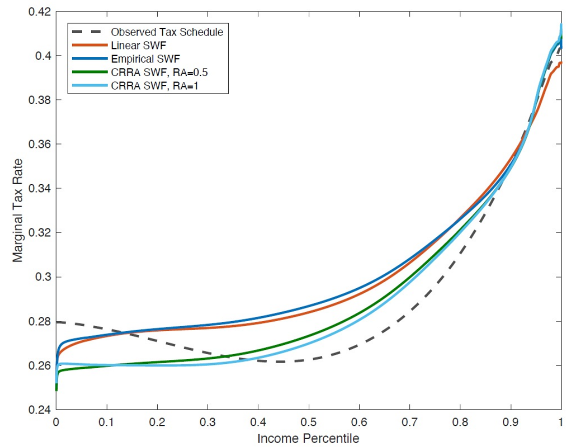
Figure A4: The Response of the Optimal Tax Schedule to Observed Shifts in the Skill Distribution (2004-2015): The Role of the Curvature of the Social Welfare Function



(i) Exogenous prices



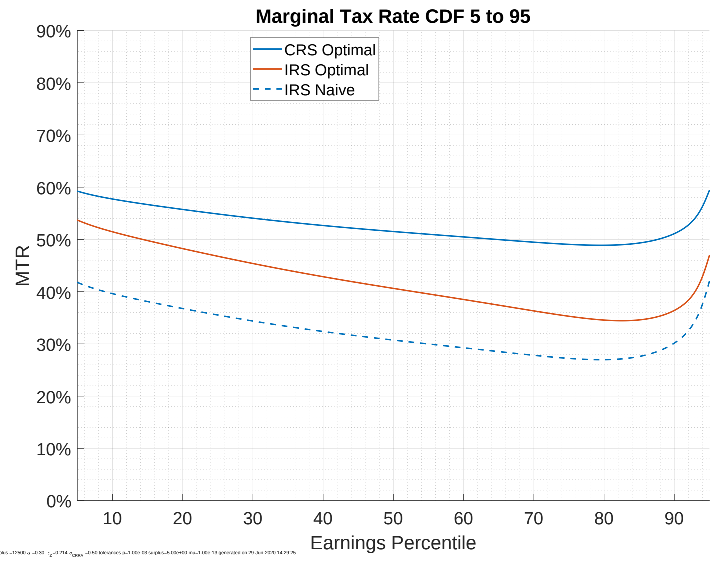
(ii) Endogenous prices, $\sigma = 0.6$



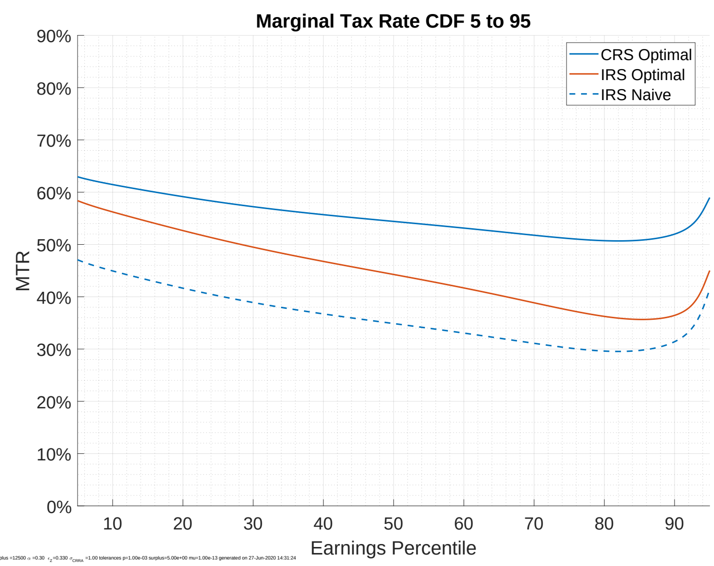
(iii) Endogenous prices, $\sigma = 2$

Notes: the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.21$; the CEX-CPI dataset is used in both panels and the initial tax schedule is taken from Hendren (2020). See Section 5.2.2 for a description of the quantitative model and counterfactuals.

