Prices, Non-homotheticities, and Optimal Taxation*

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Abstract

We characterize theoretically and quantitatively the effects of price changes on optimal tax design in the presence of non-homothetic preferences. We find that, rather than offsetting price changes, the optimal tax system *amplifies* their redistributive effects. With a quantitative model matching observed non-homotheticities and the empirical elasticity of prices to market size in the United States, we find that due to the amplification channel, (i) the optimal tax schedule is more redistributive when accounting for non-homothetic spending patterns, (ii) observed heterogeneous inflation rates, which are lower for luxuries relative to necessities in the United States, generate a regressive tax response.

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1 Introduction

How should tax policy respond when prices change? While price variation is pervasive, the optimal taxation literature has largely treated prices as fixed. Indeed, the seminal result of Diamond and Mirrlees [1971] states that optimal tax formulas can be derived as if prices were fixed at their equilibrium level, and leaves implicit the optimal response of taxes to price changes.[1] Yet, empirically price changes are ubiquitous – and importantly, they tend to correlate with household income. Recent work shows that heterogeneous inflation rates across products consumed by low- and high-income households played an important role for purchasing power inequality in the United States, with gradual increases in the relative price of necessities over time (e.g., McGranahan and Paulson [2005], Kaplan and Schulhofer-Wohl [2017], Jaravel [2019], Argente and Lee [2020], Klick and Stockburger [2021], Jaravel and Lashkari [2023], Jaravel [2024]). Despite the prevalence of price changes, to date we lack the tools to characterize their potential effects on optimal taxation.

In this paper, we develop a theoretical framework to analyze the effect of prices on optimal tax design, and we quantitatively estimate their impact. We provide an explicit characterization of the impact of prices on the marginal social value of transfers, on labor supply, and on labor supply elasticities. Under homothetic preferences, relative price changes have no implications for optimal taxes, since all households are equally affected. This neutrality breaks down once preferences are non-homothetic. Increases in the relative price of necessities disproportionately harm low-income households. A natural presumption is that a benevolent planner should offset these losses. We show, however, that although full compensation is feasible, it is never optimal. Instead, the planner optimally allows welfare at the bottom of the distribution to decline. Furthermore, if non-homotheticities are strong relative to the curvature of the social welfare function, the optimal tax response itself becomes regressive, amplifying the distributional effects of price changes. Applying our framework to the United States between 2004 and 2015, a period in which relative price changes disproportionately burdened low-income households, we show that the optimal adjustment of the tax schedule is both regressive and economically large.

To facilitate comparison with the prior literature, we work with a standard, static

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[1] Standard optimal tax formulas are first-order conditions featuring endogenous variables that depend on prices, such as the marginal utility of disposable income.

Mirrlees model: agents have preferences over multiple consumption goods and leisure, and labor is the only factor of production; preferences are weakly separable between consumption and labor, as in the Atkinson-Stiglitz benchmark. This setting allows us to capture non-homothetic spending patterns across the income distribution while focusing on a single tax instrument for redistribution, the nonlinear income tax.

The main challenge is that the channels through which prices shape redistribution are not explicit in standard optimal tax formulas, as prices appear only implicitly in the first-order conditions determining optimal marginal tax rates. Using a comparative static approach, we characterize the first-order responses of taxes to price changes in terms of observable statistics.

To isolate the role of non-homotheticities, we start by analyzing the case of linear production functions: prices are fully exogenous. We first consider the case of a linear social welfare function and identify two key channels through which prices affect the income tax schedule. We show that the impact of prices on taxes is governed by the marginal propensity to spend on the products experiencing a price change. To illustrate, consider an increase in the price of a product for which the marginal propensity to spend decreases with income, which we refer to as a "necessity product". First, a price increase on a necessity good raises the marginal price index of lower-income households relatively more than that of higher-income households, i.e. lower income households can now buy less with an additional dollar of income. Therefore, the social value of a dollar transfer from higher-income to lower-income households decreases (Channel #1). Second, as the decrease in marginal purchasing power is larger at lower income levels, the price increase generates a positive income effect on labor supply, which is higher as income increases. Thus a price increase on a necessity good increases the efficiency cost of taxation (Channel #2).[2] Since both the cost of taxation and the social value of transfers to higher-income increases, the marginal tax rates decreases everywhere, and redistribution to the poor falls.

Thus, with a linear social welfare function, the optimal tax system amplifies the redistributive effects of price changes: an increase in the price of necessity goods induces more redistribution at the top of the income distribution; the opposite is true when luxuries become more expensive. These channels do not operate when preferences are homothetic as all agents are equally impacted by price changes.

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[2] Negative income effects make taxation less costly: increasing the tax burden at a fixed marginal rate boosts labor supply and revenue. Conversely, positive income effects make taxation more costly.

We then consider the case of a concave social welfare function. In addition to Channel #1 and #2, price changes then operate through a third, more standard channel: an increase in the price of a necessity good directly reduces the real disposable income of low-income households, which, due to the concavity of the welfare function, raises the social value of transfers to these households. This creates an incentive to redistribute toward lower-income households as the price of necessities rises, partially offsetting the first two channels. We show, first, that there is a threshold level of curvature of the social welfare function below which transfers to low-income households decrease, and above which they increase, in response to a rise in the relative price of necessities. Second, even when transfers to low-income households become positive, they never fully offset the negative impact of the price change, although full compensation is feasible. For any concave welfare function, it remains optimal to reduce the welfare of low-income households and increase that of high-income households, reflecting the reduced benefits and increased costs of redistribution generated by Channels #1 and #2. While greater concavity attenuates the regressive effects of price changes at the optimum, stronger non-homotheticities amplify them. An increase in the price of a stronger necessity (i.e., a good consumed disproportionately more by low-income households) strengthens Channels #1 and #2 and therefore leads to less redistribution, regardless of the degree of concavity.

In the final part of the theory section, we consider non-linear production functions: prices become endogenous and adjust through general equilibrium effects. With non-linear production functions, the elasticity of prices with respect to aggregate consumer demand across products becomes pivotal to evaluate the interplay between optimal taxes and prices. Using a sufficient statistics specification, we capture both the canonical Diamond-Mirrlees setting – with perfect competition and potentially decreasing returns to scale – and a wide class of free entry models allowing for increasing returns to scale through firm selection (e.g., Melitz [2003]), variable markups (e.g., Feenstra and Weinstein [2017]), and innovation (e.g., Bustos [2011]).

We show theoretically that, when a product's price decreases as its market size expands (in line with the empirical evidence of Costinot et al. [2019], Jaravel [2019], and Faber and Fally [2021]), endogenous price adjustments amplify the optimal tax mechanisms identified with linear production functions. This amplification occurs through both substitution and income effects. For instance, when the relative price of necessities increases, consumers substitute away from them, which leads to further increases in their relative price through increasing returns. Moreover, the increase in the relative price of necessities leads to a fall in the real income of lower-income households, who consume relatively more necessities, implying a further decline in their relative demand and a further increase in the relative price of necessities. These amplification channels operate in any supply side model with elastic prices, although this effect remains implicit in the standard Diamond-Mirrlees tax formulas.[3]

Building on these theoretical insights, in the quantitative section of the paper we evaluate the optimal response of taxes to the price changes observed in the data in recent years. We first implement the sufficient-statistics formulas of our comparative static approach. While it only gives the first order response of taxes to price changes, we can non-parametrically fit non-homothetic spending patterns. By linking the Consumer Expenditure Survey (CEX) and the Consumer Price Index (CPI) datasets, we obtain observed price changes and households' spending across 248 product categories for the period 2004 to 2015, covering the entire consumption basket of American households. Empirically, inflation was lower in product categories with higher income elasticities. We find that, in response, it is optimal to reduce redistribution and set lower marginal tax rates. With a linear social welfare function, marginal tax rates fall by 8 percentage points at the bottom of the income distribution (relative to the observed tax schedule); they fall by 5.5 percentage points with a concave social welfare function.

Next, we make parametric assumptions on non-homotheticities, using non-homothetic CES preferences as in Hanoch [1975] and Comin et al. [2021]. We then study the quantitative importance of increasing returns to scale, non-homotheticities and price shocks for optimal tax rates and welfare across the skill distribution. By introducing parametric assumptions on preferences, these analyses complement the first-order approximations, because they characterize how our new channels affect the optimum when accounting for potential non-linearities. They also allow us to characterize the quantitative importance of these channels for the optimal tax schedule more generally, beyond the optimal response of taxes to observed price changes.

Relative to the optimal tax schedule with homothetic preferences, we find that non-homotheticities imply *more* redistribution: marginal taxes increase over the full

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[3] The redistributive effects of prices are instead *muted* through general equilibrium effects when prices increase as the market expands (in contrast with our baseline case featuring increasing returns, in line with empirical evidence).

range of the income distribution. The increase is more pronounced at the bottom of the income distribution, with an increase in marginal tax rates of about 6pp for levels of earned income below \$20,000. The increase is about 2pp at an income level of \$100,000, and then gradually decreases, reaching levels close to zero above \$300,000. Thus, the simulations show that non-homotheticities have a significant quantitative impact on optimal marginal tax rates. To document whether these tax changes and their induced price effects have meaningful distributional effects, we compute the willingness to pay of agents for the optimal tax schedule under non-homothetic preferences, relative to the optimal schedule under homothetic preferences. The equivalent variation is sizable, ranging from about +15% in the bottom decile of the income distribution to -9% in the top decile.

We show that this increase in redistribution can be explained by the change in equilibrium prices and in the marginal utility of redistribution across the skill distribution. As the relative price of the necessity bundle decreases, it is optimal to redistribute more to those with a higher marginal propensity to consume on necessities, which induces further tax changes, price changes, and changes in labor supply. The strength of these feedback loops depends on the parameters governing increasing returns and social preferences for redistribution. Considering a range of alternative parameter values for robustness, we consistently find that feedback loops play a quantitatively important role.

Related literature. The main contribution of this paper is to provide a theoretical and quantitative characterization of the impact of prices on optimal tax design. We thus relate to several strands of literature. First, in prior work the effect of prices on the tax schedule has remained implicit, as standard tax formulas depend on endogenous variables that depend on prices, such as the marginal utility of disposable income. Several papers have highlighted the implications of specific assumptions on consumers' preferences for tax design, including preference heterogeneity (e.g., Saez [2002], Diamond and Spinnewijn [2011]) and consumers' myopia (e.g., Allcott et al. [2019]). Instead, we show theoretically and quantitatively that prices play an important role even in the canonical setting where the utility function is separable between labor and all commodities, i.e. no indirect taxes need to be used, as in Atkinson and

Stiglitz [1976].[4] We explicitly characterize the impact of prices on the tax schedule, providing decompositions isolating the economic forces at play. Second, our results contribute to a growing strand of the optimal taxation literature that has isolated the general equilibrium effects of taxes, focusing on wages (e.g., Rothschild and Scheuer [2013], Sachs et al. [2020]); we complement these analyses by characterizing the general equilibrium impact on prices in the presence of non-homotheticities. Third, although imperfect competition is not our focus, our work relates to a growing literature on optimal taxation in the presence of imperfect competition, in which endogenous prices or wages play a role for redistribution from firm owners toward workers (e.g., Boar and Midrigan [2019], Eeckhout et al. [2021], Kushnir and Zubrickas [2020]).[5] Instead, we demonstrate the importance of non-homotheticities and show that prices play an important role even in the canonical setting with no profit or full profit taxation, as in Diamond and Mirrlees [1971]. We thus isolate a novel mechanism, the amplification of redistribution due to the interaction between price changes and non-homotheticities.

Furthermore, by studying price changes stemming from increasing returns to scale, this paper contributes to a growing literature on optimal tax design and endogenous productivity. Prior work highlights the role that taxes may have on entrepreneurial effort (e.g, Jaimovich and Rebelo [2017], Bell et al. [2018]) and draws implications for optimal taxation of top earners (e.g, Jones [2019], Bell et al. [2019]). In contrast, we study productivity effects that are induced by changes in demand, through returns to scale, and which inherently interact with the income tax schedule. We find that the

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[4] Our analysis is also conceptually distinct from the contribution of Naito [1999], whose channel requires segmented labor markets and multiple factors of production and can operate with homothetic utility. Our analysis can have a single factor of production (with differences in efficiency units across agents) and requires non-homothetic utility.

[5] These papers highlight the importance of rents that accrue to firm owners, which can be redistributed through taxation of income, endogenous price changes, and commodity taxes. We instead characterize different channels, which continue to apply in settings with no rents, i.e. with full profit taxation or zero profit. While Kushnir and Zubrickas [2020]'s Appendix A.3 examines the case of non-homothetic preferences, the impacts of non-homotheticities and prices remain implicit in their tax formulas through endogenous variables that depend on prices, such as the marginal utility of disposable income. In their study, the social planner uses the price level as an additional redistributing tool: a decrease in the price level benefits low-productivity agents as they can afford more consumption, but hurts high-productivity agents through a decrease in profits. We characterize a different channel: our price effects operate through non-homotheticities and changes in the marginal utility of income at different income levels. We do not need to track the distribution of profits across households, who hold a diversified portfolio of firms with zero profits. We can thus cleanly separate our analysis from complementary prior work focusing on the distribution of profits.

impact of taxes on productivity through demand and returns to scale is quantitatively large, implying substantial adjustments to the optimal tax schedule.

Outline. The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 derives the optimal income and commodity taxation formula in terms of sufficient statistics. Section 4 uses the comparative static approach to characterize the sensitivity of optimal tax rates to price shocks. The quantitative analysis is carried out in Section 5.

The Main Appendix reports additional figures and a more in-depth discussion of the social planner's problem. The Supplemental Appendix presents the proofs of all theoretical results in the main text. Finally, in a Companion Note available online, we present several extensions, including additional theoretical and quantitative results.

2 Model

To streamline our analysis, we present in the main text the simplest possible model allowing us to illustrate the mechanisms described in the introduction.

We consider a two-sector economy,[6] with sectors indexed by $k = l, h$. A unit mass of households differs in productivity type θ, distributed according to $\pi(\theta)$.

Households. Households' preferences over goods c_h, c_l and hours worked z/θ, where z denotes pre-tax income, are given by:

$$u(c_l, c_h) - \frac{1}{1+\frac{1}{\varepsilon}} \left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon}},$$

with u concave, increasing and $\mathcal{C}^3$, and $\varepsilon \leq 1$. Preferences are of the Atkinson-Stiglitz type so that consumption choices only depend on consumer prices and post tax income z^*. This specification allows us to capture non-homothetic spending patterns across the income distribution, and thus the unequal effects of price changes, while focusing on a single tax instrument for redistribution, the nonlinear income tax.[7]

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[6] All results directly generalize in an n-sector economy, the proofs in the Supplemental Appendix are derived with n sectors.

[7] With more general preferences, it would be possible to use the consumer prices of certain goods to better discriminate between different taxpayers (e.g., Saez [2002], Ferey et al. [2023]). However, we focus on characterizing how unequal price changes for consumption baskets along the income

We denote by V the indirect utility of the agent, by v the indirect sub-utility out consumption (i.e., the maximum of $u(c_l, c_h)$ at fixed post tax income z^*), and by v_{z^*} the marginal utility of income. V depends on the agent type θ, on consumer prices and on the tax schedule. Aggregate demand for good k is denoted C_k.

Firms. We adopt a supply-side formulation that nests both perfect competition and monopolistic competition with free entry. The competitive case, following the canonical Diamond-Mirrlees framework, remains central in public finance. In that case, prices are pinned down by technology and demand. However, incorporating monopolistic competition is crucial for capturing how prices respond to demand shifts. Empirical evidence (e.g., Costinot et al. [2019], Jaravel [2019], Faber and Fally [2021]) shows that prices tend to fall as market size grows—a result driven by increased entry and declining markups. Our framework is meant to accommodate both forces: technological determination of prices and endogenous markups that respond to demand.

In each sector, good k is produced either competitively or monopolistically using labor as the sole input. Under these assumptions, we can summarize production with a cost function $\chi_k(C_k, \xi_k)$, and a pricing function $p_k = \phi_k(C_k, \xi_k)$ where ξ_k is a cost shifter. Under perfect competition, the pricing function is simply the marginal cost of production: $\phi_k(C_k, \xi_k) = \partial_{C_k}\chi_k(C_k, \xi_k)$. Under monopolistic competition, we assume that firms can freely enter market k, by paying a fixed labor cost, in which case total cost is equal to total revenue: $\chi_k(C_k, \xi_k) = C_k\phi_k(C_k, \xi_k)$.[8]

An important statistic for our analysis is the elasticity of the price p_k to market size, $-C_k/p_k\, \partial_{C_k}\phi_k$. In the main text, we impose that this elasticity is constant and equal to α in all sectors. This elasticity plays a central role: tax changes shift households' incomes and thus aggregate demand for goods, which affects prices.

To illustrate our supply side model, we provide some simple parametric microfoundations as examples. First, consider a competitive case with a representative

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distribution affect the desirability of redistribution policies. The heterogeneous welfare impacts of price changes only depend on the heterogeneity in households' expenditure shares, whether they stem from idiosyncratic preferences or households' income levels.

[8] In our micro-founded model presented in the Companion Note, we assume that ex ante identical monopolistic firms pay a fixed labor cost to enter the market and a labor cost to produce differentiated varieties of the good. Competitive retailers then bundle the varieties using a constant return to scale technology: $\phi_k(C_k, \xi_k)\, C_k$ is therefore both the revenue of retailers and the total revenue of entrants. Because entry is free, total revenue – equivalent to expected revenue for a marginal entrant – has to be equal to total labor costs (entry plus production costs) – equivalent to expected costs.

firm in sector k producing its good at cost $\chi_k\left(C_k,\xi_k\right)=\xi_k C_k^{1-\alpha}/(1-\alpha)$. The pricing function is then $\phi_k\left(C_k,\xi_k\right)=\xi_k C_k^{-\alpha}$ and the elasticity of the price p_k to market size is α. In particular, $\alpha=0$ corresponds to the standard case where production functions are linear and prices are exogenous, given by ξ_k.

For the monopolistic case, consider the Melitz-Chaney model (Melitz [2003], Chaney [2008]). Producers of differentiated varieties of product k can freely enter market k by paying a fixed labor cost $\xi_{e,k}$. Upon entering, they draw their productivity type $\gamma\left(i\right)$ from a Pareto distribution $1-\Psi_k\left(\gamma\right)=\gamma^{-\gamma_k}$. To start production, firms have to pay a second fixed cost, $\xi_{p,k}$. The variable labor cost of producing $c_{k,i}$ units of variety i is $c_{k,i}/\gamma\left(i\right)$. Competitive retailers then aggregate the varieties according to $C_k=\left(\int_{i\in\mathcal{I}_k}c_{k,i}^{1-\alpha}di\right)^{\frac{1}{1-\alpha}}$, with $0<\alpha<1$, where $\mathcal{I}_k$ is the set of producing firms. We then obtain the price of good k is $p_k=C_k^{-\alpha}\varphi_k(\boldsymbol{\xi}_k)$[9] and the elasticity of the price p_k to market size is α.

Planner's problem. The social planner has access to a full set of commodity taxes and to a non-linear income tax and a full tax on profits (in the monopolistic case, profits are 0). As our agents have Atkinson-Stiglitz preferences, the role of commodity taxation is limited but we include commodity taxes for completeness.

The planner maximizes the following social welfare function,

$$\int_{\underline{\theta}}^{\bar{\theta}}G(V(\theta),\theta)\pi(\theta)d\theta,$$

with G concave and increasing in its first argument. The planner sets consumer prices $\{q_h,q_l\}$ and the income tax $T(z)$ subject to three constraints. First, the producer prices p_h,p_l are given by the functions ϕ_k, with $p_k=\phi_k(C_k,\xi_k)$, where C_k denotes the aggregate demand for k. Second, households optimally choose consumption and labor supply under q_h,q_l and $T(z)$. We denote by $f(z)$ the resulting distribution of income, with $dz/d\theta f(z)=\pi(\theta)$, and by z^* disposable income, with $z^*=z-T(z)$. Finally, the government's budget constraint is given by $\sum_{k=h,l}(q_k-p_k)C_k+\mathbb{E}_z(T(z))+\sum_{k=h,l}\left(p_kC_k-\chi_k\left(C_k,\xi_k\right)\right)\geq 0$.

With this formulation of the planner problem, we encompass both the standard Diamond-Mirrlees framework where firms are competitive and profits are fully taxed

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[9] With $\varphi_k(\boldsymbol{\xi}_k)=\frac{1}{\alpha}\xi_{p,k}\left(\frac{\alpha(1+\gamma_k)-1}{1-\alpha}\frac{\xi_{e,k}}{\xi_{p,k}}\right)^{-\frac{1}{\gamma_k}\alpha}\left(\frac{\alpha}{1-\alpha}\frac{1}{\xi_{p,k}}\right)^{\frac{1}{1-\alpha}}$.

and the monopolistic case with free entry where firms earn zero profits on average ($\chi_k(C_k, \xi_k) = C_k \phi_k(C_k, \xi_k)$). The Theory Appendix at the end of the paper discusses how the solution of the government problem is constrained efficient.

Notation. We use standard notation throughout. $\zeta \equiv \varepsilon / \left(1 - \varepsilon \partial_{\ln(z)} \ln(v_{z^*})\right)$ is the compensated labor supply elasticity and $\tilde{\zeta}$ the compensated labor supply elasticity corrected for non-linearities in the budget constraint: $\tilde{\zeta} = \zeta/(1 + z\zeta T''/(1-T'))$. Similarly, $\eta = \zeta \partial_{\ln(z)} \log(v_{z^*})$ is the income effect with a linear budget constraint and $\tilde{\eta}$ the corrected income effect (e.g, Scheuer and Werning [2016]). Regarding spending patterns, $e_k = q_k c_k(z^*, q)$ denotes expenditure on k, $s_k(z^*, q) = e_k/z^*$ its budget share, and $\partial_{z^*} e_k$ the marginal propensity to spend on k. E_k and $\bar{s}_k$ are aggregate expenditure and the aggregate spending share, and $\partial_{z^*} E_k$ the average marginal propensity to spend on k. Finally, S is the matrix of cross price derivatives of aggregate Hicksian demand, with $S_{jk} = \mathbb{E}(\partial_{q_k} c_j + \partial_{z^*} c_j c_k)$, $\mathcal{S}$ the matrix of price elasticities $\mathcal{S}_{jk} = q_k/C_j S_{jk}$, and $\sigma = -\mathcal{S}_{hh} + \mathcal{S}_{lh} = -\mathcal{S}_{ll} + \mathcal{S}_{hl}$ the elasticity of substitution.