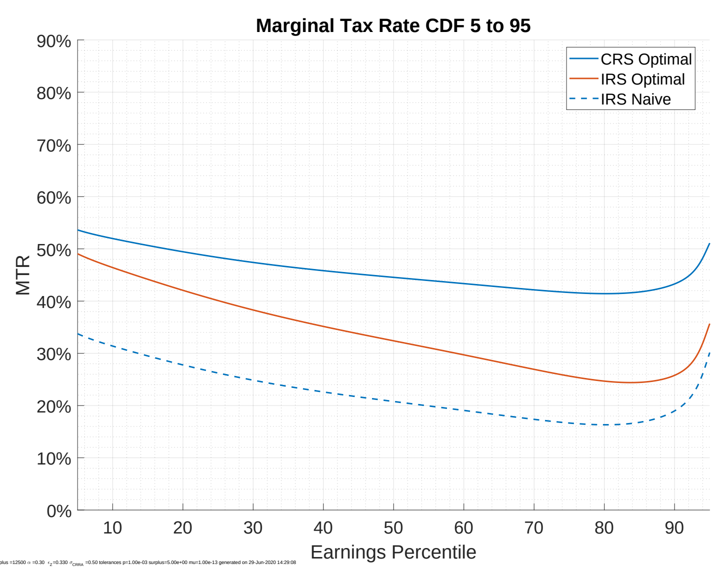
Figure A5: Returns to Scale and the Optimal Tax Schedule:
Sensitivity to Parameter Values



(a) $\alpha = 0.3$, $\varepsilon = 0.21$, CRRA=0.5



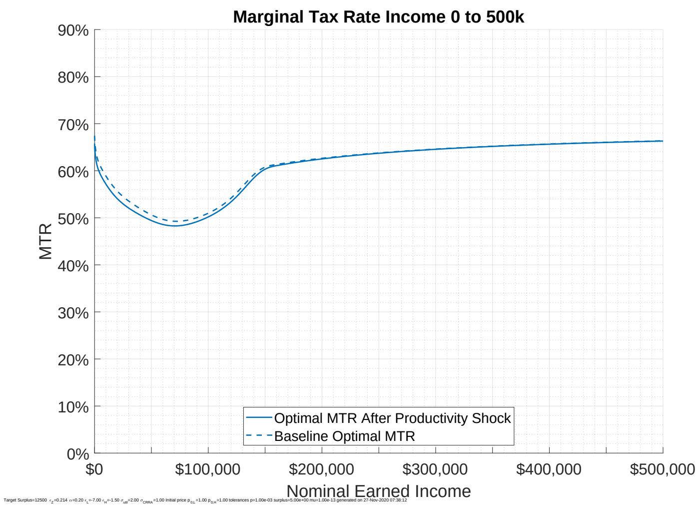
(b) $\alpha = 0.3$, $\varepsilon = 0.33$, CRRA=1



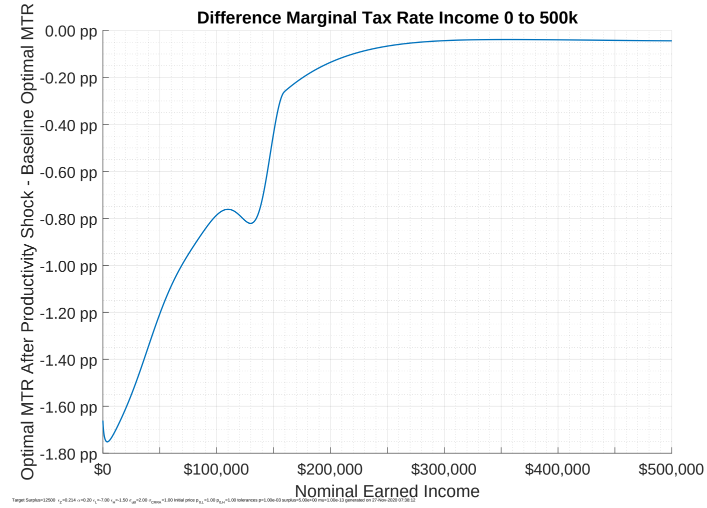
(c) $\alpha = 0.3$, $\varepsilon = 0.21$, CRRA=0.5

Notes: The "naive" correction uses the formula $1 - T'_{NAIVE}(\theta) = \frac{1}{1-\alpha}\left(1 - T'_{CRS}(\theta)\right)$.
See Section 5.3.1 for a description of the quantitative model and counterfactuals.

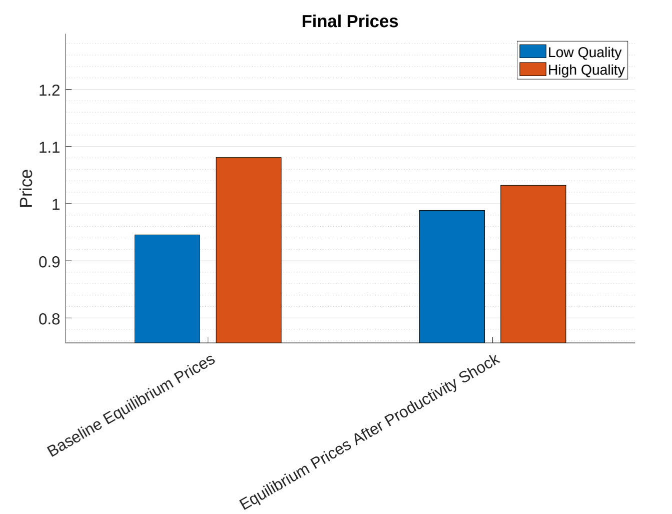
Figure A6: Lower Returns to Scale Reduce the Impact of Productivity Shocks
($\alpha = 0.2$, $\varepsilon_z = 0.21$, Pareto weights from SWF CRRA=1, PE price low-quality +2.5%, PE price high-quality -2.5%)



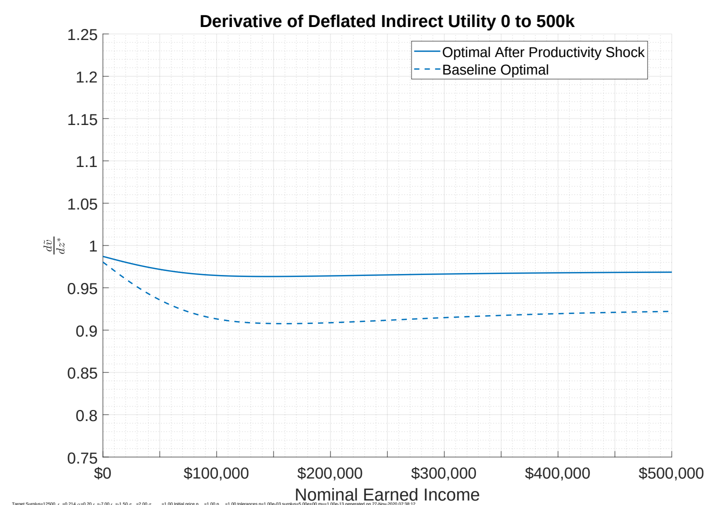
(a) Optimal MTRs Before vs. After Price Shocks



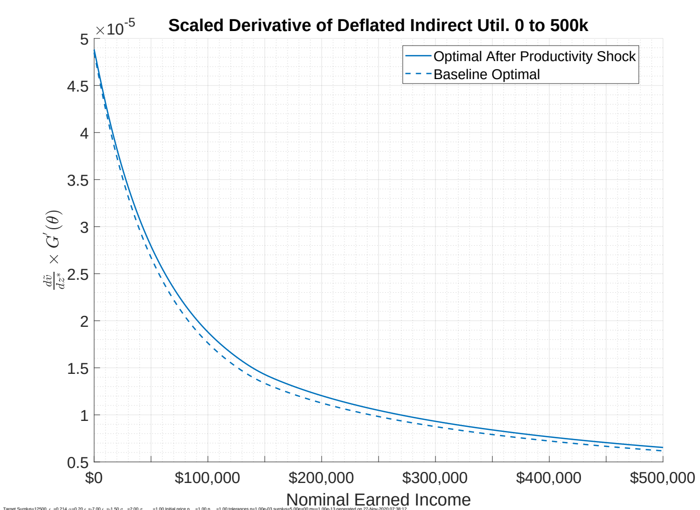
(b) Difference b/w MTRs



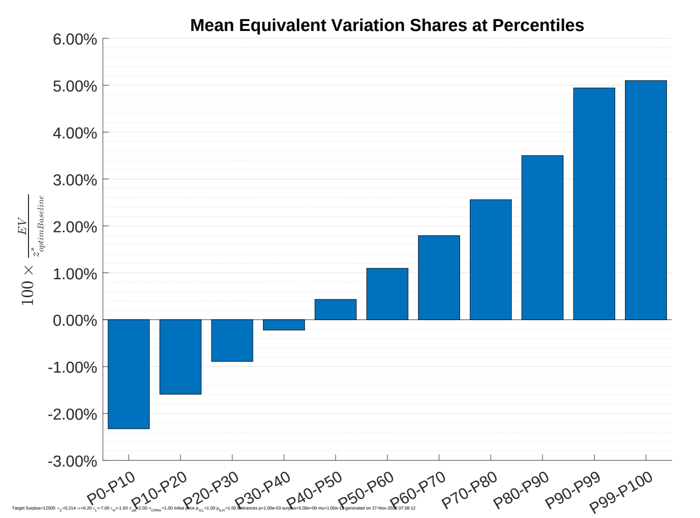
(c) Equilibrium Prices



(d) $\partial\widetilde{v}/\partial z^*$ by Earned Income Before vs. After Price Shocks



(e) $\partial\widetilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income



(f) EV, initial vs. new schedules

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1. The exogenous productivity changes are such that the partial equilibrium price of the low-quality bundle increases by 2.5% while the partial equilibrium price of the high-quality bundle decreases by 2.5%, as described in Section 5.3.3.