3 Optimal Taxation: First-Order Approach

In this section, we characterize the optimal commodity and income taxes. While we provide heuristic derivations in this section, the Supplemental Appendix reports the formal proofs. Our Companion Note provides a generalization of Proposition 1, with general household preferences in a multi-sector economy with heterogeneous returns to scale and potential spillovers across sectors.

Commodity Tax. Consider a small change in the consumer price of k, dq_k, compensated with an income tax change $dT(z) = -c_k(z^*, q) dq_k$. As explained in Saez [2002], this compensation keeps the welfare and labor supply of all agents constant.[10] Therefore, the impact on government revenue is:

$$\underbrace{dq_k C_k}_{\text{Mechanical effect}} + \underbrace{\mathbb{E}(dT(z))}_{\text{Cost of the compensation}} + \underbrace{\sum_{j=h,l} (q_j - \partial_{C_j} \chi_j(C_j, \xi_j)) dC_j}_{\text{Households' behavioral response}}.$$

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[10] Intuitively, the compensating tax change keeps the indirect utility out of consumption v unchanged despite the price shock. Therefore the initial labor supply choice of the household remains optimal.

The increase in q_k first mechanically raises revenues from the tax on k by $dq_k C_k$. Households are compensated for the consumer price increase through the income tax, so revenue from the income tax decreases: $\mathbb{E}(dT) = -C_k dq_k$. The mechanical effect and the cost of the compensation exactly offset each other. Since dq_k is compensated, aggregate consumption reacts through a substitution effect and the impact on government revenue of the households' change in consumption is $\sum_{j=h,l}(q_j - \partial_{C_j}\chi_j\left(C_j, \xi_j\right))C_j \mathcal{S}_{j,k}\left(dq_k/q_k\right)$. Given our supply side specification, we have $\partial_{C_j}\chi_j\left(C_j, \xi_j\right) = \left(1 - t_w\right)p_j$ with $t_w = \alpha$ in the monopolistic case and $t_w = 0$ in the competitive case.[11] This change in marginal cost captures the response of the supply side to the shift in demand, and the adjustment of producer prices. At the optimal consumption prices, revenue should remain unchanged, so we obtain: $\sum_{j=h,l}\left(q_j - (1 - t_w)p_j\right)C_j \mathcal{S}_{j,k}\frac{dq_k}{q_k} = 0$. Since this must hold for both dq_l and dq_h, we have $q_k = \beta p_k$ at the optimum,[12] with β an arbitrary scaling constant. Without loss of generality, we choose the scaling so that on average commodity taxes raise no revenue, which implies that we get $q_k = p_k$.[13] We therefore obtain a version of the standard Atkinson-Stiglitz result that commodity taxes are not used at the optimum.[14]

Income Tax. Next, consider the perturbation of Saez [2001], a small change of marginal tax $d\tau$ in a neighborhood dz of z and a change in tax $dzd\tau$ above. This has four effects: a mechanical change in revenue, a welfare effect, a fiscal externality due to labor supply responses, and a fiscal externality due to shifts in aggregate consumption and producer price adjustments.

Mechanical and Welfare Effects. Households above z pay an additional $dzd\tau$ in taxes. The welfare loss is $G'v_{z^*}dzd\tau/\lambda$, where λ is the Lagrange multiplier on the government budget constraint. The total effect on social welfare is:

$$\mathbb{E}_{z'>z}\left(1 - G'v_{z^*}/\lambda\right)dzd\tau.$$

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[11] In the monopolistic case, we have $\partial_{C_k}\chi_k\left(C_k, \xi_k\right) = \phi_k\left(C_k, \xi_k\right) + C_k\partial_{C_k}\phi_k\left(C_k, \xi_k\right) = \left(1 - \alpha\right)p_k$; in the competitive case, we have $\partial_{C_k}\chi_k\left(C_k, \xi_k\right) = \phi_k\left(C_k, \xi_k\right) = p_k$.

[12] Indeed, $\sum_{j=h,l} q_j C_j \mathcal{S}_{j,k} = 0$ since Hicksian demand is homogenous of degree zero in q.

[13] Conceptually, the average commodity tax should be zero. If it was instead positive (or negative), consumer prices would be on average higher than producer prices, which is an implicit income tax (or subsidy).

[14] The derivation also makes clear that when pricing inefficiencies varies across sectors, then commodity tax should be used to correct relative inefficiencies across sectors (i.e., heterogeneous α_k's). This is not the case in our benchmark specification where pricing inefficiencies are the same in both sectors.

Labor Supply Effects. The change in tax rate at z generates a compensated wage effect on labor supply, while the change in the tax burden above z creates an income effect. The change in government revenue is:

$$-f(z)\frac{T'}{1-T'}z\tilde{\zeta}dzd\tau-\mathbb{E}_{z'>z}\left(\frac{T'}{1-T'}\tilde{\eta}\right)dzd\tau.$$

Price and Demand Effects. The change in the tax schedule affects households' disposable income both mechanically and through labor supply responses. This leads to a change in aggregate demand for goods, through substitution and income effects, and in producer prices and costs. The total impact on government revenue, through the receipts of the commodity and profit taxes, is given by:

$$\begin{aligned}\sum_{k=h,l}(q_k-\partial_{C_k}\chi_k\left(C_k,\xi_k\right))dC_k&=\sum_{k=h,l}\left(p_k-\left(1-t_w\right)p_k\right)dC_k\\&=-\,t_w\left(f(z)z\tilde{\zeta}+\mathbb{E}_{z'>z}\left(1+\tilde{\eta}\right)\right)dzd\tau.\end{aligned}$$

In the derivation above, the first line uses $\partial_{C_k}\chi_k\left(C_k,\xi_k\right)=\left(1-t_w\right)p_k$ and $q_k=p_k$. The second line uses the fact that, from the budget constraint of households, $\sum_{k=h,l}p_kdc_k$ is equal to the change in disposable income. To interpret this effect, note that when $t_w>0$, prices are above marginal costs and demand for goods is inefficiently low. An increase in the income tax decreases labor income and further depresses demand, accentuating the initial inefficiency.

Summing up all of the effects derived above gives the first order conditions for the optimal tax rate. We denote by g the social welfare weights $g=G'v_{z^*}/((1-t_w)\lambda)$, where the $1-t_w$ normalization is such that $\mathbb{E}(g)=1$ when there are no income effects. Collecting our earlier result on optimal commodity taxes, we then obtain our first Proposition.

Proposition 1. *Commodity taxes are not used at the optimum. The optimal non-linear income tax schedule is characterized by:*

$$\frac{T'}{1-T'}=-t_w+\frac{1-t_w}{z\tilde{\zeta}f(z)}\left\{\mathbb{E}_{z'>z}\left(1-g\right)-\frac{1}{1-t_w}\mathbb{E}_{z'>z}\left(\left(t_w+\frac{T'}{1-T'}\right)\tilde{\eta}\right)\right\},\quad(1)$$

where $t_w=\alpha$ in the monopolistic case and $t_w=0$ in the competitive case. With $\alpha=0$, we obtain the standard optimal tax formula in both cases.

Proposition 1 first shows that, as in the standard Atkinson-Stiglitz framework, commodity taxes are not needed. This is not the case when α varies across sectors: commodity taxes then have a corrective role but are not used for redistribution.

Turning to the optimal income tax schedule, Proposition 1 suggests that when the average market size elasticity is positive and firms are not competitive ($t_w = \alpha > 0$), labor supply is subsidized and optimal tax rates are reduced. Intuitively, there is an externality from working: more labor supply increases aggregate income, i.e. market size, and leads to a fall in prices through returns to scale. If the endogenous quantities (the social welfare weights, the income distribution, and the labor supply elasticities) remain constant as α varies, then the formula tells us that the tax rate with $\alpha > 0$ is such that $1 - T' = (1 - T')_{\alpha=0}/(1 - \alpha)$. In that case, the planner implements a uniform wage subsidy $1/(1 - \alpha)$ on top of the standard non linear tax, and it appears that there is no interaction between the corrective tax (the wage subsidy) and redistributive motives. This interpretation is however naive, as prices and all endogenous quantities are likely to vary as α changes: the interaction between corrective tax and redistributive motives is hidden in the formula.

These observations highlight an important limitation of the standard optimal tax formula, which leaves the effects of prices completely implicit and therefore provides little insight about how prices affect optimal redistribution. In the next section, we provide a characterization of the role of prices for optimal taxes.

4 Understanding the Impact of Prices and Nonhomotheticities

In this section, we use a comparative statics approach to understand the mechanisms through which optimal tax rates respond to prices in the presence of nonhomotheticities. Our Companion Note extends this section's results, with general household preferences in a multi-sector economy with heterogeneous returns to scale and potential spillovers across sectors.

4.1 Assumptions

To streamline the analysis, we make the following assumptions. First, while sectors h and l were arbitrary in the previous section, we now impose that l is a "necessity" good (therefore h is a "luxury" good) to highlight the importance of non-homotheticities.[15] Second, we make an assumption on the distribution of skills.

Assumption A1. *At initial prices, l is a necessity good: $\partial_{z^*} e_l$ is decreasing in post-tax income and $\partial_{z^*} E_l \leq \bar{s}_l$, where $\bar{s}_l$ and $\partial_{z^*} E_l$ are the aggregate spending share and average marginal propensity to spend on l.*

Assumption A2. $\underline{\theta}\pi(\underline{\theta}) = 0$ *and* $\left(1 + \theta\pi'\left(\theta\right)/\pi\left(\theta\right)\right)\epsilon/\left(1+\epsilon\right) \leq 1$ *for all* θ.

These assumptions allow for clean theoretical results in the following section but are not substantive restrictions. They do not affect the tax formulas of Proposition 2: $A1$ is used to sign the tax response and $A2$ to establish monotonicity of the welfare response. We relax $A1$ in our quantitative analysis in Section 5 to match observed spending patterns for all products. The empirical income distribution in the United States satisfies $A2$, as discussed in Section 5.

Next we normalize the income effect of labor supply to 0 at initial prices:

Assumption A3. *There are no income effects at initial prices, i.e.* $v_{z^*} = 1\,\forall z^*$.

This assumption is common in the optimal taxation literature and provides a useful benchmark to facilitate comparisons between our results and prior work; it is relaxed in the Companion Note. To clarify its role, note that household preferences can be written as $\Psi\left(u\left(c_l, c_h\right)\right) - \left(1+\epsilon^{-1}\right)^{-1}\left(z/\theta\right)^{1+\epsilon^{-1}}$. Here, the function Ψ parametrizes the income effect of labor supply without affecting consumption demand $c_k\left(z^*, p\right)$.[16] This normalization simplifies the labor supply side of the model, allowing us to focus on how non-homotheticities in consumption preferences shape optimal redistribution, compared to prior work where the benchmark formulas similarly do not feature income effects (e.g., Diamond [1998], Saez [2001]). Importantly, the assumption does not restrict the concavity of the social welfare weight, which can be arbitrarily adjusted through G ($g = G' v_{z^*}/((1-t_w)\lambda)$).

In the remainder of this section, we mostly analyze how the optimal tax schedule and household welfare respond to changes in the relative price of the necessity good. To isolate redistribution effects from aggregate efficiency concerns, we consider a

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[15] The comparative statics approach we use in this section allows us to provide an explicit characterization of the impact of non-homotheticities and market size effects on optimal taxation, in terms of observable statistics. With non-homotheticities, it is not possible to obtain a closed form solution of the optimal tax schedule in partial or general equilibrium. With homothetic utility, quasilinear preferences in consumption, and a linear social welfare function, a closed form solution can be obtained in some cases (e.g., Eeckhout et al. [2021]).

[16] Details on how to choose Ψ are provided in Section 5: see footnote 33.

compensated price change: the price of the necessity rises while that of the luxury falls, keeping the average price level constant. Formally, we consider an increase $d\ln\bar{p}_l$, such that $d\ln p_l = \bar{s}_h d\ln\bar{p}_l$ and $d\ln p_h = -\bar{s}_l d\ln\bar{p}_l$.[17]

4.2 Response to a Price Change with Linear Production Functions: Channels #1 and #2

To isolate the role of non-homotheticities, we first focus on the response of the optimal income tax schedule to a price change when prices are exogenous. This is the case when $\alpha = 0$, which imposes $p_k = \phi_k(\xi_k)$ and $\chi_k(C_k, \xi_k) = \phi_k(\xi_k) C_k$ with both competitive and monopolistic firms (linear production functions). We show that price changes affect optimal taxes through two channels: a change in the value of redistribution (Channel #1) and a change in the efficiency cost of taxation (Channel #2).

Response with a Linear Social Welfare Function

We first consider the case of a linear social function, i.e. $G(V(\theta), \theta) = \lambda_\theta V(\theta)$,[18] which allows us to derive closed form solutions. Prices affect both the value and the cost of redistribution around θ (i.e. around the $F(z(\theta))$ percentile of the income distribution). While these effects are only implicit in Proposition 1, we now make them explicit for a marginal change in the price of good k, p_k. Recall that in Proposition 1 the optimal tax rate is defined implicitly by

$$\frac{T'}{1-T'} = \frac{1}{z\tilde{\zeta} f(z)} \left\{ \underbrace{\mathbb{E}_{z'>z}(1-g)}_{\text{Welfare effects}} - \underbrace{\mathbb{E}_{z'>z}\left(\frac{T'}{1-T'}\tilde{\eta}\right)}_{\text{Behavioral effects}} \right\}. \tag{2}$$

Under $A3$, we have $z\tilde{\zeta} f(z) = \epsilon / (\epsilon + 1)\, \theta\pi(\theta)$, independent from prices and taxes, so we only need to derive the change in the Welfare and Behavioral Effects terms, the two channels of the adjustment of the tax system. We now provide heuristic derivations these channels, presenting the proofs in Supplemental Appendix A.2.

Welfare Effects (Channel #1). Under $A3$, the derivative of social welfare weights

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[17] We report the comparative statics for uniform price increases in the Supplemental Appendix.

[18] Note that λ_θ can be used arbitrarily to parametrize the level of redistribution at initial prices.

with respect to prices p_k satisfies $p_k \partial_{p_k} g = -g \partial_{z^*} e_k$, with $\partial_{z^*} e_k$ the marginal propensity to spend on good k. Indeed, at the initial prices the social value of a dollar transfer to an agent with income z is given by the social welfare weight g. With this additional dollar, the agent spends $\partial_{z^*} e_k$ on good k. When the price of k increases, the purchasing power of the agent is therefore reduced at the margin by $\partial_{z^*} e_k$. Since an agent at z can buy less with an additional dollar, the value of a dollar transfer is reduced. This channel can be thought of as a "terms of trade effect for redistribution."

The change in the optimal income tax for type θ is determined by the change in individual purchasing power for type θ *relative to* the average change in purchasing power, $\mathbb{E}\left(g \partial_{z^*} e_k\right)$. The planner decreases the tax rate at θ – to redistribute more to agent with income $z(\theta') > z(\theta)$ – if a dollar transfer above θ buys relatively more welfare than below θ, that is if the (marginal) purchasing power after the price change decreases relatively less above than below. For the necessity good l, the marginal propensity to spend decreases with income, so the tax rate is lowered everywhere in response to an increase in p_l.

Therefore, the adjustment in tax rates $\frac{p_k d}{dp_k}\left\{\frac{T'}{1-T'}\right\}$ through Channel #1, the change in the value of redistribution at θ, is given by:

$$\frac{1}{z\tilde{\zeta} f(z(\theta))} \mathbb{E}_{z>z(\theta)} \Big(g \left(\partial_{z^*} e_k - \mathbb{E}\left(g \partial_{z^*} e_k\right)\right) \Big).$$

By contrast, when preferences are homothetic, there is no effect on the tax rate since the change in purchasing power is uniform along the income distribution.

Behavioral Effects (Channel #2). Under $A3$, the derivative of the income effect with respect to consumer prices p_k satisfies $p_k \partial_{q_k} \tilde{\eta} = -z\tilde{\zeta}((1-T')\partial_{z^* z^*} e_k$. Under $A1$, the marginal propensity to spend on good l decreases, $\partial_{z^* z^*} e_k < 0$. An increase in the price of l causes a fall in the households' (marginal) purchasing power, which is smaller at higher income levels. Therefore a dollar transfer to an agent makes work more valuable – since they now have a higher real wage at the margin – and stimulates labor supply through an income effect.

Consequently, an increase in the price of the necessity good l increases the cost of taxation at θ: raising the tax rate at θ lowers the income of all agents with $\theta' > \theta$, and reduces their labor supply. Through this mechanism, the tax rate should be reduced at θ.

As above, what ultimately determines the change in the optimal income tax is the increase in the cost of taxation at θ relative to the average change in the cost of taxation across the distribution. Thus, the adjustment in optimal tax rates due to Channel #2, the change in the cost of redistribution at θ, is given by:

$$\frac{1}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(T'z\tilde{\zeta}\partial_{z^*z^*}e_k - g\mathbb{E}\left(T'z\tilde{\zeta}\partial_{z^*z^*}e_k\right)\right).$$

As the first channel, this channel does not operate when preferences are homothetic, since $\partial_{z^*z^*}e_k = 0$.

Taking stock. Summing and rearranging the two channels above,[19] we obtain a Proposition characterizing the response of the tax schedule to consumer price changes. This Proposition also shows formally that the adjustment of the tax system amplifies the redistributive effects of price changes.

Proposition 2. *With a linear social function and under A3, the response of the optimal tax rate at θ to an increase in the price of k when $\alpha = 0$ is:*

$$\frac{p_k d}{dp_k}\left\{\frac{T'}{1-T'}\right\} = \frac{1}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right) - \frac{T'}{1-T'}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right). \quad (3)$$

With homothetic preferences ($\partial_{z^*}e_k = \bar{s}_k$), we have $d_{p_k}T' = 0$. With non-homothetic preferences, under A1 the response of the optimal tax schedule to changes in the price of the necessity ($k = l$) and luxury ($k = h$) goods satisfies :

$$\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} < 0 \quad \text{and} \quad \frac{p_h d}{dp_h}\left\{\frac{T'}{1-T'}\right\} = -\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} > 0 \quad \forall\theta.$$

The advantage of Proposition 2 is twofold. First, it allows us to quantify the effect of prices on the tax rate as a function of *observable* quantities. For example, we do not need to explicitly specify social welfare weights to evaluate the impact of prices.[20]

Second, we can sign the impact of prices on taxes. When the marginal propensity to spend on good k decreases (i.e, k is a “necessity” good), the tax rate decreases everywhere in response to an increase in p_k. The tax burden decreases at the top of the

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[19] As shown in the proof of Proposition 2 in the Supplemental Appendix, we use the optimality of the schedule – in equation 2– to re-express the social welfare weight in terms of the tax schedule.

[20] While social preferences do not appear in our formulas, they still play an implicit role. Note that the derivative of the tax rate $d_{p_k}T'$ is of order $(1-T')^2$. The stronger the preference for redistribution, the higher the (initial) tax rate, and the lower the sensitivity of the tax rate to changes in prices.

distribution and increases at the bottom: the planner redistributes to higher-income households. This result might seem surprising, because the optimal tax schedule amplifies the redistributive effects of price changes instead of offsetting them, but our analysis explains why: when k is a necessity good, the social value of a dollar transfer decreases less for higher-income than lower-income households (Channel #1), and the income effects increase more at the top (Channel #2).

It is then simple to characterize the welfare consequences of price changes. By itself, an increase in the relative price of necessities reduces the utility of households at the bottom of the income distribution, as necessities are a larger part of their budget. As the planner responds to the price increase by decreasing tax rates, the transfer to low-income households is reduced, which lowers their utility further. Through the same channels, high-income households strictly benefit from the price increase and the welfare gains are increasing in income. Note that it would be feasible to compensate all households[21] in a budget neutral fashion, but this is not optimal because of Channel #1 and #2. These observations are formalized in the corollary below.

Corollary 1. *For an increase in the relative price of necessities $dln\bar{p}_l$, the compensating scheme $dT\left(z\left(\theta\right)\right) = -\left(s_l - \bar{s}_l\right) z^*\left(\theta\right) dln\bar{p}_l$ is feasible but only optimal when preferences are homothetic. With non-homothetic preferences, under $A1 - A3$ and $G(V(\theta),\theta) = \lambda_\theta V(\theta)$, we have $dV\left(\underline{\theta}\right)/d\bar{p}_l < 0$ and $dV\left(\bar{\theta}\right)/d\bar{p}_l > 0$; $dV\left(\theta\right)/d\bar{p}_l$ is increasing in θ.*

Rawlsian social preferences. The formulas of Proposition 2 can be adapted when social preferences are Rawlsian. In that case, we have:

$$\frac{p_k d}{dp_k}\left\{\frac{T'}{1-T'}\right\} = \frac{T'}{1-T'}\left(\mathbb{E}\left(\partial_{z^*} e_k \mid z' > z(\theta)\right) - \partial_{z^*} e_k\right)$$

Even in the extreme case were the social planner only values the welfare of the poorest agent, an increase in the price of necessities leads to more redistribution towards higher income households. This is entirely due to the impact of the price change on labor supply. An increase in the price of necessities increases the income effect on labor supply ($\partial_{q_l}\tilde{\eta} > 0$) and decreases the income tax.

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[21] By "compensating", we mean keeping their utility equal to their pre-price change level. The feasibility of the compensation is direct using Saez [2002]: the mechanical cost of the compensation is zero for relative price changes, and it leaves both welfare and labor supply unchanged.

Response with a Non-Linear Social Welfare Function

We now consider the case of a non-linear social welfare function. While the concavity of the welfare function does not affect the initial level of redistribution,[22] it impacts the redistributive effects of price changes. With a concave social welfare function, an increase in prices has a direct "income effect" on social welfare weights: reducing the (real) disposable income of an agent directly increases the social value of a transfer to this agent. More precisely, the derivative of social welfare weights with respect to prices p_k now satisfies $p_k \partial_{p_k} g = -g\left(\partial_{z^*} e_k + G''\left(V, \theta\right)/G'(V, \theta)\, e_k\right)$: the second term captures the income effect on social weights. With a concave G, the higher e_k the stronger the incentive for the social planner to compensate lower-income households when they face higher prices.[23]

As the analysis with a concave welfare function is complex, we present first a simple model to convey the intuition. Consider a finite type version of our model: the poor (p), middle-class (m) and rich (r) households have types $0 = \theta_p < \theta_m < \theta_r$. Households have the same preferences as in the continuous type version: household i has a marginal propensity to spend on the necessity product, $\partial_{z^*} e_{l,i}$, satisfying $\partial_{z^*} e_{l,p} > \partial_{z^*} e_{l,m} = \partial_{z^*} E_l > \partial_{z^*} e_{l,r}$. With a linear social welfare function ($G''\left(V\right) = 0$), the change in welfare of household i, $dV_i/d\bar{p}_l$, satisfies $dV_p/d\bar{p}_l = dV_m/d\bar{p}_l < 0 < dV_r/d\bar{p}_l$.

With a concave social function G satisfying $G''\left(V\right) < 0$, we denote the change in welfare by $dV_i^G/d\bar{p}_l$, it satisfies $dV_i^G/d\bar{p}_l = dV_i/d\bar{p}_l/\left(1 + \mathcal{G}\right)$ with $\mathcal{G} > 0$.[24] The parameter $\mathcal{G}$ depends only on the curvature of G ($-G''\left(V\right)/G'\left(V\right)$) and captures the income effect on Pareto weights: the price change and the tax reform reduce the welfare of poor and middle income households, which raises their social welfare weights. As a result, $dV_p/d\bar{p}_l < dV_p^G/d\bar{p}_l < 0$: a concave social welfare function mutes the incentive to redistribute to the high income household. With enough curvature (large $\mathcal{G}$) the welfare loss of the poor can made arbitrarily small. However, the planner still does not offset their loss, even though such compensation is feasible, and the price change remains regressive for any concave G.

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[22] As long as $G'\left(V\left(\theta\right)\right) \propto \lambda_\theta$,the initial tax rate is the same.

[23] Note that with homothetic preferences, the derivative of welfare weights with respect to prices cancel when $z^* G''\left(V, \theta\right)/G'(V, \theta) = -1$. This is no longer the case with non-homothetic preferences and there is no G such that the derivatives of the weights cancels for all price changes and income levels.

[24] The derivations for the simple example can be found in Supplemental Appendix A.2.2.

We now show that this remains true with continuous types. In the Supplemental Appendix (p. A14), we also provide the comparative statics formulas that we use to quantitatively evaluate the impact of price changes with a concave welfare function.

Proposition 3. *Under* $A1 - A3$ *and with* $-G''(V,\theta)/G'(V,\theta)$ *positive and non increasing (e.g., a CARA or CRRA function), the compensating scheme* $dT(z(\theta)) = -(s_l - \bar{s}_l)\, z^* d\ln\bar{p}_l$ *is feasible for an increase in the relative price of necessities, but only optimal under homothetic preferences. With non-homothetic preferences, the change in welfare and tax rate of agent* θ, $dV^G/d\bar{p}_l(\theta)$, $dT'^G/d\bar{p}_l(\theta)$ *satisfy:*

$$\frac{dV}{d\bar{p}_l}(\underline{\theta}) < \frac{dV^G}{d\bar{p}_l}(\underline{\theta}) < 0,$$

$$\frac{dV^G}{d\bar{p}_l}(\theta) - \frac{dV^G}{d\bar{p}_l}(\underline{\theta}) < \frac{dV}{d\bar{p}_l}(\theta) - \frac{dV}{d\bar{p}_l}(\underline{\theta}),$$

$$\frac{dT'^G}{d\bar{p}_l}(\theta) > \frac{dT'}{d\bar{p}_l}(\theta).$$

where $dV/d\bar{p}_l(\theta)$ *and* $dT'/d\bar{p}_l(\theta)$ *are the welfare and tax change with a linear social function satisfying* $\lambda_\theta \propto G'(V(\theta),\theta)$.

As in Proposition 2, it is feasible to compensate all households in a budget-neutral manner for relative price changes but the planner still optimally allows the welfare of lower-income households to decline.[25] Intuitively, fully compensating agents for a price change keeps their disposable income unchanged. This neutralizes the "income effect" of prices on social welfare weights, but leaves unaffected the valuation effects of prices and their effect on labor supply derived in Proposition 2. For an increase in the relative price of necessities, full compensation cannot be optimal: there remains an incentive to redistribute to higher income households through Channel#1 and #2. However, the concavity of the welfare function dampens the impact of these channels: lower income households lose less than with a linear function[26] ($dV/d\bar{p}_l(\underline{\theta}) < dV^G/d\bar{p}_l(\underline{\theta}) < 0$) and utility increases more slowly across types ($dV^G/d\bar{p}_l(\theta) - dV^G/d\bar{p}_l(\underline{\theta}) < dV/d\bar{p}_l(\theta) - dV/d\bar{p}_l(\underline{\theta})$). Overall, the tax reform is more progressive ($dT'^G/d\bar{p}_l > dT'/d\bar{p}_l$). The following corollary characterizes the tax reform

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[25] With homothetic preferences, we do have $dV^G/d\bar{p}_l(\theta) = 0$ for all θ and $dT/d\ln\bar{p}_l = -(s_l - \bar{s}_l)\, z^* = 0$ both welfare and taxes are left unchanged.

[26] Under $A3$, $dV^G/d\bar{p}_l$ and $dV/d\bar{p}_l$ are money metric and directly comparable. They are the equivalent variation of the price change and induced tax reform.

for bottom and top earners:

Corollary 2. *Under the assumptions of Proposition 3, consider a class of social welfare function* G_γ *such that* $G'_\gamma(V,\theta) = G'(V,\theta)$ *and* $-G''_\gamma(V,\theta)/G'_\gamma(V,\theta) = -\gamma G''(V,\theta)/G'(V,\theta)$.

Then, $dV^{G_\gamma}/d\bar{p}_l(\underline{\theta})$ *is increasing in* γ *with* $\lim_{\gamma\to\infty} dV^{G_\gamma}/d\bar{p}_l(\underline{\theta}) = 0$ *and* $\lim_{\gamma\to 0} dV^{G_\gamma}/d\bar{p}_l(\underline{\theta}) = dV/d\bar{p}_l(\underline{\theta})$.

Moreover, if $\bar{\theta} = \infty$*, the distribution of type is bounded by a Pareto distribution,* $\theta\pi'(\theta)/\pi(\theta) \leq -1-\gamma$ *for* θ *large enough, and* $G(V)$*, independent of* θ*, is either CARA or CRRA, then* $\lim_{\theta\to\infty} dV^G/d\bar{p}_l(\theta) = \lim_{\theta\to\infty} dV/d\bar{p}_l(\theta)$ *and* $\lim_{\theta\to\infty} dT'^G/d\bar{p}_l(\theta) = \lim_{\theta\to\infty} dT'/d\bar{p}_l(\theta)$.

The first part of the corollary shows that arbitrarily increasing the curvature of the social welfare function (raising γ while keeping the initial welfare weights constant[27]) strengthens redistribution toward bottom earners. For sufficiently high γ, the welfare loss of the lowest types following a necessity price increase becomes arbitrarily small: bottom earners are almost fully compensated. In particular, beyond a threshold level of curvature, households at the bottom of the income distribution receive positive transfers. Below this threshold, the tax reform remains regressive.

The second part shows that the concavity of the social welfare function does not matter for the change in welfare of high income households. Intuitively, if the social welfare weights are small at the top of the income distribution, the planner does not directly value a change in utility of high income households. The only determinant of taxes are the one described by Channel#1 and #2 and, as a direct consequence, the income tax rate at the top is left unchanged by the concavity of the welfare function.

While Proposition 3 shows that stronger concavity of the social function mutes the regressive impact of price changes, the following Corollary shows that stronger non-homotheticities amplifies it.

Corollary 3. *Consider two economies, indexed by* A, B*, satisfying* $A1 - A3$ *and that only differ in the consumption preferences of households. If* $\partial_{z^*} e_{l,A} - \partial_{z^*} e_{l,B}$ *is non increasing in* z^* *and* $\partial_{z^*} E_{l,A} - \bar{s}_{l,A} \leq \partial_{z^*} E_{l,B} - \bar{s}_{l,B}$*, then* $dV^G_A/d\bar{p}_l(\underline{\theta}) \leq dV^G_B/d\bar{p}_l(\underline{\theta}) < 0$.

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[27] The class G_γ could be of the form $G_\gamma(V,\theta) = \lambda_\gamma(\theta)\frac{V^{1-\gamma}}{1-\gamma}$ with $\gamma > 0$ and λ_γ such that $G'_\gamma(V,\theta) = G'_{\gamma'}(V,\theta)$ for any γ, γ'.

The result states that if the necessity good is consumed relatively more by low-income households in economy A than B ($\partial_{z^*} e_{l,A} - \partial_{z^*} e_{l,B}$ is non increasing), or if, in aggregate, households consume relatively less of the necessity good at the margin ($\partial_{z^*} E_{l,A} - \bar{s}_{l,A} \leq \partial_{z^*} E_{l,B} - \bar{s}_{l,B}$), then a necessity price increase generates less redistribution toward low-income households in economy A.[28] Note that it would still be feasible to achieve the same level of redistribution in both economies. However, doing so would not be optimal, since redistributing toward higher-income households is more efficient in economy A (through Channel#1 and #2) even though lower-income households are more affected by the price change.

The result can also be restated in a multi-good setting. If A and B denote two different necessities within the same economy, with the same properties as above, then an increase in the price of A leads to less redistribution than an increase in the price of B.

4.3 Response to a Price Change with Non-Linear Production Functions

We now turn to the case of non-linear production functions. Non-linearity introduces interaction between demand and supply. As prices change, so does demand for the two goods, which generates a supply side response, further changes in demand, and so on. In this section, we investigate this feedback loop. We show that when $\alpha > 0$, the response of taxes to an increase in the price of necessity is amplified and leads to further redistribution towards higher-income households.

To formally define an exogenous price change with a non-linear production function, we normalize the supply shifter ξ_k such that an increase in ξ_k corresponds to an increase in the price of k, p_k. We consider a cost shifter $p_k^* = 1/\partial_{\xi_k} \phi_k$, which implies $\partial_{p_k^*} \phi_k = 1$ and $\partial_{p_k^*} \chi_k = (1 - \alpha + t_w)^{-1} C_k$.[29] As before, we define an increase in the relative price of the necessity $d\ln\bar{p}_l$, such that $d\ln p_l^* = \bar{s}_h d\ln\bar{p}_l$ and $d\ln p_h^* = -\bar{s}_l d\ln\bar{p}_l$.

In addition, we introduce τ_l which captures the impact of non-homothecities on

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[28] Concretely, suppose that the expenditure share of l for low income household is the same in both economies ($s_{l,A}(\underline{\theta}) = s_{l,B}(\underline{\theta})$) so the welfare impact of the price change is the same. The transfer to low income household is lower in economy A than in economy B.

[29] With monopolistic competition ($t_w = \alpha$), this is obvious since $\chi_k = C_k \phi_k$ so $\partial_{p_k^*} \chi_k = C_k$. With competitive firms ($t_w = 0$), we can rewrite the pricing function as $\phi_k(\xi_k, C_k) = \tilde{\phi}_k(\xi_k) C_k^{-\alpha} = \partial_{C_k} \chi_k(\xi_k, C_k)$ so $\chi_k(\xi_k, C_k) = \phi_k(\xi_k, C_k) C_k/(1-\alpha) + \underline{\chi_k}$, where the potential fixed cost $\underline{\chi_k}$is assumed to be independent from ξ_k.

the sensitivity of aggregate demand to prices:

$$\tau_l\left(z\right) \equiv (1-t_w)(1-T')\left(\frac{1}{z\tilde{\zeta}f(z)}\mathbb{E}_{z'>z}\left(\partial_{z^*}e_l - \partial_{z^*}E_l\right) + \partial_{z^*}e_l - \partial_{z^*}E_l\right) < 0.$$

To understand the role of τ_l, consider the impact of an increase in the relative price of l when $\alpha = 0$ (no supply side adjustments). As shown in the previous section, optimal tax rates decrease in response to the price increase. We show in Lemma A3 of Supplemental Appendix A.2.3 that the resulting change in relative demand for necessity satisfies:

$$\frac{dlnC_l}{dln\bar{p}_l} - \frac{dlnC_h}{dln\bar{p}_l} = \underbrace{-\sigma}_{\text{Substitution}} \underbrace{- \frac{\zeta}{\bar{s}_h\bar{s}_l}\mathbb{E}\left(\frac{z}{E}\left(\tau_l + \partial_{z^*}E_l - \bar{s}_l\right)^2\right)}_{\text{Income}}.$$

With homothetic preferences, the income effect is zero and demand for l decreases only through substitution. With non-homothetic preferences, the price increase also creates a negative income effect for lower-income households, as necessities constitute a larger portion of their consumption basket.[30] Since tax rates decrease, the income of poorer households is further lowered. Because lower income households have a higher propensity to spend on l, these income effects further reduce the aggregate expenditure share of necessities.

With these definitions, we can now characterize the response of the tax rate to price changes with non-linear production functions. We first consider the partial equilibrium response, when prices do not endogenously respond, so that $dp_k/dp_k^* = \partial p_k/\partial p_k^* = 1$. We then derive the general equilibrium response, when prices adjust to their new equilibrium level.

Proposition 4. *Under A3 and $G(V(\theta),\theta) = \lambda_\theta V(\theta)$, the partial equilibrium response of the income tax to a change in the relative price of necessities is:*

$$\frac{\partial}{\partial ln\bar{p}_l}\left\{\frac{T'}{1-T'}\right\} = \frac{1-t_w}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*}e_l - \partial_{z^*}E_l\right) - \left(\frac{T'}{1-T'} + t_w\right)\left(\partial_{z^*}e_l - \partial_{z^*}E_l\right).$$

Under A1, $\partial_{\bar{p}_l}T'$ is negative for all θ. With homothetic preferences, $\partial_{\bar{p}_l}T' = 0$.

---

[30] The income effect on consumption due to heterogeneity in spending is captured in the first term in τ_l. The second term captures the change in real wages when the price of l increases.

In general equilibrium, the response of the income tax to a change in the relative price of necessities is:

$$\underbrace{\frac{dT'}{d\bar{p}_l}}_{\text{GE response}} = \left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1} \underbrace{\frac{\partial T'}{\partial \bar{p}_l}}_{\text{PE response}},$$

with $\Omega = \frac{1}{1-t_w}\frac{\zeta}{\bar{s}_h\bar{s}_l}\left(\mathbb{E}_z\left((\tau_l+\partial_{z^*}E_l-\bar{s}_l)^2\right)+\frac{\alpha\zeta}{1-t_w-\alpha\zeta}\left(\mathbb{E}_z(\tau_l+\partial_{z^*}E_l-\bar{s}_l)\right)^2\right)>0$. *When* $\alpha>0$, $\frac{dT'}{d\bar{p}_l}<\frac{\partial T'}{\partial\bar{p}_l}<0$; *when* $\alpha<0$, $\frac{\partial T'}{\partial\bar{p}_l}<\frac{dT'}{d\bar{p}_l}<0$.

The main insight of Proposition 4 is that that supply-side adjustments amplify the tax response to a price increase when $\alpha > 0$ and mutes it when $\alpha < 0$. With $\alpha > 0$, the amplification is driven by the equilibrium response of the relative price of l, given by:

$$\frac{dln\left(p_l/p_h\right)}{dln\bar{p}_l} = -\frac{1}{1-\underbrace{\alpha\sigma}_{\substack{\text{Amplification}\\\text{through subsititution effects}}}\underbrace{-\alpha\Omega}_{\substack{\text{Amplification}\\\text{through income effects}}}}.$$

The increase in the relative price of l is amplified through income and substitution effects in general equilibrium. First, as the relative price of l rises, households substitute toward h. With $\alpha \geq 0$, the expansion of sector h lowers its price, which creates more substitution away from l. This is the only channel of amplification when preferences are homothetic, since the shares of h and l remain constant as income shifts.

With non-homothetic preferences, income effects generate additional amplification, summarized by $\Omega > 0$. Income effects operates through two channels, driven respectively by changes in relative prices and in the average price index.

The first term in Ω, $\mathbb{E}_z\left((\tau_l+\partial_{z^*}E_l-\bar{s}_l)^2\right)$, captures the decline in the share of l in response to an increase in the relative price of l. Lower income households are more exposed to the rise in price, and, as they consume more l at the margin, the direct impact of the shock is to reduce the aggregate expenditure share of l. In addition, tax rates decrease everywhere, as it is more valuable to redistribute to higher income households. This amplifies the reallocation of income towards the luxury good. Through these two effects the aggregate share of l decreases, and its relative price increases through returns to scale ($\alpha > 0$), triggering additional reallocation towards the luxury good.

The second term in Ω, $\frac{\alpha\zeta}{1-t_w-\alpha\zeta}\left(\mathbb{E}_z(\tau_l+\partial_{z^*}E_l-\bar{s}_l)\right)^2$, captures a further reduction in the share of l stemming from an endogenous fall in the average price index. As the relative price of h falls, it can be shown that real wages rise on average across agents, hence labor supply increases.[31] Higher labor supply raises nominal incomes and demand for both the necessity and the luxury goods, which leads to a fall in both prices. Thus, on average households' real incomes also grow and they reallocate their expenditures towards the luxury good. The relative price of the luxury good therefore decreases with $\alpha > 0$ and the planner responds by lowering tax rates. Lower taxes further stimulate aggregate labor supply and increase real incomes, generating a further fall in the price index, and, as households become richer on average, more reallocation towards luxuries, reinforcing the cycle.

Overall, an increase in the relative price of necessities triggers a general-equilibrium feedback loop that amplifies redistribution toward higher-income households when $\alpha > 0$. The amplification is stronger when the price elasticity α or the elasticity of substitution σ are larger. Moreover, the amplification is stronger when non-homotheticities are more pronounced, as they accentuate reallocation towards necessities through income effects in Ω.[32]

5 Quantitative Analysis

We now present the setting and specifications for our quantitative analysis (Section 5.1), implement our comparative static approach with a first-order approximation (Section 5.2), and finally make parametric assumptions on non-homotheticities to study the optimal tax schedule and feedback loops between redistribution and endogenous prices (Section 5.3).

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[31] Keeping taxes fixed, the change in real wage for household θ is $\hat{w}_t^*(\theta) = -\left(\partial_{z^*}e_l(\theta) - \bar{s}_l\right) d\log\bar{p}_l$, so $\mathbb{E}\left(\hat{w}_t^*(\theta)\right) = -\left(\partial_{z^*}E_l - \bar{s}_l\right) d\log\bar{p}_l > 0$ under A2.

[32] More precisely, comparing two economies A and B where $\partial_{z^*}e_h^A - \partial_{z^*}e_h^B$ is increasing and A and B are otherwise identical, then we have $\Omega^A \geq \Omega^B$ and the amplification through income effects is stronger.

5.1 Setting

Starting from the general model with multiple goods in the Supplemental Appendix A.2.4, we consider a standard additively separable specification (e.g., Saez [2001]):

$$U(z^*, z, \boldsymbol{p}, \theta) = v\left(z^*, \boldsymbol{p}\right) - \frac{1}{1+\frac{1}{\varepsilon}} \left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon}},$$

where $v\left(z^*, \mathbf{p}\right)$ is the indirect utility function given prices and disposable income. Following the nonparametric evidence of Kleven and Schultz [2014], we set $\varepsilon = 0.214$; for robustness, we consider $\varepsilon = 0.33$ as in the meta-analysis of Chetty [2012]. We calibrate the skill distribution $f(\theta)$ nonparametrically to match the income distribution at the observed tax schedule, using data from Hendren [2020].

Returns to scale. Returns to scale are governed by parameter α. Several empirical papers document that increasing demand leads to higher productivity and lower prices (in the long run), and recent papers provide causal estimates for α. Jaravel [2019] finds that a 1% increase in demand reduces the consumer price index by 0.42%, and by 0.62% once changes in product variety are included. Beerli et al. [2020] and Bartelme et al. [2019] document sizable economies of scale in manufacturing, with market-size elasticities of TFP ranging from 0.13 to 0.46. Given this range of estimates, we set $\alpha = 0.30$ in our baseline specification and study sensitivity. For our comparative statics exercise below, we also analyze the case $\alpha = 0$ (linear production functions) which is the benchmark of the public finance literature and allows us to isolate the impact of non-homotheticities in consumption preferences.

For the comparative static analysis in Section 5.1, α can be viewed as the "local" returns to scale. When studying the optimal tax schedule in Section 5.3, we specify the global relationship between the price p_i of the good produced in sector i and equilibrium quantities in that sector, setting $p_i = \gamma_i Q_i^{-\alpha}$. We use the observed equilibrium to calibrate the set of parameters γ_i, as discussed in the Companion Note (Section C.1.2).

Preferences. We set the indirect utility function $v\left(z^*, \mathbf{p}\right)$ to be either homothetic or non-homothetic in the analysis below to isolate the impact of non-homotheticities on the optimal schedule.

A non-homothetic utility function introduces curvature in the agent's indirect utility from consumption. We normalize the curvature of utility at fixed prices, so that we mechanically reach the same optimum with homothetic and non-homothetic utility under constant returns to scale.[33] This approach ensures that the comparison between the homothetic and non-homothetic specifications captures the channel of interest (endogenous prices and their impact on the marginal utility of disposable income), rather than assumptions about curvature *per se*.

For the comparative static analysis, we directly use the formulas derived in Section 4. With linear production functions, we only need to know the local marginal propensities to consume across goods for agents across the income distribution, $\partial_{z^*} e_i$. We measure marginal propensities to consume non-parametrically from expenditure shares across 248 product categories, linking the CPI price data set to consumption patterns in the CEX, following Jaravel [2019].

As shown in Proposition 4, the demand elasticity of substitution σ between products plays an important role for the feedback loops in general equilibrium. We take estimates from the literature as bounds for the elasticity of substitution between our product categories. Based on estimates of the elasticity of substitution between goods and services, two broad categories of consumption which are likely to be less substitutable than our 248 categories, we set $\sigma = 0.6$ as a lower bound (e.g., Comin et al. [2021] and Cravino and Sotelo [2019]). Given estimates on the substitutability between products within the same detailed product category, we take $\sigma = 2$ as our upper bound (e.g., DellaVigna and Gentzkow [2019], and Handbury [2019]).[34]

To study the optimal tax schedule beyond the comparative static approach, we need parametric assumptions on the utility function, which are described in Subsection 5.3.2.

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[33] We work with a "deflated indirect utility function" $\widetilde{v}(z^*, \boldsymbol{p}) \equiv v^{-1}(v(z^*, \mathbf{p}), \mathbf{p}_{CRS})$,where $\mathbf{p}_{CRS}$ are the prices prevailing under constant returns (which are normalized to one in the simulations, without loss of generality). We have $\widetilde{v}(z^*, \mathbf{p}_{CRS}) = z^*$,which is identical to the homothetic specification. Section C.2.2 in the Companion Note discusses the properties of the deflated non-homothetic indirect utility function.

[34] A limitation of the results with endogenous price changes presented below is that we use a single elasticity of substitution across all goods. The existing literature offers no widely accepted estimates of heterogeneous elasticities of substitution across detailed product categories: developing such estimates would be a fruitful avenue in future work. This limitation does not affect the results we obtain in partial equilibrium or when production functions are linear.

Social preferences for redistribution. For the comparative static approach in Section 5.2, the formulas derived in Section 4 show that social preferences for redistribution can be recovered from the initial tax schedule. Taking the observed tax schedule as optimal obviates the need for specifying the social welfare function explicitly. For the analysis of the optimal tax schedule in Section 5.3, the planner's social welfare function, $G\left(U(\theta, \mathbf{p})\right)$, is assumed to be CRRA, with a relative risk aversion coefficient of one in our baseline specification and 0.5 for sensitivity.

5.2 Comparative Statics

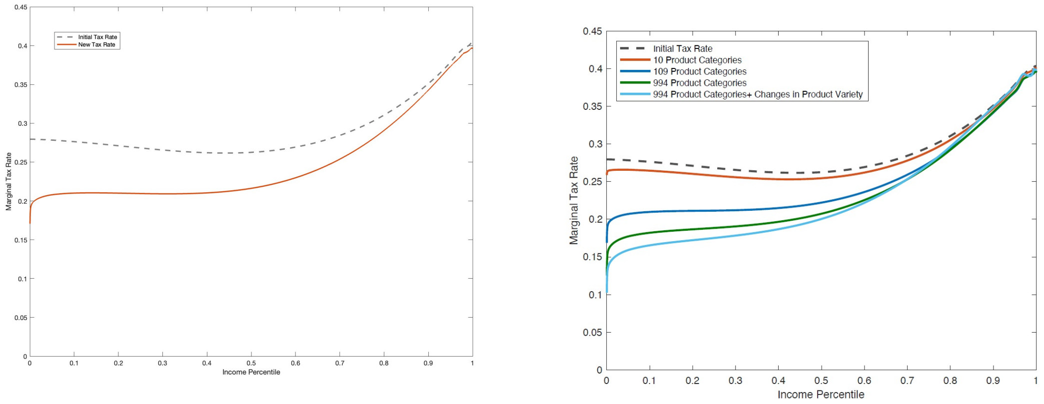
Using the comparative static approach introduced in Section 4, we now quantify the response of the tax schedule to exogenous price shocks.

Starting from the observed tax schedule, we apply the formulas from Section 4 to observed price changes between 2004 and 2015. We obtain price shocks for the 248 product categories covering the full consumption basket of U.S. households by linking the CPI price data set to the consumption patterns of the CEX. Inflation is lower in product categories with higher income elasticities: how large is the impact on the optimal tax schedule? To isolate the role of non-homotheticities, we first assume linear production functions ($\alpha = 0$, as in Section 4.2). We then explore how general equilibrium effects alter this response under increasing returns to scale ($\alpha > 0$, as in Section 4.3). Finally, we examine how the curvature of social preferences shapes the results.

Main results, linear production functions. Panel A of Figure 1 shows the response of the optimal tax schedule to observed price shocks from 2004 to 2015, using all 248 CEX product categories. Cumulative inflation during this period ranges from 29.4% at the bottom of the income distribution to 24.5% at the top (Appendix Figure A1.A). As in the theory section, we focus on relative prices, keeping the average price level constant: relative prices rise by 3.3% at the bottom of the income distribution and fall by 1.6% at the top (Appendix Figure A1.B).

We compute the tax schedule response using Proposition 2, which remains valid in a multi-sector economy, as shown in Supplemental Appendix A.2.4. Under linear production, the tax response depends only on marginal propensities to spend and captures Channels #1 and #2 discussed in Section 4.2. This allows us to isolate the impact of non-homothetic preferences. The response is substantial: marginal

Figure 1 Optimal Tax Schedule and Observed Price Shocks, Linear Production Functions


A. CEX data

B. NielsenIQ scanner data

Notes: both panels of this figure focus on the equilibrium response of the optimal tax schedule to price shocks, as in Proposition 2. The initial tax schedule is taken from Hendren [2020]. In Panel A, the CEX-CPI data set is used to compute inflation rates from 2004 to 2015 across the income distribution, while Panel B uses the NielsenIQ scanner data.

tax rates fall by about 10pp at the bottom of the income distribution and gradually converge back to the observed schedule at the top. Because inflation is lower in product categories for which higher-skill agents have a higher marginal propensity to consume, it is optimal to redistribute toward them. This can be done efficiently by lowering marginal tax rates at the bottom of the income distribution.[35] Inflation inequality therefore induces a sizable regressive response of the tax schedule.

To understand the magnitude of the tax response at the bottom, note that for an arbitrary price change, the formula of Proposition 2 can be re-expressed as:

$$dT'(\underline{\theta}) = -\underbrace{\frac{g(\underline{\theta})}{g(\underline{\theta})-1}T'\left(1-T'\right)}_{\text{Preferences for redistribution}}\underbrace{\left(dlnp\left(\underline{\theta}\right)-dln\bar{p}\right)}_{\text{Marginal Price Index}},$$

where the marginal price index for household $\underline{\theta}$ is $dlnp\left(\underline{\theta}\right)=\sum_{k=1}^{n}\partial_{z^*}e_k\left(\underline{\theta}\right)dlnp_k$, and the average marginal price index is $dln\bar{p}=\sum_{k=1}^{n}\partial_{z^*}E_k dlnp_k$. Supplemental Appendix A.2.2 reports the derivation.

Using this formula, we can plug in values to understand what drives the sizable tax response at the bottom of the income distribution. First, we can use observed tax

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[35] This mechanism is standard: high marginal tax rates at the bottom are paid by all agents earning higher levels of income, without distorting their marginal incentives to work, and all revenue is rebated to the lower-income households through the intercept of the tax schedule.

rates to back out preferences for redistribution ($g(\underline{\theta})$). In the data (Hendren [2020]), the pre-shock marginal tax rate range from 27% at the bottom to 40% at the top.[36] These tax rates correspond to a social welfare weight at the bottom of $g(\underline{\theta}) = 1.1$,[37] implying that the planner values the poorest households only 10% more than the average. This yields a redistribution term of approximately 2.2.

Second, to gauge the role of non-homothetic preferences, we approximate the change in marginal price index using the change in relative price index observed in the CEX.[38] As prices increase by 3.3% more at the bottom than on average, the formula implies a fall of the bottom tax rate of 7.2 percentage points, close to the magnitude oberved in Figure 1.[39] This sizable response is primarily driven by low social preference for redistribution – i.e., $g(\underline{\theta})$ is close to one. Increasing $g(\underline{\theta})$ to 1.3 would more than halve the tax rate response to price changes.

At the top of the income distribution, the same formula applies:

$$dT'(\bar{\theta}) = -\frac{g(\bar{\theta})}{g(\bar{\theta})-1}T'\left(1-T'\right)\left(dlnp\left(\bar{\theta}\right) - dln\bar{p}\right).$$

Here, $T' = 40\%$, $g(\bar{\theta}) = 0.65$, and relative prices fall by 1.6% (Figure A1.B). As a result, the response of the tax rate is much smaller, with a reduction of 0.7 percentage points. If $g(\bar{\theta})$ was closer to one (e.g., $g(\bar{\theta}) = 0.9$), the sensitivity would be similar to that at the bottom.

Response to inflation inequality in scanner data. Using NielsenIQ scanner data, panel B of Figure 1 investigates the role of aggregation bias and product variety for the optimal tax policy response to inflation inequality. We now cover only fast-moving consumer goods (about 15% of total expenditure and 40% of expenditures on goods), but at a much higher level of granularity than the product categories from

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[36] These rates are low compared to the theoretical optimal tax rates obtained in Saez [2001] using social welfare function with a CRRA coefficient of 1, ranging from 80% at the bottom to 60% at the top.

[37] Hendren [2020] reports that social welfare weights at the bottom of the distribution are between 1.1 and 1.2 depending on the elasticity ε. Note that the social welfare weight can be recovered from the optimal tax formula of Proposition 1, setting $\alpha = 0$, $\frac{d}{dz}\left\{z\tilde{\zeta}T'/\left(1-T'\right)f(z)\right\} = -\left(1-g\left(z\right)\right)f\left(z\right)$.

[38] In this calculation, the relative price index is measured in terms of budget shares which are directly observed. Figure 1 uses the estimated marginal budget shares. Figure A1.B reports changes in prices indices across the income distribution, relative to average.

[39] Plugging into the formula above, the calculation is: $dT'(\underline{\theta}) = -\frac{1.1}{1.1-1}\cdot 0.27\cdot(1-0.27)\cdot 3.3 = -7.2$.

the CEX-CPI linked data set and allowing for changes in product variety. Appendix Figure A1 reports inflation inequality patterns in the NielsenIQ data.

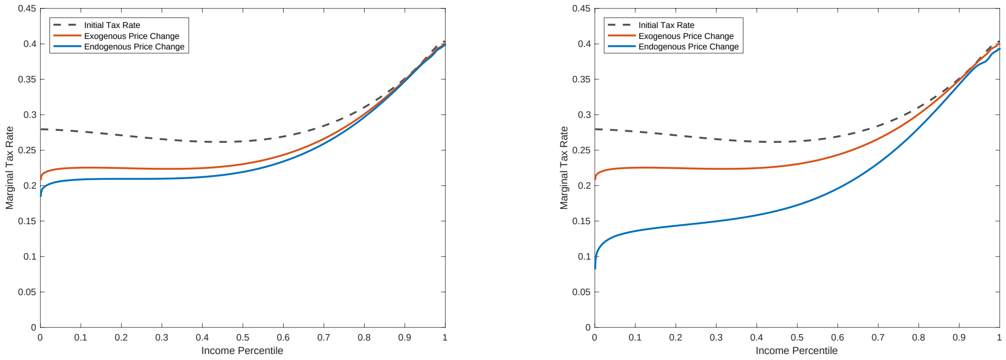
We first focus on price indices computed at different levels of aggregation, using products available across consecutive years, and that do not account for changes in product variety. With the 994 most detailed product categories, called "product modules", we find that it is optimal for marginal tax rates to fall by about 12 percentage points at the bottom of the income distribution in response to the price changes observed between 2004 and 2015. With 109 larger product categories, called "product groups", the fall in the tax schedule is only about 7.5 percentage points. With the ten broad "departments", measured inflation inequality is much smaller and the fall in tax rates is under 2 percentage points.

Furthermore, we introduce a correction for changes in product variety, using a CES price index as in Feenstra [1994]. In the data, product variety expands faster in product categories purchased by high-income households, which further reduces the price index faced by high-skill agents. Consequently, marginal tax rates fall by an additional 2.5 percentage points at the bottom.

Main results, non-linear production functions. We now turn to the case of non linear production functions. We set $\alpha = 0.30$ and consider the monopolistic case. Figure 2 presents the results. We report the changes in the tax schedule in response to price shocks, depending on the value of σ and contrasting the responses in partial and general equilibrium.

Using the first formula of Proposition 4, we compute the partial equilibrium response. This response, shown in red and labeled "exogenous price change" in the figure, is independent of σ and only captures Channels #1 and #2. The pattern of the tax response is qualitatively similar to the case with linear production but quantitatively muted: marginal tax rates at the bottom fall by around 6pp, compared to 10pp in Figure 1. This muted response arises from the interaction between returns to scale and the strength of redistribution preferences. Introducing $\alpha > 0$ implies that stronger redistribution preferences are required to rationalize the same observed tax schedule. To understand why, we have to come back to Proposition 1: with $\alpha > 0$, a wage subsidy is necessary to incentivize labor supply, exploiting increasing returns to scale to reduce prices. This shifts the effective retention rate to $(1 - \alpha)(1 - T')$, which is lower than in the linear case. As a result, even at the same observed tax

Figure 2 Optimal Tax Schedule and Observed Price Shocks, Nonlinear Production Functions



A. $\sigma = 0.6$

B. $\sigma = 2$

Notes: In both panels, the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.21$. The "exogenous price change" results, depicted in red, are obtained by applying Proposition 2, while the "endogenous price change" results, depicted in blue, follow from Proposition 4.

rate, the planner is implicitly favoring low-income households more under non-linear production. Empirically, we find that this stronger redistribution motive is reflected in a higher social welfare weight at the bottom, $g(\underline{\theta}) = 1.2 > 1.1$, which dampens the impact of price shocks on the optimal tax rate relative to the linear case. Thus, under increasing returns, inflation inequality still generates regressive tax responses, but their magnitude is partially offset by a stronger underlying redistributive motive.

Moreover, using the second part of Proposition 4, we find that the response of the tax schedule is amplified in general equilibrium. The results are reported in blue in the figure, with the label "endogenous price change". With $\sigma = 0.6$ in Panel A, the planner reduces marginal tax rates by an additional two percentage points at the bottom of the income distribution. With $\sigma = 2$ in Panel B, the amplification is much larger and the optimal marginal tax rate is reduced to only 10% at the bottom of the income distribution. To understand the magnitude of the amplification, recall from Proposition 4 that the general equilibrium response is the partial equilibrium response scaled by $(1 - \alpha(\sigma + \Omega))^{-1}$. In our estimation, Ω is small compared to σ, so this scaling term is well approximated by $(1 - \alpha\sigma)^{-1}$ and is equal to 1.22 with $\sigma = 0.6$ and 2.5 with $\sigma = 2$, i.e the amplification ranges from about 20% to 150% depend on the elasticity of substitution. In general equilibrium consumers reallocate their expenditures toward the goods that become relatively cheaper, which amplifies the price changes through increasing returns and further reduces the relative price of products with a high income elasticity. These endogenous price changes create an additional motive for the social planner to redistribute toward higher-skill agents, which leads to further price changes, and so on. These results show that the general equilibrium response of prices plays a quantitatively important role for optimal tax policy.[40]

Main results, the role of non-linear social preferences. Next, Figure 3 quantifies the role of the curvature of the social welfare function for the response of the tax schedule, illustrating the theoretical insights from Proposition 3. While Figure 2 gives the results with linear social welfare weights, we now introduce curvature by taking the inverse optimum weights at the observed tax schedule as our concave social welfare function. Specifically, under a linear social welfare function $G(V, \theta) = \lambda(\theta)V$, $\lambda(\theta)$ is chosen such that the observed schedule is optimal. By contrast, under a concave social welfare function, $G(V, \theta) = G(V)$, G itself is chosen such that the observed schedule is optimal and is not allowed to depend on θ. This specification is the most natural benchmark, as it does not embed any inherent preference for inequality across types.[41] Figure 3 reports the results with a linear social welfare function (in blue) and the concave social welfare function (in red). The degree of optimal redistribution toward the rich is slightly muted with the concave social welfare function, because the social value of redistributing income toward high-skill agents falls endogenously as they get more transfers. With $\sigma = 0.6$, marginal tax rates increase by about 2.5pp at the bottom of the income distribution with the concave social welfare function, compared with the baseline case with a linear social welfare function. With $\sigma = 2$ in Panel B, optimal marginal tax rates are about 7pp larger than with a linear social welfare function, but the fall in optimal marginal tax rates induced by price shocks remains sizable.

To further investigate the role of the curvature of the social welfare function, we consider a mixed social welfare function of the form $G(V, \theta) = \lambda(\theta)\frac{V^{1-\gamma}}{1-\gamma}$, with γ the CRRA coefficient and setting $\lambda(\theta)$ such that the observed tax schedule is optimal. We choose values of γ that imply stronger curvature than for the natural concave

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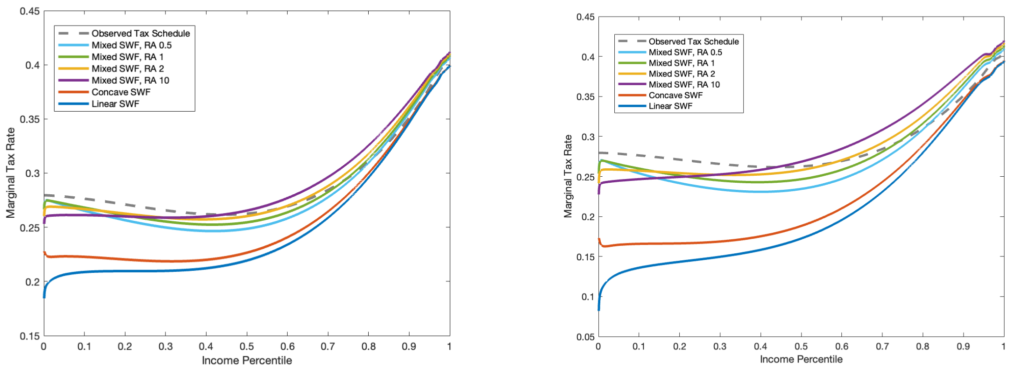
[40] In the Diamond-Mirrlees case, the partial equilibrium response are the same as in Figure 1. The amplification of the response through general equilibrium effects is the same as in the monopolistic case presented in Figure 2: the partial equilibrium response is scaled by 1.22 with $\sigma = 0.6$ and 2.5 with $\sigma = 2$.

[41] Indeed, it is the only social welfare function under which utilities are equalized across households in the first-best allocation. Perfect equality is generally not optimal in the first best for social welfare functions of the form $G(V, \theta) = \lambda(\theta)G(V)$ with non constant $\lambda(\theta)$.

Figure 3 The Role of the Curvature of the Social Welfare Function


A. $\sigma = 0.6$ B. $\sigma = 2$

Notes: In both panels, the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.21$. Each panel reports the optimal tax schedule under different social welfare functions. The dark blue lines in the figure are the same as in Figure 2.

benchmark, namely $\gamma = 0.5$, 1, 2, and 10.[42] As curvature increases, redistribution toward the rich falls further. For CRRA coefficients below 2, the result that marginal tax rates fall in response to the price shocks is attenuated but not overturned: the marginal tax rates remain about one to five percentage points below the observed tax schedule in the first six deciles, and gradually converge to the observed schedule at higher percentiles. Consequently, transfers to low-income households remain negative for CRRA below 2. For CRRA values above 2, transfers to low-income households become positive,[43] and at CRRA = 10 households are almost fully compensated for the price change. Nevertheless, the change in households' welfare remains negative at the bottom of the distribution and positive at the top, in line with the predictions of our theoretical results.

Overall, these results show that non-linearities in social preferences for redistribution may play a significant role for the optimal response of the tax schedule. With the concave social welfare function, the fall in taxes remains substantial for both values of σ. Appendix Figure A2 shows that similar results apply with alternative values of the labor supply elasticity.

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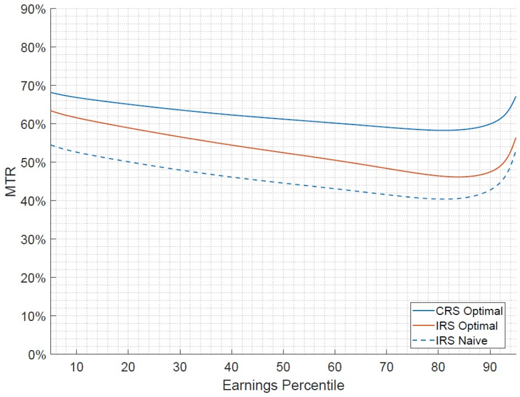
[42] As a consequence, $\lambda(\theta)$ diverges at the top of the distribution, implying that the planner places arbitrarily large weight on high-income households. This is unappealing, as it makes observed inequality reflect the planner's intrinsic bias toward the rich rather than the cost of redistribution.

[43] The concave welfare function is relatively well approximated by a CRRA with coefficient 0.1, so we need to increase the curvature by a factor of 20 to start making the tax response progressive.

Figure 4 Returns to Scale and the Optimal Tax Schedule



Notes: This figure plots optimal marginal tax rates under constant returns to scale (CRS, $\alpha = 0$) and increasing returns to scale (IRS, $\alpha = 0.3$). The social welfare function is logarithmic (CRRA=1) and the elasticity of labor supply is $\varepsilon = 0.21$. With increasing returns, the "naive" correction uses the formula $1 - T'_{NAIVE}(\theta) = \frac{1}{1-\alpha}\left(1 - T'_{CRS}(\theta)\right)$. The optimal tax schedule solves the full optimization problem, accounting for endogenous changes in the value of redistribution across the income distribution.

5.3 Optimal Tax Schedule

We now analyze the quantitative importance of increasing returns to scale and non-homotheticities for optimal tax rates and welfare. We first document the impact of increasing returns to scale in a homothetic model, then isolate the impact of non-homotheticities.

5.3.1 The Interaction between Returns to Scale and Redistributive Motives

We first investigate the impact of returns to scale on the optimal tax schedule under homothetic utility, i.e. $v\left(z^*, \mathbf{p}\right) \equiv \frac{z^*}{p}$. We consider a setting with a single sector, such that α can be interpreted as "aggregate" returns to scale. A "naive" interpretation of Proposition 1 is that, relative to the CRS tax schedule, the planner should uniformly subsidize nominal wages $1-T'$ at a constant rate $1/(1-\alpha)$ throughout the distribution.

The solid blue line in Figure 4 shows the baseline optimal tax schedule under CRS and a logarithmic social welfare function. The optimal marginal tax rates start around 68% at the bottom of the income distribution, fall gradually to 58% at the 80th percentile, and then increase toward 68% at the top. The dashed blue line depicts the tax schedule with the naive correction for increasing returns to scale, with $\alpha = 0.30$, whereby the net-of-tax wage is increased by 43% everywhere. This result already conveys that it is important to take into account returns to scale for optimal tax design: the effect on optimal tax rates is large.

The solid red line shows the optimal tax schedule with returns to scale and a logarithmic social welfare function. The fall in marginal tax rates is smaller than with the naive correction. This result shows that the curvature of the social welfare function plays a quantitatively important role in determining the correction for increasing returns to scale, i.e. there is an important *interaction* with redistributive motives. It is optimal for the cost of the work subsidy to be predominantly paid by high-skill agents, hence marginal tax rates do not fall as much as with the naive correction.

By contrast, with a linear social welfare function, set to match welfare weights at the CRS optimum, the "naive" correction is correct. To isolate the role of non-homotheticities independently of the curvature of the social welfare function, we take the specification with Pareto weights as our baseline in the next subsection. The Pareto weights are set as $\lambda(\theta) \equiv (U_{optim}(\theta))^{-\widetilde{\sigma}}$, where $U_{optim}(\theta)$ is the solution of the optimal taxation problem with homothetic utility, constant returns to scale ($\alpha = 0$), and the CRRA parameter $\widetilde{\sigma}$ for the social welfare function.

5.3.2 The Role of Non-Homotheticities

We now turn to a specification with non-homothetic utility, using non-homothetic CES (nhCES) preferences as in Hanoch [1975] and Comin et al. [2021].

Parametric assumptions on non-homothetic preferences. The indirect utility function $v(z^*, \boldsymbol{p})$ is given by $v \equiv v(z^*, \boldsymbol{p}) \equiv F(\mathbf{Q})$, where $\mathbf{Q}$ is the consumption vector of the agent over the set of products $i \in \mathcal{I}$. Indirect utility v is implicitly defined by:

$$\sum_{i \in \mathcal{I}} \left(\Omega_i v^{\varepsilon_i}\right)^{\frac{1}{\sigma}} Q_i^{\frac{\sigma-1}{\sigma}} = 1.$$

NhCES preferences have convenient features, in particular $\frac{\partial \log(Q_i/Q_j)}{\partial \log(v)} = (\varepsilon_i - \varepsilon_j)$ and $\frac{\partial \log(Q_i/Q_j)}{\partial \log(p_j/p_i)} = \sigma \quad \forall i, j \in \mathcal{I}$. This tractable specification allows us to separately examine the impact on the tax schedule of the "utility elasticities" $\{\varepsilon\}_{i \in \mathcal{I}}$, which govern non-homothetic spending patterns, and the elasticity of substitution σ.

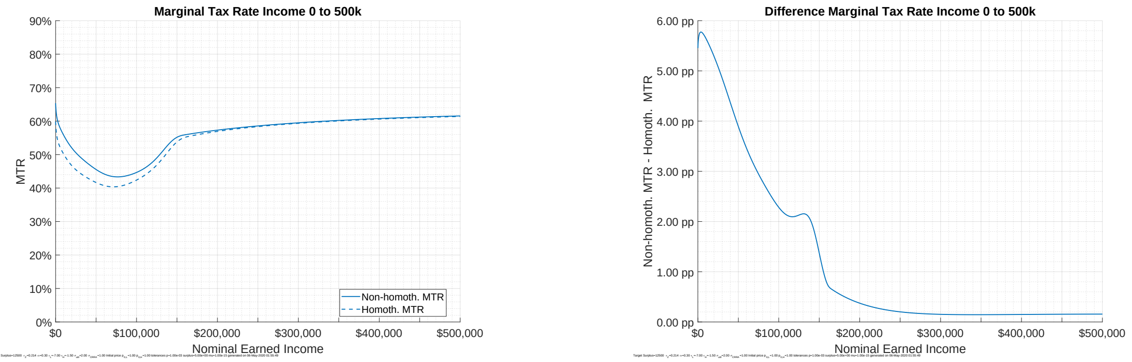
For tractability, in our calibration we consider two products, labeled "high quality" and "low quality" products. In line with evidence on the substitutability between products within the same detailed product category (DellaVigna and Gentzkow [2019], and Handbury [2019]), we set $\sigma = 2$. We then specify the elasticities $\{\varepsilon\}_{H,L}$ to match the dissimilarity index of consumption shares observed across the income distribution in the Consumer Expenditure Survey in 2014. We compute the dissimilarity index at the level of the product categories available in the CEX interview files, called universal classification codes (UCC). We focus on 2014 as the data on the observed tax schedule from Hendren [2020] is available for that year. We obtain $\varepsilon_L = -7$ and $\varepsilon_H = -1.5$, implying that low-income households have a large marginal propensity to spend on the low-quality goods.

Main results. Figure 5 characterizes the impact of non-homotheticities in our baseline specification relative to the homothetic case, with $\alpha = 0.3$ and Pareto weights from the logarithmic social welfare function. Panels A and B show the effect of introducing non-homotheticities on optimal marginal tax rates. Due to non-homotheticities, marginal tax rates increase over the full range of the income distribution. The increase is larger at the bottom of the income distribution, with an increase in marginal tax rates of about 6pp for levels of earned income below \$20,000. The increase is about 2pp at an income level of \$100,000, and then gradually decreases, reaching levels close to zero above \$300,000. Non-homoheticities thus have a significant quantitative impact on optimal marginal tax rates.

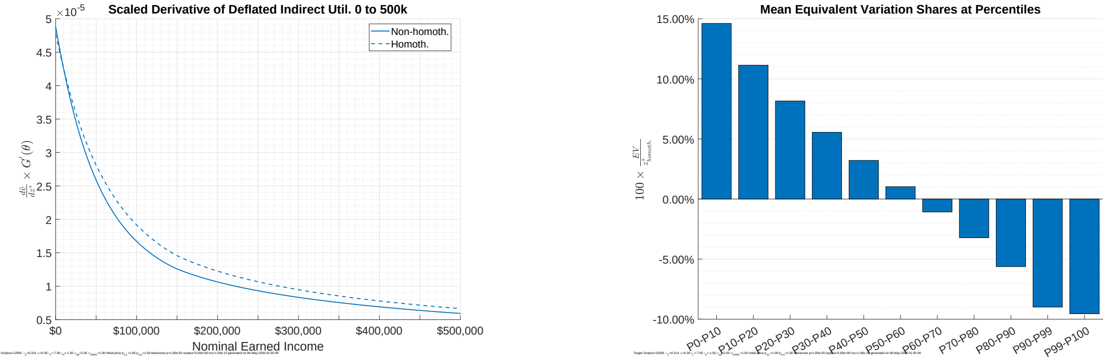
The mechanism explaining the change in marginal tax rates operates through the change in equilibrium prices and in the marginal utility of redistribution across the skill distribution. In the homothetic specification with increasing returns, the price index increases by about 3.6% at the optimal tax schedule, because preferences for redistribution induce higher taxes than at the observed schedule, which reduces labor supply and market size and thus drives an increase in the price. With non-homotheticities, prices of the high-quality and low-quality products diverge: the price of the high-quality good increases by 14%, while the low-quality product becomes 10% cheaper. Indeed, additional redistribution (relative to the observed schedule) leads to an increase in the relative market size of the product which has a higher marginal propensity to consume from low-income households, i.e. the low-quality product in our specification. This result shows that the response of the optimal tax schedule to non-homotheticities lead to large endogenous price changes in equilibrium.

In our simulation, the induced change in the marginal utility of disposable income across agents is substantial. While under homothetic utility the marginal utility is about 0.965 ($= 1/p$) throughout the distribution, with non-homotheticities the

Figure 5 The Response of the Optimal Tax Schedule to Non-Homotheticities
($\alpha = 0.3$, $\varepsilon = 0.21$, Pareto weights from SWF CRRA=1)


A. Optimal Non-Homothetic vs. Homothetic MTRs

B. Difference b/w Optimal Non-Homothetic and Homothetic MTRs


C. $\partial\widetilde{v}/\partial z^* \cdot G'(\theta)$ by Earned Income

D. EV relative to Optimal Homothetic Tax Schedule

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1. Panel A reports the optimal tax schedule with homothetic and non-homothetic preferences. Panel B plots the difference between these two tax schedules across the income distribution. Panel C shows the derivative of (deflated) indirect utility, $d\widetilde{v}(z^*, \boldsymbol{p})/dz^*$, across the income distribution. Panel D report the equivalent variation (EV) giving the willingness to pay of agents for the non-homothetic optimal tax schedule instead of the homothetic optimal tax schedule, expressed as a percentage of their disposable income under the homothetic optimal tax schedule.

marginal utility is 0.99 at the bottom, falls gradually to 0.85 around $150,000, and then increases slightly. The fall in marginal utility is largest for the agents with the highest marginal propensity to consume on the high-quality good, which in equilibrium occurs for earned income levels around $150,000 in our simulation. Panel C combines each agent's marginal utility of disposable income with Pareto weights and shows a steeper decline in welfare weights across the distribution with the non-homothetic specification, because of the price effects.

Finally, panel D summarizes the willingness to pay of agents for the optimal tax schedule under non-homothetic preferences, relative to the optimal schedule under homothetic preferences.[44] The equivalent variation is close to 15% in the bottom decile of the income distribution and decreases monotonically throughout the distribution, turning negative in the seventh income decile. In the top decile, the welfare loss from the new schedule, and its induced price effects, is about 9%. These estimates show that adjusting the tax schedule for non-homotheticities generates substantial distributional effects, with large welfare gains at the bottom of the distribution. Although panels A and B depicted an increase in marginal tax rates at the bottom of the distribution, overall the change in the tax schedule benefits low-income households more. Indeed, setting higher marginal tax rates at the bottom of the income distribution raises the overall amount of redistribution in a more efficient way than increasing marginal tax rates at the top, and the induced price effects benefit agents with a high average spending share on the low-quality product.

Thus, the baseline simulation shows that non-homotheticities can have meaningful quantitative implications for optimal taxation. The results account for all feedback loops between the desirability of redistributing more and the induced price changes in general equilibrium (Proposition 4). As the relative price of the low-quality product decreases, it is optimal to redistribute more to those with a higher marginal propensity to consume on this product, which induces further tax changes and changes in labor supply. The strength of these feedback loops depend on the parameters governing increasing returns and social preferences for redistribution, which we turn to next.

Sensitivity to increasing returns. Figure 6 reports the simulation results with larger increasing returns, setting $\alpha = 0.4$, close to the baseline estimate of 0.42 in Jaravel [2019]. The results and channels described for the baseline specification are all amplified by the larger increasing returns. Optimal marginal tax rates increase by 11.5 percentage points at the bottom of the income distribution (panel A). The price of the high quality good increases by 22%, while the price of the low-quality good falls by 18%. The new tax schedule and the induced price effects create welfare gains of 35% at the bottom of the skill distribution, and welfare losses of 16% at the top (panel B).

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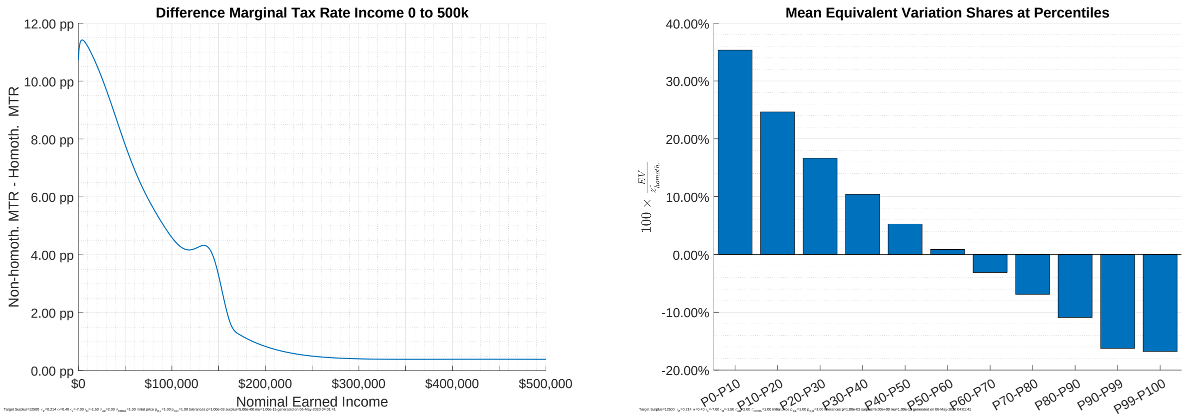
[44] We study the equivalent variation defined by: $\widetilde{v}\left(z_H^*(\theta) + EV(\theta), \mathbf{p}_H\right) - \psi\left(\frac{z_H(\theta)}{\theta}\right) = u_{NH}(\theta)$,where H denotes the equilibrium under the optimal tax schedule with homothetic preferences, while NH corresponds to the equilibrium with non-homothetic preferences.

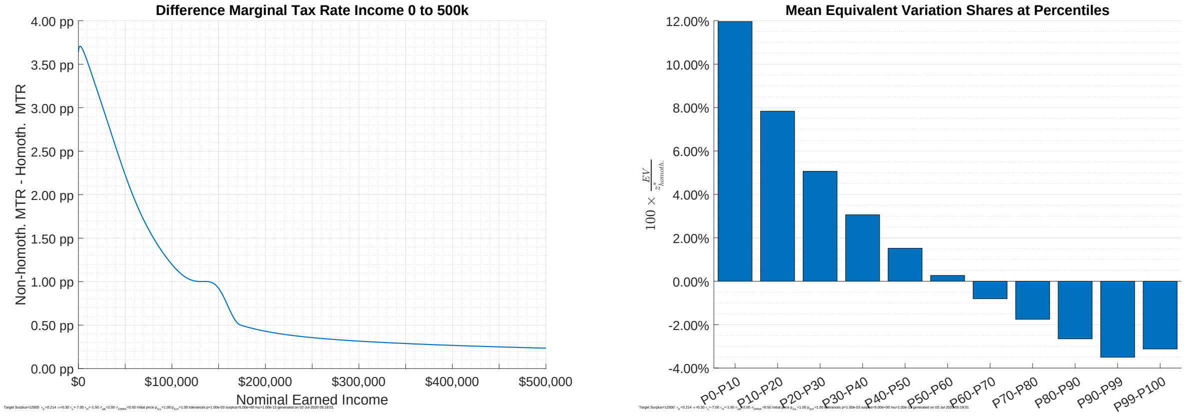
Figure 6 Higher Returns to Scale Magnify the Impact of Non-Homotheticities
($\alpha = 0.4$, $\varepsilon = 0.21$, Pareto weights from SWF CRRA=1)


A. Difference b/w Optimal Non-Homothetic and Homothetic MTRs

B. EV relative to Optimal Homothetic Tax Schedule

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=1.

Figure 7 Lower Social Preferences for Redistribution Reduce the Impact of Non-Homotheticities
($\alpha = 0.3$, $\varepsilon = 0.21$, Pareto weights from SWF CRRA=0.5)


A. Difference b/w Optimal Non-Homothetic and Homothetic MTRs

B. EV relative to Optimal Homothetic Tax Schedule

Notes: The quantitative model uses Pareto weights computed at the optimal homothetic tax schedule obtained under a social welfare function with CRRA=0.5.

Sensitivity to preferences for redistribution. With $\alpha = 0.30$, Figure 7 investigates the effects of nonhomotheticities when preferences for redistribution are weaker. The Pareto weights are taken from the optimal schedule with constant returns to scale and a social welfare function with a CRRA coefficient of 0.5, rather than 1 as previously.

With this specification, the impact of non-homotheticities on the optimal tax schedule is muted: the marginal tax rate increases by 3.75pp at the bottom of the distribution (panel A). The price of the high quality product increases by about 3.75%, while the price of the low-quality product falls by about 3.5%. The willingness to pay for the tax schedule accounting for non-homotheticities remains meaningful, especially at the bottom of the income distribution, with a welfare gain of 12% in the bottom decile and a welfare loss of about 3% in the top decile (panel B).

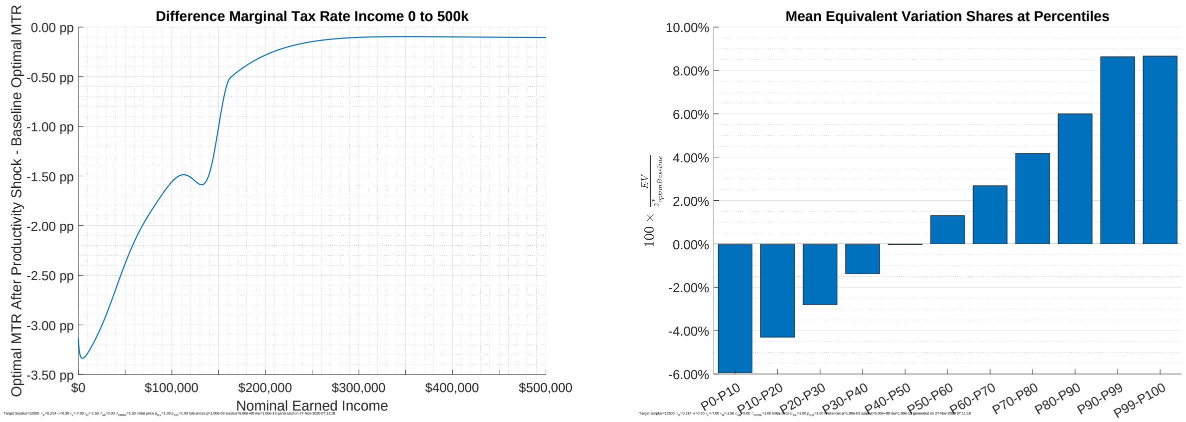
These results illustrate the interplay between social preferences for redistribution and endogenous prices. A weaker taste for redistribution endogenously leads to smaller changes in market size, hence smaller price changes in equilibrium and a smaller adjustment to optimal marginal tax rates.

The impact of exogenous price shocks. Finally, we analyze how price shocks (e.g., from productivity shocks) affect the optimal tax schedule. We already characterized this response using a first-order approximation in Section 5.2. We now briefly present complementary results with no approximation, accounting for feedback loops created by large price changes.

Specifically, we consider an exogenous 5% change in the relative price of the high-quality and low-quality bundles, reporting the results in Figure 8. We find that this price shock leads to a fall in marginal tax rates of 3.25 percentage points at the bottom of the income distribution. The equivalent variation, capturing the willingness to pay of agents for the revised optimal tax schedule, ranges from -6% at the bottom to +9% at the top of the income distribution. In equilibrium, the relative price of the high-quality bundle falls by 13%, more than twice the exogenous relative price shock. We thus find that the amplification effects and their welfare implications are sizable.

Extensions. The Companion Note presents additional robustness checks and characterizes the optimal response of the tax schedule to exogenous shifts in the income distribution, accounting for the endogenous response of prices.

Figure 8 The Response of the Optimal Tax Schedule to Productivity Shocks ($\alpha = 0.3$, $\varepsilon = 0.21$, Pareto from SWF CRRA=1, PE price low-quality +2.5%, PE price high-quality -2.5%)



A. Difference b/w MTRs

B. EV, initial vs. new tax schedules

Notes: The model uses Pareto weights computed at the optimal homothetic tax schedule under a social welfare function with CRRA=1. The exogenous productivity changes are such that the partial equilibrium price of the low-quality bundle increases by 2.5% while the price of the high-quality bundle decreases by 2.5%.

6 Conclusion

In this paper, we have shown that optimal commodity and income taxation is sensitive to exogenous price shocks, the elasticity of prices to market size, and non-homothetic preferences. We provided an explicit analytical characterization of the response of the optimal tax schedule to price shocks, in both partial and general equilibrium. Using simulations based on observed patterns of inflation inequality, we found that these channels have a sizable quantitative impact on optimal marginal tax rates and welfare across the skill distribution.

Going forward, our framework could be used to study the response of optimal taxation to a variety of supply shocks that could affect prices (e.g., technology, trade, immigration, or market concentration). We conjecture that the mechanisms we highlighted might become even richer in a model with trade. Another interesting avenue for future work would be to estimate heterogeneity in returns to scale across sectors and the consequences for the optimal tax schedule.

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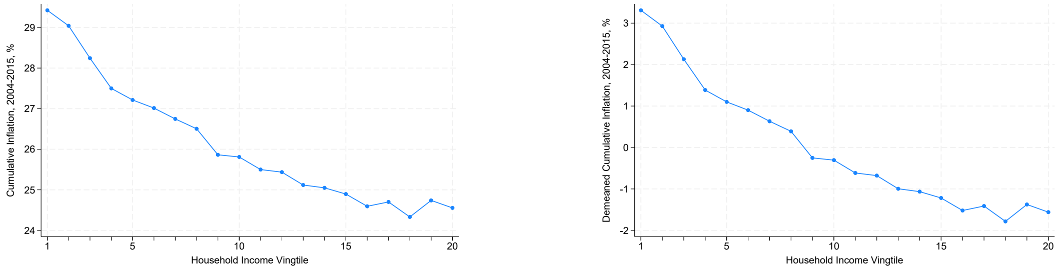
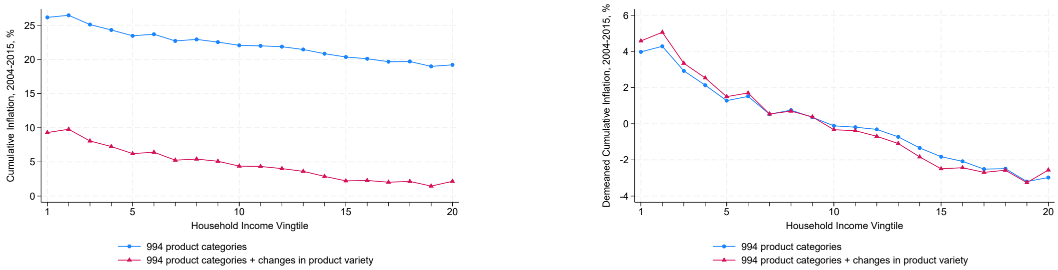
Appendix Figures

Figure A1 Inflation Inequality in the United States, 2004-2015



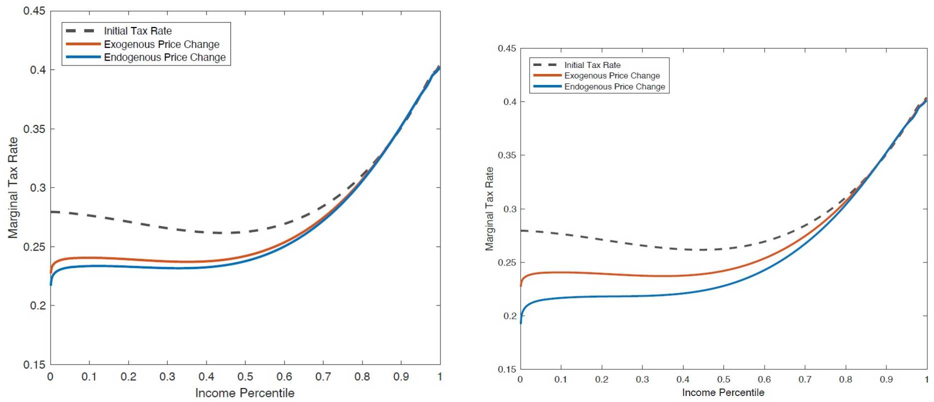
A. Raw Price Changes, CEX-CPI Data B. Relative Price Changes, CEX-CPI Data



C. Raw Price Changes, NielsenIQ Data D. Relative Price Changes, NielsenIQ Data

Notes: Panel A of this figure reports Laspeyres inflation rates across the income distribution between 2004 and 2015, considering 248 product categories observed in CEX-CPI data. Panel B reports the same patterns after demeaning price changes. Panel C reports Laspeyres inflation rates using the 994 most detailed product categories in the NielsenIQ data, called "product modules" and report inflation rates with and without the CES utility correction for changes in product variety. Panel D reports the same patterns after demeaning price changes.

Figure A2 Sensitivity Analysis for the Response to Observed Price Shocks
Results with Labor Supply Elasticity $\varepsilon = 0.33$



A. $\sigma = 0.6$ B. $\sigma = 2$

Notes: in all specifications, the IRS parameter is set to $\alpha = 0.3$ and the labor supply elasticity to $\varepsilon = 0.33$. The CEX-CPI dataset is used in both panels.

Theory Appendix: the Role of Missing Taxes

In this appendix, we discuss the “missing tax” in our model.

Our benchmark specification does not have, in general, an efficient supply side. Therefore, the solution of the government problem will be *constrained efficient*. With more tax instruments, the social planner could regulate firms and improve the allocation. For example, the planner could directly choose the number of firms in each market to minimize the total cost of production, which includes the variable cost of production and the entry cost.

Note however that the planner can regulate supply in a revenue neutral fashion: for a given industrial policy τ that depends on aggregate quantities, there is a new reduced-form pricing function $p_k = \phi^{\tau}_k(C_k, \xi_k)$ which depends on the regulatory regime. The solution of the planner problem characterizes the optimal choice of consumption and income taxes for a given industrial policy, which may or may not be optimal.

Suppose that under the industrial policy τ the pricing function is $\phi^{\tau}_k(C_k, \xi_k)$, so the cost is $C_k\phi^{\tau}_k(C_k, \xi_k)$, and the fiscal cost is $C_k\psi^{\tau}_k(C_k, \xi_k)$ for an arbitrary function ψ^{τ}_k. By imposing a sales tax $t_k(C_k, \xi_k) = \psi^{\tau}_k(C_k, \xi_k)$ the industrial policy is budget neutral and the income taxation problem is equivalent with a new pricing function $\tilde{\phi}^{\tau}_k(C_k, \xi_k) = \phi^{\tau}_k(C_k, \xi_k) + t_k(C_k, \xi_k)$.

In that sense, industrial policies and redistribution are separable: for a given regulatory rule of the supply side, we take the induced pricing function (and the market size elasticity) as given and derive the optimal redistributive policy. Our results will therefore be valid whether or not industrial policies are optimal, or missing altogether.

In the quantitative analysis, we use the estimated market size elasticity in the United States between 2004 and 2015, which depends implicitly on the regulatory regime in that period.

A Supplemental Appendix for Online Publication: Proofs

In this section, we provide proofs for the theoretical results of Sections 3 and 4. We also derive the comparative statics formulas that underpin the quantitative results of Section 5.2, with many sectors.

In our quantitative analysis, we consider an economy with n sectors. Here, we formulate a simple extension of the model of Section 3, which allows us to generalize the results of Proposition 1, 2 and 4 and provide the theoretical ground for the quantitative results of Section 5.2.

The economy has n sectors indexed by k. There is a mass 1 of households with different productivity types θ distributed according to $\pi(\theta)$.

Households. Households' preferences over goods and hours worked z/θ are:

$$u(c_1, ..., c_n) - \frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon}},$$

with u concave, increasing and $\mathcal{C}^3$, and $\varepsilon \leq 1$. Given separability of preferences between consumption and labor, the household problem, under consumer prices $\mathbf{q} = \{q_1, ..., q_n\}$ and the income tax schedule $\mathbf{T}$, can be written as:

$$\begin{aligned} V\left(\theta; \mathbf{T}, \mathbf{q}\right) &= \sup_{z, z^*} v\left(z^*, \mathbf{q}\right) - \frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\varepsilon}}, \\ \text{such that } z^* &= z - T\left(z\right) \\ v\left(z^*, \mathbf{q}\right) &= \sup_{\mathbf{c}} u(c_1, ..., c_n), \quad \mathbf{q} \cdot \mathbf{c} = z^*. \end{aligned}$$

The consumption problem on the third line defines an indirect utility of consumption $v\left(z^*, \mathbf{q}\right)$ and a Marshallian demand function $c_k\left(z^*, \mathbf{q}\right)$. Since u is concave and $\mathcal{C}^3$, the implicit function theorem directly shows that v and c_k are $\mathcal{C}^2$. The labor supply function is $z\left(\theta; \mathbf{T}, \mathbf{q}\right)$ and post tax income is $z^*\left(\theta; \mathbf{T}, \mathbf{q}\right) = z\left(\theta; \mathbf{T}, \mathbf{q}\right) - T\left(z\left(\theta; \mathbf{T}, \mathbf{q}\right)\right)$.

Firms. We adopt the same supply-side specification as in the main text. In each sector, goods are produced using labor as the sole input. The cost of producing C_k units of good k is $\chi_k\left(C_k, \xi_k\right)$ and the price of k is given by $\phi_k\left(C_k, \xi_k\right)$, where ξ_k is an exogenous supply shifter. We consider two cases: in the competitive case, we have $\phi_k\left(C_k, \xi_k\right) = \partial_{C_k}\chi_k\left(C_k, \xi_k\right)$; in the monopolistic case, we have $\chi_k\left(C_k, \xi_k\right) = C_k\phi_k\left(C_k, \xi_k\right)$. We assume, as in the main text, that the elasticity of price with respect to market size, $\alpha = -C_k\partial_{C_k}\phi_k\left(C_k, \xi_k\right)/\phi_k\left(C_k, \xi_k\right)$, is constant, equal across sectors and independent from C_k and ξ_k.

Planning Problem. The government maximizes $\int G(V(\theta), \theta)\pi(\theta)d\theta$, with G increasing and concave in V using a nonlinear income tax $\mathbf{T}$, and commodity taxes, $q_k - p_k$, and a profit tax. Preferences of the household satisfy the single crossing property: $(z/\theta)^{\epsilon^{-1}}/\left(\theta\partial_{z^*}v\left(z^*, \boldsymbol{q}\right)\right)$ is decreasing in type θ. Therefore, the planning problem can be expressed as a direct mechanism where global incentive compatibility constraints are replaced with a local constraint and a monotonicity condition on $z\left(\theta\right)$.

$$\begin{aligned}
&\mathcal{W} = \sup_{V(\theta), z(\theta), \boldsymbol{q}} \int G(V(\theta), \theta)\pi(\theta)d\theta \\
&s.t. \quad V'(\theta) = \frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{1+\frac{1}{\epsilon}} \quad \text{and} \quad z(\theta) \text{ is non-decreasing} \qquad \text{(A1)} \\
&with \quad V(\theta) = v\left(z^*\left(\theta\right), \boldsymbol{q}\right) - \frac{1}{1+\frac{1}{\epsilon}}\left(\frac{z}{\theta}\right)^{1+\frac{1}{\epsilon}} \\
&\qquad \int\left(z\left(\theta\right) - z^*\left(\theta\right)\right)\pi(\theta)d\theta + \sum_{k=1}^{n}\left(q_kC_k - \chi_k\left(C_k, \xi_k\right)\right) = 0
\end{aligned}$$

where the consumption function solves $\boldsymbol{c}\left(z^*, \boldsymbol{q}\right) = \text{argmax}_{\boldsymbol{c}}u\left(\boldsymbol{c}\right)$ $s.t.$ $\boldsymbol{q}\cdot\boldsymbol{c} = z^*$, $v\left(z^*, \boldsymbol{q}\right) = u\left(\boldsymbol{c}\left(z^*, \boldsymbol{q}\right)\right)$ and $C_k = \int c_k(z^*\left(\theta\right), \boldsymbol{q})\pi(\theta)d\theta$.

A.1 Proof of Proposition 1

In this subsection we prove Proposition 1 in an economy with n goods; the two-good economy of the main text is included as a special case.

Proof: After integration by parts of the planning problem A1, the corresponding

Lagrangian is:

$$\mathcal{L} = \int G(V(\theta), \theta)\pi(\theta)d\theta - \int \left(\mu'(\theta)V(\theta) + \mu(\theta)\frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{1+\frac{1}{\epsilon}} \right) d\theta$$
$$-\lambda \left(\int \left(z^*(\theta) - z(\theta)\right)\pi(\theta)d\theta - \sum_{k=1}^{n} \left(q_k C_k - \chi_k\left(C_k, \xi_k\right)\right) \right),$$

where $\mu(\theta)$ are the multipliers on the incentive constraints and λ is the multiplier on the resource constraint.

We start with the FOC with respect to consumer prices q_i. Denote $c^h(v, \mathbf{q})$ is the Hicksian demand function at prices q for a given sub-utility v, we have:

$$\left.\frac{dc_j}{dq_i}\right|_{z,V} = \left.\frac{dc_j}{dq_i}\right|_{v} = \frac{\partial c_j^h}{\partial q_i}$$
$$\left.\frac{dz^*}{dq_i}\right|_{z,V} = \left.\frac{dz^*}{dq_i}\right|_{v} = c_i$$

We therefore have, denoting $\partial_{q_i} C_j^h = \int \partial_{q_i} c_j^h \pi(\theta)d\theta$:

$$\frac{d\mathcal{L}}{dq_i} = \lambda \left(C_i + \sum_j \left(q_j - \partial_{C_j}\chi_j\left(C_j, \xi_j\right)\right)\partial_{q_i} C_j^h - C_i \right),$$

which gives for all i, $\sum_j \left(q_j C_j - (1 - t_w)\, p_j C_j\right)\mathcal{S}_{j,i} = 0$,with $t_w = 0$ in the competitive case ($\partial_{C_j}\chi_j\left(C_j, \xi_j\right) = p_j$), and $t_w = \alpha$ in the monopolistic case ($\partial_{C_j}\chi_j\left(C_j, \xi_j\right) = \phi_j\left(C_j, \xi_j\right) + C_j \partial_{C_j}\phi_j\left(C_j, \xi_j\right) = (1 - \alpha)\, p_j$). Recall that $\mathcal{S}$ is the matrix of cross price elasticities. Given that $\mathcal{S}$ is generically of rank $n - 1$ with left kernel $\boldsymbol{qC}$, we have: $\boldsymbol{q} \propto \boldsymbol{p}$, so choosing $p = q$ is optimal.

Next, we derive the FOC associated with V. $V(\theta)$ impacts consumption and producer prices through $z^*(\theta)$ with $dz^*(\theta)/dV(\theta) = 1/v_{z^*}$. We thus have:

$$\begin{aligned}
0 &= G'(V(\theta),\theta)\pi(\theta) - \mu'(\theta) - \frac{\lambda\pi(\theta)}{v_{z^*}}\left(1 - \sum_i \left(q_i - \partial_{C_i}\chi_i\left(C_i,\xi_i\right)\right)\partial_{z^*}c_i(\theta)\right) \\
&= G'(V(\theta),\theta)\pi(\theta) - \mu'(\theta) - \frac{\lambda\pi(\theta)}{v_{z^*}}\left(1 - \sum_i \left(q_i - \left(1-t_w\right)p_i\right)\partial_{z^*}c_i(\theta)\right) \\
&= G'(V(\theta),\theta)\pi(\theta) - \mu'(\theta) - \frac{\lambda\pi(\theta)}{v_{z^*}}\left(1 - t_w\sum_i q_i\,\partial_{z^*}c_i(\theta)\right) \\
\Rightarrow& \mu'(\theta)\frac{v_{z^*}}{\lambda} = -\left(1 - t_w - \frac{G'(V(\theta),\theta)v_{z^*}}{\lambda}\right)\pi(\theta).
\end{aligned}$$

Finally, defining $\tilde{\mu} = \mu v_{z^*}/\lambda$, we have, denoting $MRS = \frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{\epsilon^{-1}}/v_{z^*}$ the marginal rate of substitution:

$$\tilde{\mu}'(\theta) + \tilde{\mu}\,\partial_{z^*}MRS\;z'(\theta) = -\left(1 - t_w - \frac{G'(V(\theta),\theta)v_{z^*}}{\lambda}\right)\pi(\theta).$$

Finally, the FOC associated with z, using the same steps as above to derive the response of consumption and prices, is:

$$\tilde{\mu}\,\partial_\theta MRS = \pi(\theta)((1-t_w)MRS - 1).$$

Since $MRS = 1 - T'(z(\theta))$, and $z\tilde{\zeta}\partial_\theta MRS = -z'(\theta)(1 - T'(z(\theta)))$, we therefore have, denoting $f(z(\theta)) = \pi(\theta)/z'(\theta)$:

$$\tilde{\mu}(\theta) = f(z)z\tilde{\zeta}\left(\frac{T'}{1-T'} + t_w\right).$$

Finally, using $-z\tilde{\zeta}\,\partial_{z^*}MRS = \tilde{\eta}$, we get:

$$f(z)z\tilde{\zeta}\left(\frac{T'}{1-T'} + t_w\right) + \int_{z(\theta)}^{z(\bar{\theta})}\tilde{\eta}\left(\frac{T'}{1-T'} + t_w\right)f(z)dz = \int_{z(\theta)}^{z(\bar{\theta})}\left(1 - t_w - \frac{G'v_{z^*}}{\lambda}\right)f(z)dz$$

Using $g = G'v_{z^*}/((1-t_w)\lambda)$, we obtain the formula of Proposition 1.□

A.2 Proofs for Section 4: Propositions 2, 3, and 4, and Corollary 1

In this section, we provide proofs for our comparative statics results of Section 4. In the last subsection, we provide the comparative statics formulas underpinning the results of Section 5.2.

A.2.1 Intermediary Lemma

To streamline the presentation of the proofs, we first present an intermediary lemma valid in an $n-$sector economy. For an exogenous supply shock ξ_k, we derive the change in welfare for agent θ, $dV\left(\theta\right)/d\xi_k$, expressed in terms of the resulting equilibrium price changes, $dp_l/d\xi_k$. Much of the algebra required for Propositions 1, 2 and 3, as well as Corollary 1 is the same. The purpose of the lemma is to consolidate these repetitive derivations into a unified result.

As in the main text, we assume that there are no income effects of labor supply at initial prices (assumption $A3$). At initial prices $\mathbf{p} = \{p_1, ..., p_N\}$, we have $\partial_{z^*} v\left(z^*, \mathbf{p}\right) = 1, \forall z^*$. Recall that the utility of the agent can be rewritten $U(c_1, ..., c_n, z, \theta) = \Psi\left(u\left(c_1, ..., c_n\right)\right) - \left(1+\epsilon^{-1}\right)^{-1}\left(z/\theta\right)^{1+\epsilon^{-1}}$, where the function Ψ can be used to calibrate the income effect of labor supply at initial prices: here we choose $\Psi'\left(v\left(z^*, \mathbf{p}\right)\right) = \partial_{z^*} v\left(z^*, \mathbf{p}\right)^{-1}$.

Lemma A1. Under assumption $A3$, the change in welfare for agent θ, $dV\left(\theta\right)/d\xi_k$, in response to an exogenous supply shift $d\xi_k$, conditional on the change of prices $dp_l/d\xi_k$, is given by:

$$\mathcal{D}\left[\frac{dV}{d\xi_k}\right](\theta) = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\sum_{l=1}^{n}\left(\tau_l\left(\theta\right) + \partial_{z^*}E_l\right)\frac{1}{p_l}\frac{dp_l}{d\xi_k},\ \left(1-t_w\right)\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{d\xi_k}\pi d\theta = -\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right),$$

Where for a function $f\left(\theta\right)$,

$$\mathcal{D}\left[f\right]\left(\theta\right) = \frac{\epsilon}{\left(1+\epsilon\right)^2}\frac{\theta\pi(\theta)}{\left(1-T'\right)^2}\frac{\theta}{z(\theta)}\frac{d}{d\theta}\left\{f\right\}\left(\theta\right) + \left(1-t_w\right)\int_{\theta}^{\bar{\theta}} g\left(\gamma\left(\theta'\right)f\left(\theta'\right) - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)f\left(\theta'\right)d\theta'\right)\pi d\theta'$$

and $\gamma\left(\theta\right)$, $\tau_l\left(\theta\right)$ are given by:

$$\begin{aligned}
\gamma\left(\theta\right) &\equiv -\frac{G''\left(V\left(\theta\right),\theta\right)}{G'\left(V\left(\theta\right),\theta\right)}, \\
\tau_l\left(\theta\right) &\equiv \left(1-t_w\right)\left(1-T'\right)\left(\frac{1+\epsilon}{\epsilon}\frac{1}{\theta\pi(\theta)}\int_{\theta}^{\bar{\theta}}\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\pi d\theta' + \left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\right).
\end{aligned}$$

Proof: Recall from the proof of Proposition 1 that $p_l = q_l$. In addition, the optimal tax system is determined by the following envelope conditions and first order condition:

$$\begin{aligned}
\frac{d\hat{\mu}(\theta)}{d\theta} &= -\left(\frac{1-t_w}{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)} - \frac{G'(V(\theta),\theta)}{\lambda}\right)\pi(\theta) \\
\frac{dV(\theta)}{d\theta} &= \frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{1+\frac{1}{\epsilon}} \\
\hat{\mu}(\theta)\left(1+\frac{1}{\epsilon}\right)\frac{1}{\theta^2}\left(\frac{z(\theta)}{\theta}\right)^{\frac{1}{\epsilon}} &= \pi(\theta)\left(1-\left(1-t_w\right)\frac{1}{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)}\frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{\frac{1}{\epsilon}}\right), \qquad \text{(A2)}
\end{aligned}$$

with $\hat{\mu}(\theta) = \mu(\theta)/\lambda$ and $\hat{\mu}(\underline{\theta}) = \hat{\mu}(\bar{\theta}) = 0$. Finally, the budget constraint needs to be satisfied:

$$\int\left(z\left(\theta\right)-z^*\left(\theta\right)\right)\pi\left(\theta\right)d\theta + \sum_{l=1}^{n}\left(p_lC_l - \chi_l\left(\xi_l, C_l\right)\right) = 0.$$

We first start by differentiating the marginal value of income $v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)$:

$$\begin{aligned}
\frac{d}{d\xi_k}\left\{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\right\} &= v_{z^*z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\frac{dz^*}{d\xi_k} + \sum_{l=1}^{n}\frac{\partial}{\partial p_l}\left\{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\right\}\frac{dp_l}{d\xi_k} \\
&= v_{z^*z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\frac{dz^*}{d\xi_k} + \sum_{l=1}^{n}\frac{\partial}{\partial z^*}\left\{v_{p_l}\left(z^*\left(\theta\right),\mathbf{p}\right)\right\}\frac{dp_l}{d\xi_k} \\
&= v_{z^*z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\frac{dz^*}{d\xi_k} - \sum_{l=1}^{n}\frac{\partial}{\partial z^*}\left\{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)c_l\right\}\frac{dp_l}{d\xi_k} \\
&= v_{z^*z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\left(\frac{dz^*}{d\xi_k} - \sum_{l=1}^{n}c_l\frac{dp_l}{d\xi_k}\right) - v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\sum_{l=1}^{N}\partial_{z^*}c_l\frac{dp_l}{d\xi_k} \\
&= -v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)\sum_{l=1}^{n}\partial_{z^*}c_l\frac{dp_l}{d\xi_k},
\end{aligned}$$

where the second line uses Schwarz's identity, the third Roy's identity, and the fifth uses $v_{z^*z^*}\left(z^*\left(\theta\right),\mathbf{p}\right) = 0$. Using the fact that t_w is constant (equal to α in the monopolistic case, equal to 0 in the competitive case), differentiating the first equation of system A2, we obtain:

$$\frac{d}{d\theta}\left\{\frac{d\hat{\mu}(\theta)}{d\xi_k}\right\}=-\left(\frac{1-t_w}{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)}\sum_{l=1}^{n}\partial_{z^*}c_l\frac{dp_l}{d\xi_k}+\frac{G'(V(\theta),\theta)}{\lambda}\left(\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}+\frac{1}{\lambda}\frac{d\lambda}{d\xi_k}\right)\right)\pi(\theta),$$

with $\gamma\left(\theta\right)=-G''(V(\theta),\theta)/G'(V(\theta),\theta)$. Using the fact that $d_{\xi_k}\hat{\mu}(\underline{\theta})=d_{\xi_k}\hat{\mu}(\bar{\theta})=0$, we have in addition:

$$\begin{aligned}\frac{1}{\lambda}\frac{d\lambda}{d\xi_k}&=-\int\left(\frac{1-t_w}{v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)}\sum_{l=1}^{n}\partial_{z^*}c_l\frac{dp_l}{d\xi_k}+\frac{G'(V(\theta),\theta)}{\lambda}\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}\right)\pi(\theta)d\theta\\&=-\left(1-t_w\right)\int\left(\sum_{l=1}^{n}\partial_{z^*}c_l\frac{dp_l}{d\xi_k}+g\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}\right)\pi(\theta)d\theta,\end{aligned}$$

where the second line uses $v_{z^*}\left(z^*\left(\theta\right),\mathbf{p}\right)=1$ and $G'(V(\theta),\theta)//\lambda=\left(1-t_w\right)g$ (by definition of g). We therefore have:

$$\begin{aligned}\frac{d}{d\theta}\left\{\frac{d\hat{\mu}(\theta)}{d\xi_k}\right\}=&-\left(1-t_w\right)\{\sum_{l=1}^{n}\left(\partial_{z^*}c_l-g\left(\theta\right)\int\partial_{z^*}c_l\pi(\theta')d\theta'\right)\frac{dp_l}{d\xi_k}\\&+g\left(\theta\right)\left(\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}-\int g\gamma\left(\theta'\right)\frac{dV(\theta')}{d\xi_k}\pi(\theta')d\theta'\right)\}\pi(\theta)\end{aligned}$$

Differentiating the second and last equation of system A2, we obtain:

$$\begin{aligned}\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\}&=\left(1+\frac{1}{\epsilon}\right)\frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{1+\frac{1}{\epsilon}}\frac{1}{z\left(\theta\right)}\frac{dz(\theta)}{d\xi_k}\\\frac{d\hat{\mu}(\theta)}{d\xi_k}&=-\frac{\epsilon}{1+\epsilon}\theta\pi(\theta)\left(\left(1-t_w\right)\sum_{l=1}^{n}\partial_{z^*}c_l\frac{dp_l}{d\xi_k}+\frac{1}{\epsilon}\theta\left(\frac{\theta}{z(\theta)}\right)^{\frac{1}{\epsilon}}\frac{1}{z\left(\theta\right)}\frac{dz(\theta)}{d\xi_k}\right)\\&=-\frac{\epsilon}{1+\epsilon}\theta\pi(\theta)\left(\left(1-t_w\right)\sum_{l=1}^{n}\partial_{z^*}c_l\frac{dp_l}{d\xi_k}+\frac{1}{1+\epsilon}\theta^2\left(\frac{\theta}{z(\theta)}\right)^{1+2\frac{1}{\epsilon}}\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\}\right).\end{aligned}$$

Putting everything together, we obtain:

$$\begin{aligned}&\frac{\epsilon}{1+\epsilon}\theta_0\pi(\theta_0)\left(\left(1-t_w\right)\sum_{l=1}^{n}\partial_{z^*}e_l\frac{1}{p_l}\frac{dp_l}{d\xi_k}+\frac{1}{1+\epsilon}\theta_0^2\left(\frac{\theta_0}{z(\theta_0)}\right)^{1+2\frac{1}{\epsilon}}\frac{d}{d\theta}\left\{\frac{dV(\theta_0)}{d\xi_k}\right\}\right)\\=&-\left(1-t_w\right)\int_{\theta_0}^{\bar{\theta}}\left(\sum_{l=1}^{n}\left(\partial_{z^*}e_l-g\left(\theta\right)\partial_{z^*}E_l\right)\frac{1}{p_l}\frac{dp_l}{d\xi_k}+g\left(\theta\right)\left(\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}-\int g\gamma\left(\theta'\right)\frac{dV(\theta')}{d\xi_k}\pi(\theta')d\theta'\right)\right)\pi(\theta)d\theta.\end{aligned}$$

Using the optimality of the initial schedule, we have:

$$\begin{aligned}
\frac{\epsilon}{1+\epsilon}\theta_0\pi(\theta_0)\frac{1}{1+\epsilon}\theta_0^2\left(\frac{\theta_0}{z(\theta_0)}\right)^{1+2\frac{1}{\epsilon}}\frac{d}{d\theta}\left\{\frac{dV(\theta_0)}{d\xi_k}\right\} =& -(1-t_w)\int_{\theta_0}^{\bar{\theta}} g\left(\theta\right)\left(\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}-\int g\gamma\left(\theta'\right)\frac{dV(\theta')}{d\xi_k}\pi(\theta')d\theta'\right)\pi(\theta)d\theta \\
&-(1-t_w)\sum_{l=1}^{n}\left(\int_{\theta_0}^{\bar{\theta}}\left(\left(\partial_{z^*}e_l-g\left(\theta\right)\partial_{z^*}E_l\right)\right)\pi(\theta)d\theta+\frac{\epsilon}{1+\epsilon}\theta_0\pi(\theta_0)\partial_{z^*}e_l\right)\frac{1}{p_l}\frac{dp_l}{d\xi_k} \\
=& -(1-t_w)\int_{\theta_0}^{\bar{\theta}} g\left(\theta\right)\left(\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}-\int g\gamma\left(\theta'\right)\frac{dV(\theta')}{d\xi_k}\pi(\theta')d\theta'\right)\pi(\theta)d\theta \\
&-(1-t_w)\sum_{l=1}^{n}\left(\int_{\theta_0}^{\bar{\theta}}\left(\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\right)\pi(\theta)d\theta+\frac{\epsilon}{1+\epsilon}\theta_0\pi(\theta_0)\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\right)\frac{1}{p_l}\frac{dp_l}{d\xi_k} \\
&-\frac{\epsilon}{1+\epsilon}\theta_0\pi(\theta_0)\frac{1}{1-T'\left(z\left(\theta_0\right)\right)}\sum_{l=1}^{n}\partial_{z^*}E_l\frac{1}{p_l}\frac{dp_l}{d\xi_k}
\end{aligned}$$

The last line uses the fact that the initial schedule is optimal, so that:

$$(1-t_w)\int_{\theta_0}^{\bar{\theta}} g\left(\theta\right)\pi(\theta)d\theta=(1-t_w)\int_{\theta_0}^{\bar{\theta}}\pi(\theta)d\theta-\frac{\epsilon}{1+\epsilon}\theta_0\pi(\theta_0)\left(\frac{1}{1-T'\left(z\left(\theta_0\right)\right)}-(1-t_w)\right).$$

Using the definition of τ_l, we therefore have

$$\begin{aligned}
\frac{\epsilon}{(1+\epsilon)^2}\frac{\theta_0\pi(\theta_0)}{\left(1-T'\left(z\left(\theta_0\right)\right)\right)^2}\left(\frac{\theta_0}{z(\theta_0)}\right)\frac{d}{d\theta}\left\{\frac{dV(\theta_0)}{d\xi_k}\right\} =& -(1-t_w)\int_{\theta_0}^{\bar{\theta}} g\left(\theta\right)\left(\gamma\left(\theta\right)\frac{dV(\theta)}{d\xi_k}-\int g\gamma\left(\theta'\right)\frac{dV(\theta')}{d\xi_k}\pi d\theta'\right)\pi d\theta \\
&-\frac{\epsilon}{1+\epsilon}\frac{\theta_0\pi(\theta_0)}{1-T'\left(z\left(\theta_0\right)\right)}\sum_{l=1}^{n}\left(\tau_l\left(\theta_0\right)+\partial_{z^*}E_l\right)\frac{1}{p_l}\frac{dp_l}{d\xi_k},
\end{aligned}$$

which proves the first formula of the Lemma.
Next, we differentiate the budget constraint:

$$\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{dz(\theta)}{d\xi_k}-\frac{dz^*(\theta)}{d\xi_k}\right)\pi\left(\theta\right)d\theta+\sum_{l=1}^{n}\left(p_l-\partial_{C_l}\chi_l\left(\xi_l,C_l\right)\right)\frac{dC_l}{d\xi_k}+\sum_{l=1}^{n}\frac{dp_l}{d\xi_k}C_l-\partial_{\xi_k}\chi_k\left(\xi_k,C_k\right)=0.$$

Recall that we have $C_l=\int c_l\left(z^*\left(\theta\right),\mathbf{p}\right)\pi\left(\theta\right)d\theta$ and that $c_h\left(v,\mathbf{p}\right)$ is the Hicksian demand function. Using the standard Slutsky decomposition, we have:

$$\frac{dC_l}{d\xi_k}=\int_{\underline{\theta}}^{\bar{\theta}}\partial_{z^*}c_l\left(\theta\right)\left(\frac{dz^*(\theta)}{d\xi_k}-\sum_{m=1}^{n}c_m\left(\theta\right)\frac{dp_m}{d\xi_k}\right)\pi\left(\theta\right)d\theta+\sum_{m=1}^{n}\int_{\underline{\theta}}^{\bar{\theta}}\partial_{p_m}c_l^h\left(\theta\right)\pi\left(\theta\right)d\theta\frac{dp_m}{d\xi_k}.$$

Using $p_l-\partial_{C_l}\chi_l\left(\xi_l,C_l\right)=t_wp_l$, $\sum_{l=1}^{n}p_l\partial_{p_m}c_l^h\left(\theta\right)=0$ and $\sum_{l=1}^{n}p_l\partial_{z^*}c_l\left(\theta\right)=1$, we obtain:

$$\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{dz(\theta)}{d\xi_k}-(1-t_w)\left(\frac{dz^*(\theta)}{d\xi_k}-\sum_{l=1}^{N}c_l\left(\theta\right)\frac{dp_l}{d\xi_k}\right)\right)\pi\left(\theta\right)d\theta-\partial_{\xi_k}\chi_k\left(\xi_k,C_k\right)=0.$$

Using Roy's identity and the envelope condition, we have

$$\frac{dV(\theta)}{d\xi_k} = v_{z^*}\left(\frac{dz^*(\theta)}{d\xi_k} - \sum_{l=1}^{n} c_l\left(\theta\right)\frac{dp_l}{d\xi_k} - \left(1-T'\right)\frac{dz(\theta)}{d\xi_k}\right),$$

$$\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\} = \left(1+\frac{1}{\epsilon}\right)\frac{1}{\theta}\left(1-T'\left(z(\theta)\right)\right)\frac{dz(\theta)}{d\xi_k},$$

so we can re-express the government budget constraint in terms of household's welfare:

$$\int_{\underline{\theta}}^{\bar{\theta}}\left(\left(\frac{1}{1-T'}-\left(1-t_w\right)\right)\frac{\epsilon}{1+\epsilon}\theta\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\}-\left(1-t_w\right)\frac{1}{v_{z^*}}\frac{dV(\theta)}{d\xi_k}\right)\pi d\theta - \partial_{\xi_k}\chi_k\left(\xi_k, C_k\right) = 0.$$

Finally, using the optimality of the initial schedule we have

$$\int_{\underline{\theta}}^{\bar{\theta}}\left(\left(\frac{1}{1-T'}-\left(1-t_w\right)\right)\frac{\epsilon}{1+\epsilon}\theta\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\}-\left(1-t_w\right)\frac{1}{v_{z^*}}\frac{dV(\theta)}{d\xi_k}\right)\pi d\theta = -\left(1-t_w\right)\int_{\underline{\theta}}^{\bar{\theta}} g\left(\theta\right)\frac{dV(\theta)}{d\xi_k}\pi d\theta,$$

which implies

$$\left(1-t_w\right)\int_{\underline{\theta}}^{\bar{\theta}} g\left(\theta\right)\frac{dV(\theta)}{d\xi_k}\pi\left(\theta\right)d\theta = -\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right),$$

which proves the lemma.□

A.2.2 Proofs for Section 4.2

Here, we provide a proof of our results when the production function is linear. In that case, prices are fully exogenous $p_k = \phi_k\left(\xi_k\right)$ and $\chi_k = \phi_k\left(\xi_k\right)C_k$. Re-normalizing the shock ($\tilde{\xi}_k = \phi_k\left(\xi_k\right)$), we can directly re-write the system of Lemma A1 as:

$$\mathcal{D}\left[\frac{dV}{d\ln p_k}\right](\theta) = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\left(\tau_k\left(\theta\right)+\partial_{z^*}E_k\right), \quad \int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{d\ln p_k}\pi d\theta = -p_k C_k.$$

As in the main text, we define an increase in the relative price of the necessity, keeping the average price level constant, as $d\ln\bar{p}_l$, such that $d\ln p_l = \bar{s}_h d\ln\bar{p}_l$ and $d\ln p_h = -\bar{s}_l d\ln\bar{p}_l$. We also define a homogeneous increase in price $d\ln\bar{p}$, such that $d\ln p_l = d\ln p_h = d\ln\bar{p}$. In a two-good economy, a relative increase in the price of necessity and an homogeneous increase in price summarizes all the price changes.

Linear Social Welfare Function (Proposition 2 and Corollary 1).

<u>Proof (Proposition 2):</u> If $G(V(\theta),\theta) = \lambda_\theta V(\theta)$, we have $\gamma(\theta) = 0\ \forall\theta$ (since $G''(V,\theta) = 0$). We therefore have, using Lemma A1:

$$\mathcal{D}\left[\frac{dV}{dlnp_k}\right](\theta) = \frac{\epsilon}{(1+\epsilon)^2}\frac{\theta\pi(\theta)}{(1-T')^2}\frac{\theta}{z(\theta)}\frac{d}{d\theta}\left\{\frac{dV}{dlnp_k}\right\} = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\left(\tau_k(\theta)+\partial_{z^*}E_k\right).$$

In addition, we have from the envelope condition:

$$\frac{d}{d\theta}\left\{\frac{dV(\theta)}{dlnp_k}\right\} = \left(1+\frac{1}{\epsilon}\right)\frac{1}{\theta}\left(1-T'\right)\frac{dz(\theta)}{dlnp_k}.$$

From the optimality of labor supply ($\frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{\epsilon^{-1}}/v_{z^*}\left(z^*(\theta),\mathbf{p}\right) = 1-T'$), we have:

$$\frac{1}{z}\frac{dz(\theta)}{dlnp_k} = -\frac{\epsilon}{1-T'}\frac{dT'}{dlnp_k} - \epsilon\partial_{z^*}e_k.$$

Plugging these expressions in our formula, we obtain:

$$\begin{aligned}\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{(1-T')}\left(-\frac{1}{1-T'}\frac{dT'}{dlnp_k} - \partial_{z^*}e_k\right) &= -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\left(\tau_k(\theta)+\partial_{z^*}E_k\right)\\ \frac{p_k d}{dp_k}\left\{\frac{T'}{1-T'}\right\} &= \frac{1}{1-T'}\left(\tau_k(\theta) - \left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\right).\end{aligned}$$

Finally, using the definition of $\tau_k(\theta)$, we have:

$$\begin{aligned}\frac{p_k d}{dp_k}\left\{\frac{T'}{1-T'}\right\} &= \left(\frac{1+\epsilon}{\epsilon}\frac{1}{\theta\pi(\theta)}\int_\theta^{\bar{\theta}}\left(\partial_{z^*}e_l - \partial_{z^*}E_l\right)\pi d\theta' - \frac{T'}{1-T'}\left(\partial_{z^*}e_l - \partial_{z^*}E_l\right)\right)\\ &= \frac{1}{z\tilde{\zeta}f(z(\theta))}\int_{z(\theta)}^{z(\bar{\theta})}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)f(z)dz - \frac{T'}{1-T'}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right),\end{aligned}$$

which proves our formula. Since $z\tilde{\zeta}f(z(\theta)) \geq 0$, the expression has the same sign as $\mathcal{F}(z(\theta))$, defined below:

$$\begin{aligned}\mathcal{F}(z(\theta)) &\equiv \int_{z(\theta)}^{z(\bar{\theta})}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)f(z)dz - z\tilde{\zeta}f(z(\theta))\frac{T'}{1-T'}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\\ &= \int_{z(\theta)}^{z(\bar{\theta})}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)f(z)dz - \left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\int_{z(\theta)}^{z(\bar{\theta})}(1-g)f(z)dz.\end{aligned}$$

Inspecting the expression in the second line, we have $\mathcal{F}(z(\underline{\theta})) = \mathcal{F}\left(z\left(\bar{\theta}\right)\right) = 0$. Assume now that $\partial_{z^*}e_k$ is decreasing (k is a necessity good) and define θ^* such that

$\partial_{z^*} e_k (z^*(\theta^*), \mathbf{p}) = \partial_{z^*} E_k$ (note that since $z(\theta)$ is increasing due to the single crossing property and $dV/d\theta > 0$ from the envelope condition, $z^*(\theta)$is increasing in θ). We have:

$$\mathcal{F}'(z(\theta)) = -(\partial_{z^*} e_k - \partial_{z^*} E_k) g f(z) - \left(1 - T'\right) \partial_{z^* z^*} e_k \int_{z(\theta)}^{z(\bar{\theta})} (1-g) f(z) dz.$$

For $\theta \geq \theta^*$, we have $\partial_{z^*} e_k < \partial_{z^*} E_k$ and since $\partial_{z^* z^*} e_k \leq 0$, we have $\mathcal{F}'(z(\theta)) > 0$ for $\theta \geq \theta^*$ ($\int_{z(\theta)}^{z(\bar{\theta})} (1-g) f(z) dz \geq 0$, as g is non increasing. Since $\mathcal{F}\left(z\left(\bar{\theta}\right)\right) = 0$, this implies $\mathcal{F}(z(\theta)) < 0$ for $\theta \geq \theta^*$.
Note in addition that we can rewrite $\mathcal{F}(z(\theta))$ as:

$$\mathcal{F}(z(\theta)) = -\int_{z(\underline{\theta})}^{z(\theta)} (\partial_{z^*} e_k - \partial_{z^*} E_k) f(z) dz + (\partial_{z^*} e_k - \partial_{z^*} E_k) \int_{z(\underline{\theta})}^{z(\theta)} (1-g) f(z) dz.$$

For $\theta \leq \theta^*$, we have $\partial_{z^*} e_k - \partial_{z^*} E_k > 0$ and decreasing in θ, so that:

$$\begin{aligned} \mathcal{F}(z(\theta)) &< -(\partial_{z^*} e_k - \partial_{z^*} E_k) \int_{z(\underline{\theta})}^{z(\theta)} f(z) dz + (\partial_{z^*} e_k - \partial_{z^*} E_k) \int_{z(\underline{\theta})}^{z(\theta)} (1-g) f(z) dz \\ &= -(\partial_{z^*} e_k - \partial_{z^*} E_k) \int_{z(\underline{\theta})}^{z(\theta)} g f(z) dz < 0. \end{aligned}$$

Thus, $\mathcal{F}(z(\theta)) < 0$ for $\theta < \theta^*$, which implies $\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} < 0$. By direct inspection, since $\partial_{z^*} e_l - \partial_{z^*} E_l = -(\partial_{z^*} e_h - \partial_{z^*} E_h)$, we have $\frac{p_h d}{dp_h}\left\{\frac{T'}{1-T'}\right\} = -\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} > 0$. □

We now turn to our welfare analysis and provide a proof of Corollary 1. We also show that welfare decreases after an homogeneous price increase and that the decrease in welfare does not depend on consumption patterns. In particular the welfare decrease is the same when household have homothetic or non-homothetic preferences. The result is intuitive: an homogeneous price increase is equivalent to an homogeneous reduction in households' real wage independently from their consumption preferences. Households reduce their labor supply and, from Proposition 2, tax rates are left unchanged: the real income of all households falls.

Complement to Corollary 1. For an homogeneous increase in price, $d\ln\bar{p}$, such that $d\ln p_l = d\ln p_h = d\ln\bar{p}$, we have $dV(\theta)/d\bar{p}$ is negative, decreasing, and independent from consumption preferences.
Proof (Corollary 1 and Complement): For the proof that full compensation is

feasible, see the proof of Proposition 3. Using the formula of lemma A1, we have that for an increase in the relative price of necessities, the change in welfare satisfies:

$$\frac{\epsilon}{(1+\epsilon)^2}\frac{\theta\pi(\theta)}{(1-T')^2}\frac{\theta}{z(\theta)}\frac{d}{d\theta}\left\{\frac{dV}{dln\bar{p}_l}\right\}=-\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\left(\tau_l\left(\theta\right)+\partial_{z^*}E_l-\bar{s}_l\right),\ \int_{\underline{\theta}}^{\bar{\theta}}g\frac{dV}{dln\bar{p}_l}\pi d\theta=0,$$

where we used $\gamma=0$ and $dV\left(\theta\right)/d\bar{p}_l=\bar{s}_h dV\left(\theta\right)/dp_l+\bar{s}_l dV\left(\theta\right)/dp_h$. The second equation directly shows that $\mathbb{E}\left(gdV\left(\theta\right)/d\bar{p}_l\right)=0$. Since $\partial_{z^*}E_l\leq\bar{s}_l$, to show that $dV\left(\theta\right)/d\bar{p}_l$ is increasing it is enough to show that $\tau_l\left(\theta\right)$ is negative. We will then have $d_\theta\left\{dV\left(\theta\right)/d\bar{p}_l\right\}>0$, which implies, given $\mathbb{E}\left(gdV\left(\theta\right)/d\bar{p}_l\right)=0$, that $dV\left(\underline{\theta}\right)/d\bar{p}_l<0$ and $dV\left(\bar{\theta}\right)/d\bar{p}_l>0$.
Recall that we have:

$$\tau_l\left(\theta\right)=\left(1-T'\right)\left(\frac{1+\epsilon}{\epsilon}\frac{1}{\theta\pi(\theta)}\int_{\theta}^{\bar{\theta}}\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\pi d\theta'+\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\right).$$

Since $(1-T')\geq 0$ and $\theta\pi(\theta)\geq 0$, $\tau_l\left(\theta\right)$ has the same sign as $\tilde{\tau}_l\left(\theta\right)=\int_{\theta}^{\bar{\theta}}\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)\pi d\theta'+\frac{\epsilon}{1+\epsilon}\theta\pi(\theta)\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)$.
As in the proof of Proposition 2, define θ^* such that $\partial_{z^*}e_l\left(z^*\left(\theta^*\right),\mathbf{p}\right)=\partial_{z^*}E_l$. For $\theta>\theta^*$, we have $\partial_{z^*}e_l<\partial_{z^*}E_l$, which implies $\tilde{\tau}_l\left(\theta\right)<0$. In addition, we have $\tilde{\tau}_l'\left(\theta\right)=-\left(1-\frac{\epsilon}{1+\epsilon}\frac{\theta\pi'(\theta)}{\pi(\theta)}\right)\pi(\theta)\left(\partial_{z^*}e_l-\partial_{z^*}E_l\right)+\left(1-T'\right)z'\left(\theta\right)\partial_{z^*z^*}e_l<0$ for $\theta\leq\theta^*$. Indeed, $\partial_{z^*}e_l-\partial_{z^*}E_l\geq 0$, $\partial_{z^*z^*}e_l\leq 0$ (and $z'\left(\theta\right)\geq 0$ for the tax schedule to be incentive compatible). Since $\tilde{\tau}_l\left(\underline{\theta}\right)=0$ is decreasing on $\left(\underline{\theta},\theta^*\right)$ and negative for $\theta>\theta^*$, we have $\tilde{\tau}_l\left(\theta\right)$ negative everywhere, which implies $d_\theta\left\{dV\left(\theta\right)/d\bar{p}_l\right\}>0$.

For an homogeneous price increase and simplifying the formulas from Lemma A1, we have:

$$\begin{aligned}\frac{d}{d\theta}\left\{\frac{dV}{dln\bar{p}}\right\}&=-\left(1+\epsilon\right)\left(1-T'\right)\frac{z(\theta)}{\theta}\\ \int_{\underline{\theta}}^{\bar{\theta}}g\frac{dV}{dln\bar{p}}\pi d\theta&=-p_lC_l-p_hC_h=-\int_{\underline{\theta}}^{\bar{\theta}}z\left(\theta\right)\pi d\theta.\end{aligned}$$

This implies that $dV\left(\theta\right)/d\bar{p}$ is decreasing and independent from consumption preferences. Next, we have:

$$\begin{aligned}&\frac{d}{d\theta}\left\{\frac{dV}{dln\bar{p}}\right\}=-\left(1+\epsilon\right)\left(1-T'\right)\frac{z(\theta)}{\theta}=-\left(1-T'+\epsilon zT''\right)z'\left(\theta\right)\\ &\Rightarrow\frac{dV\left(\theta\right)}{dln\bar{p}}=\frac{dV\left(\underline{\theta}\right)}{dln\bar{p}}-\left(z\left(\theta\right)-\left(1+\epsilon\right)T\left(z\left(\theta\right)\right)+\epsilon z\left(\theta\right)T'\left(z\left(\theta\right)\right)\right).\end{aligned}$$

Finally, recall from the proof of Lemma A1 that we have:

$$\begin{aligned}\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{ddln\bar{p}}\pi d\theta &= -\int_{\underline{\theta}}^{\bar{\theta}}\left(\frac{1}{1-T'}\frac{\epsilon}{1+\epsilon}\theta\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\} - \frac{dV(\theta)}{d\xi_k}\right)\pi\left(\theta\right)d\theta \\ &= \int_{\underline{\theta}}^{\bar{\theta}}\left(\epsilon\left(1-T'\right)z(\theta)+z\left(\theta\right)\right)\pi\left(\theta\right)d\theta + \frac{dV\left(\underline{\theta}\right)}{dln\bar{p}} \\ \Rightarrow \frac{dV\left(\underline{\theta}\right)}{dln\bar{p}} &= -\int_{\underline{\theta}}^{\bar{\theta}}\epsilon\left(1-T'\right)z(\theta)\pi\left(\theta\right)d\theta < 0.\end{aligned}$$

The second line uses $\int_{\underline{\theta}}^{\bar{\theta}} T\left(z\left(\theta\right)\right)\pi d\theta = 0$. We therefore have $dV\left(\underline{\theta}\right)/d\bar{p} < 0$ and $dV\left(\theta\right)/d\bar{p}$decreasing, which implies that $dV\left(\theta\right)/d\bar{p}$ is everywhere negative.$\square$

Top and bottom tax rates. To conclude this section, we provide formulas for the top and bottom tax rates that we use in our discussion of the quantitative results of Section 5.2. We assume that the weights g are decreasing and denote $g\left(\underline{\theta}\right) > 1 > g\left(\bar{\theta}\right)$ their limit at the bottom and top of the distribution. $\partial_{z^*}e_l\left(\underline{\theta}\right) > \partial_{z^*}E_l > \partial_{z^*}e_l\left(\bar{\theta}\right)$ are the marginal propensity to spend on l at the bottom and top of the distribution. For an increase in the price of l, our tax formula is:

$$\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} = \frac{T'}{1-T'}\left(\frac{\int_{\theta}^{\bar{\theta}}\left(\partial_{z^*}e_l - \partial_{z^*}E_l\right)\pi(\theta)d\theta}{\int_{\theta}^{\bar{\theta}}\left(1-g\right)\pi(\theta)d\theta} - \left(\partial_{z^*}e_l - \partial_{z^*}E_l\right)\right).$$

Using l'Hopital's rule, we have:

$$\begin{aligned}\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\}\left(\underline{\theta}\right) &= -\frac{g\left(\underline{\theta}\right)}{g\left(\underline{\theta}\right)-1}\frac{T'}{1-T'}\left(\partial_{z^*}e_l\left(\underline{\theta}\right) - \partial_{z^*}E_l\right) \\ \frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\}\left(\bar{\theta}\right) &= -\frac{g\left(\bar{\theta}\right)}{1-g\left(\bar{\theta}\right)}\frac{T'}{1-T'}\left(\partial_{z^*}E_l - \partial_{z^*}e_l\left(\bar{\theta}\right)\right),\end{aligned}$$

which gives the formulas for the top and bottom tax rates.

Non-Linear Social Welfare Function (Proposition 3 and Corollary 2-3) Before proving Proposition 3, let us briefly discuss the tax formula when the social welfare function is non-linear. Reexpressing the formula from Lemma A1, in terms of tax rates, we have:

$$\begin{aligned} z\tilde{\zeta} f\left(z\left(\theta\right)\right)\frac{p_k d}{dp_k}\left\{\frac{T'}{1-T'}\right\} &= \int_{z(\theta)}^{z(\bar{\theta})} g\left(\gamma v_{z^*}\left(\frac{dT}{dlnp_k}+e_k\right) - \int_{z(\underline{\theta})}^{z(\bar{\theta})} g\gamma v_{z^*}\left(\frac{dT}{dlnp_k}+e_k\right) fdz\right) fdz \\ &\quad + z\tilde{\zeta} f\left(z\left(\theta\right)\right)\frac{p_k d}{dp_k}\left\{\frac{T'_{lin}}{1-T'_{lin}}\right\} \\ z\tilde{\zeta} f\left(z\left(\theta\right)\right)\frac{p_k d}{dp_k}\left\{\frac{T'_{lin}}{1-T'_{lin}}\right\} &= \mathbb{E}_{z>z(\theta)}\left(\partial_{z^*} e_k - \partial_{z^*} E_k\right) - z\tilde{\zeta} f\left(z\left(\theta\right)\right)\frac{T'}{1-T'}\left(\partial_{z^*} e_k - \partial_{z^*} E_k\right). \end{aligned}$$

The tax formula is the sum of two terms. The second one is the change in tax rate when the social welfare function is linear, $\frac{p_k d}{dp_k}\left\{\frac{T'_{lin}}{1-T'_{lin}}\right\}$. As before, through this term and Channel #1 and #2, an increase in the price of necessities induces more redistribution towards the rich. The first term captures an income effect of redistribution on Pareto weights. If the planner implements the tax reform arising with a linear social welfare function, the tax burden increases for low income households, which decreases their utility and raises their Pareto weight. If the Pareto weights of lower income households increases more than average, then the first term is positive, which counteracts the impact of Channel #1 and #2 and pushes for more redistribution towards low-income households. We show however in Proposition 3 that this counterbalancing effect does not fully offset the impact of Channel #1 and #2, even when it is feasible to compensate all households.

We now turn to the proof of Proposition 3. As in the linear case, we also show that the welfare change after an homogeneous price increase does not depend on consumption patterns. The argument is the same as in the linear case.

Complement to Proposition 3. For an homogeneous increase in price, $d\ln\bar{p}$, such that $d\ln p_l = d\ln p_h = d\ln\bar{p}$, we have that $dV^G\left(\theta\right)/d\bar{p}$ is independent from consumption preferences.

Proof (Proposition 3 and Complement): Using the formula of lemma A1, we have for an increase in the relative price of the necessity good:

$$\begin{aligned} \mathcal{D}\left[\frac{dV}{dln\bar{p}_l}\right]\left(\theta\right) =& \kappa\left(\theta\right)\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\}, \int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{dln\bar{p}_l}\pi d\theta = 0, \\ \kappa\left(\theta\right)\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\} =& -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\left(\tau_l\left(\theta\right) + \partial_{z^*} E_l - \bar{s}_l\right), \end{aligned}$$

where $\frac{dV^{lin}}{dln\bar{p}_l}$ is the welfare change with a linear social welfare function described in Corollary 1, $\kappa\left(\theta\right)=\frac{\epsilon}{(1+\epsilon)^2}\frac{\theta\pi(\theta)}{(1-T')^2}\frac{\theta}{z(\theta)}$.
First, note that implementing $\frac{dV}{dln\bar{p}_l}=0$ implies $-dT\left(z\left(\theta\right)\right)-\left(s_l-\bar{s}_l\right)z^{*}d\text{ln}\bar{p}_l=0$. With this tax change, we have $dz\left(\theta\right)=-z\tilde{\zeta}\left(dT'\left(z\left(\theta\right)\right)/\left(1-T'\right)+\left(\partial_{z^*}e_l-\bar{s}_l\right)\right)=0$ and the total cost of the reform is:

$$-\int\left(s_l-\bar{s}_l\right)z^{*}d\text{ln}\bar{p}_l\pi d\theta=0,$$

so compensating all households is feasible (as it is budget neutral). Such compensation is however not optimal as it does not solve the first equation of the system. From the proof of Corollary 1, we know that $\frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}>0$ for all θ. We consider the auxiliary system:

$$\begin{aligned}\mathcal{D}\left[\frac{dV_0}{dln\bar{p}_l}\right]\left(\theta\right)=&\kappa\left(\theta\right)\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\},\ \int_{\underline{\theta}}^{\bar{\theta}}g\gamma\left(\theta\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta=0,\\ \mathcal{D}\left[\frac{dV_1}{dln\bar{p}_l}\right]\left(\theta\right)=&0,\ \int_{\underline{\theta}}^{\bar{\theta}}g\frac{dV_1}{dln\bar{p}_l}\pi d\theta=-\int_{\underline{\theta}}^{\bar{\theta}}g\frac{dV_0}{dln\bar{p}_l}\pi d\theta.\end{aligned}$$

We then have: $\frac{dV}{dln\bar{p}_l}=\frac{dV_0}{dln\bar{p}_l}+\frac{dV_1}{dln\bar{p}_l}$.
We first consider the term $\frac{dV_0}{dln\bar{p}_l}$. Since $\frac{dV^{lin}}{dln\bar{p}_l}>0$, we necessarily have that $dV/d\bar{p}_l$ is strictly negative at $\underline{\theta}$. If not, $-\int_{\theta}^{\bar{\theta}}g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta'$ is non decreasing and therefore non negative at $\underline{\theta}$. Therefore, $\frac{d}{d\theta}\left\{\frac{dV_0}{dln\bar{p}_l}\right\}\left(\underline{\theta}\right)\geq\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\}\left(\underline{\theta}\right)>0$, so $dV_0/dln\bar{p}_l$ is positive in a neighborhood around $\underline{\theta}$.
We show that this leads to a contradiction. Define θ_0 the first θ such that $dV_0/dln\bar{p}_l$ is 0. θ_0 must exist since $\int_{\underline{\theta}}^{\bar{\theta}}g\gamma\left(\theta\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta=0$, so there exists $dV_0/dln\bar{p}_l<0$. Then, since $\int_{\underline{\theta}}^{\bar{\theta}}g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta'=0$, $\int_{\theta_0}^{\bar{\theta}}g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta'<0$. Since in addition $\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\}$ is positive at θ_0, we have $\frac{d}{d\theta}\left\{\frac{dV_0}{dln\bar{p}_l}\right\}$ strictly positive at θ_0. Since $dV_0/dln\bar{p}_l\left(\theta_0\right)=0$, by definition, this implies $dV_0/dln\bar{p}_l\left(\theta\right)<0$ in $\left(\theta_0-\epsilon,\theta_0\right)$, which contradicts the fact that $dV_0/dln\bar{p}_l\left(\theta\right)>0$ on $\left(\theta,\theta_0\right)$. Therefore, we have $dV/d\bar{p}_l$ is strictly negative at $\underline{\theta}$. Using the same logic, $dV_0/dln\bar{p}_l$ cannot be negative around $\bar{\theta}$.

In addition, $\int_{\theta}^{\bar{\theta}}g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta'>0$ on $\left(\underline{\theta},\bar{\theta}\right)$. Indeed, suppose not and denote again θ_0 the smallest θ_0 such that $\int_{\theta_0}^{\bar{\theta}}g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta'=0$. $dV_0/dln\bar{p}_l$ cannot be negative at θ_0 since then $\int_{\theta_0}^{\bar{\theta}}g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta'$ would be increasing at θ_0, contradicting the fact that it is positive for $\theta<\theta_0$. Therefore, $dV_0/dln\bar{p}_l\left(\theta\right)$ is non negative at θ_0 – however, using the same reasoning as before, this would imply $dV_0/dln\bar{p}_l\left(\theta\right)>0$ for

$\theta > \theta_0$. As before, suppose not and consider θ_1 such that $dV_0/dln\bar{p}_l\left(\theta_1\right) = 0$. We have $\int_{\theta_0}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta' = 0$ and $\int_{\theta_0}^{\theta_1} g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$, so $\int_{\theta_1}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta' < 0$. This implies $\frac{d}{d\theta}\left\{\frac{dV_0}{dln\bar{p}_l}\right\}\left(\theta_1\right) > 0$ so $dV_0/dln\bar{p}_l\left(\theta_1\right)$ is negative below θ_1, which contradicts that θ_1 exists.

Therefore, $dV_0/dln\bar{p}_l\left(\theta\right) > 0$ for $\theta > \theta_0$, so that $\int_{\theta_0}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$, which contradicts that $\theta_0$exists.

Next, we show that $\int_{\theta}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$, for all θ, implies $\int_{\theta}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' \geq 0$. Since $dV_0/dln\bar{p}_l$ is positive at $\bar{\theta}$, we have $\int_{\theta}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$ for θ high enough. Suppose that there exists θ_0 such that $\int_{\theta_0}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' = 0$ and consider the highest θ_0 such that it is the case. We therefore have $\int_{\theta}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$ for $\theta > \theta_0$ and:

$$\begin{aligned}\int_{\theta_0}^{\bar{\theta}} g\gamma\left(\theta\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta &= \int_{\theta_0}^{\bar{\theta}}\gamma'\left(\theta\right)\int_{\theta'}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' d\theta + \gamma\left(\theta_0\right)\int_{\theta_0}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta \\ &= \int_{\theta_0}^{\bar{\theta}}\gamma'\left(\theta\right)\int_{\theta'}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' d\theta \leq 0,\end{aligned}$$

where the second line uses the fact that $\gamma'\left(\theta\right) \leq 0$ and $\int_{\theta}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$ for $\theta > \theta_0$. This contradicts the fact that $\int_{\theta_0}^{\bar{\theta}} g\gamma\left(\theta\right)\frac{dV_0}{dln\bar{p}_l}\pi d\theta > 0$, so $\int_{\theta}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' \geq 0$ for all θ.

Next, we analyze $dV_1/dln\bar{p}_l$. First, if $\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' = 0$, then $\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV_1}{dln\bar{p}_l}\pi d\theta' = 0$ and $dV_1/dln\bar{p}_l = 0$ solves the system's equations. Therefore $dV/dln\bar{p}_l = dV_0/dln\bar{p}_l$. Suppose $\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$, which implies $\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV_1}{dln\bar{p}_l}\pi d\theta' < 0$. Suppose further $\int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta' < 0$. Then we necessarily have that $\gamma\frac{dV_1}{dln\bar{p}_l} < \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ in a neighborhood of $\underline{\theta}$. Indeed, using the same reasoning as above, if $\gamma\frac{dV_1}{dln\bar{p}_l}\left(\underline{\theta}\right) > \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$, then we have $\gamma\frac{dV_1}{dln\bar{p}_l} > \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ for all θ, which would be a contradiction. We can use the same reasoning as before, if not there is a smallest θ_0 such that $\gamma\frac{dV_1}{dln\bar{p}_l}\left(\theta_0\right) = \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$; but at θ_0 we necessarily have $\frac{d}{d\theta}\left\{\frac{dV_1}{dln\bar{p}_l}\right\}\left(\theta_0\right) > 0$. Indeed, since $\gamma\frac{dV_1}{dln\bar{p}_l}\left(\theta\right) > \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ in $(\underline{\theta}$,$\theta_1)$, we have

$$\int_{\theta_0}^{\bar{\theta}} g\left(\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta' < 0,$$

so $\frac{d}{d\theta}\left\{\frac{dV_1}{dln\bar{p}_l}\right\}\left(\theta_0\right) > 0$.

Since γ is positive decreasing and $\frac{dV_1}{dln\bar{p}_l}\left(\theta_0\right) < 0$, $\frac{d}{d\theta}\left\{\frac{dV_1}{dln\bar{p}_l}\right\}\left(\theta_0\right) > 0$ at z_0, this means that $\gamma\frac{dV_1}{dln\bar{p}_l}$ is increasing at θ_0, which is a contradicts the fact that $\gamma\frac{dV_1}{dln\bar{p}_l} >$

$\int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ below θ_0. Finally, if $\gamma\frac{dV_1}{dln\bar{p}_l}\left(\underline{\theta}\right) = \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$, at $\underline{\theta}$ we would have that $\frac{d}{d\theta}\left\{\frac{dV_1}{dln\bar{p}_l}\right\} = 0$ and $\frac{dV_1}{dln\bar{p}_l} < 0$. Since γ is positive decreasing, $\gamma\frac{dV_1}{dln\bar{p}_l} > \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ in a neighborhood of $\underline{\theta}$ and we can use the same reasoning to get $\gamma\frac{dV_1}{dln\bar{p}_l} > \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ for all θ, which is a contradiction.

Therefore, we have $\gamma\frac{dV_1}{dln\bar{p}_l} < \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ in a neighborhood of $\underline{\theta}$.

Next, we have that $D(\theta) = \int_{\theta}^{\bar{\theta}} g\left(\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta'$ is positive on the interval $(\underline{\theta}, \bar{\theta})$. Consider the smallest θ_0 such that it is 0 at θ_0 and negative in a neighborhood above. First, note we cannot have $\gamma\frac{dV_1}{dln\bar{p}_l} < \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ at θ_0 or in a neighborhood above, since $D(\theta_0)$ would be locally increasing. This means that $\gamma\frac{dV_1}{dln\bar{p}_l} \geq \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ in the neighborhood above θ_0, which implies by the same reasoning as at $\underline{\theta}$ that $\gamma\frac{dV_1}{dln\bar{p}_l} \geq \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'$ for all $\theta > z$ and implies (since $\gamma\frac{dV_1}{dln\bar{p}_l}$ cannot be constant) that $D(\theta)$ is positive everywhere above θ_0, a contradiction.

Therefore, when $\int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta' < 0$, we have that $\frac{dV_1}{dln\bar{p}_l}$ is non positive and decreasing. Since the equation determining $\frac{dV_1}{dln\bar{p}_l}$ is linear, when $\int_{\underline{\theta}}^{\bar{\theta}} g\gamma\frac{dV_1}{dln\bar{p}_l}\pi d\theta' > 0$, $\frac{dV_1}{dln\bar{p}_l}$ would be non negative and increasing. Thus, to have $\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV_1}{dln\bar{p}_l}\pi d\theta' < 0$, we need $\frac{dV_1}{dln\bar{p}_l}$ non positive and decreasing and

$$\int_{\theta}^{\bar{\theta}} g\left(\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta' > 0.$$

Since $\frac{dV}{dln\bar{p}_l} = \frac{dV_0}{dln\bar{p}_l} + \frac{dV_1}{dln\bar{p}_l}$ and $\frac{dV_1}{dln\bar{p}_l}\left(\underline{\theta}\right) < 0$, $\frac{dV_0}{dln\bar{p}_l}\left(\underline{\theta}\right) < 0$ then $\frac{dV}{dln\bar{p}_l}\left(\underline{\theta}\right) < 0$. Furthermore, since $\int_{\theta}^{\bar{\theta}} g\left(\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV_1}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta' > 0$, $\int_{\theta}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' > 0$ and $\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV_0}{dln\bar{p}_l}\pi d\theta' = 0$, we have $\int_{\theta}^{\bar{\theta}} g\left(\gamma\left(\theta'\right)\frac{dV}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma\left(\theta'\right)\frac{dV}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta' > 0$, so $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l} < \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$. Since $0 = \int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{dln\bar{p}_l}\pi d\theta = g\frac{dV}{dln\bar{p}_l}\left(\underline{\theta}\right) + \int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l}\int_{\theta}^{\bar{\theta}} g\pi d\theta = g\frac{dV^{lin}}{dln\bar{p}_l}\left(\underline{\theta}\right) + \int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}\int_{\theta}^{\bar{\theta}} g\pi d\theta$, we have $\frac{dV}{dln\bar{p}_l}\left(\underline{\theta}\right) > \frac{dV^{lin}}{dln\bar{p}_l}\left(\underline{\theta}\right)$. In addition since $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l} < \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$ we have $\frac{dV}{dln\bar{p}_l}\left(\theta\right) - \frac{dV}{dln\bar{p}_l}\left(\underline{\theta}\right) < \frac{dV^{lin}}{dln\bar{p}_l}\left(\theta\right) - \frac{dV^{lin}}{dln\bar{p}_l}\left(\underline{\theta}\right)$ and $\frac{dT'}{dln\bar{p}_l} > \frac{dT'^{lin}}{dln\bar{p}_l}$ which proves Proposition 3. For an homogeneous price change, we have:

$$\mathcal{D}\left[\frac{dV}{dln\bar{p}}\right]\left(\theta\right) = \kappa\left(\theta\right)\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}}\right\}, \int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{dln\bar{p}}\pi d\theta = -\int_{\underline{\theta}}^{\bar{\theta}} z\pi d\theta,$$

$$\kappa\left(\theta\right)\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}}\right\} = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}$$

Note that $\frac{dV^{lin}}{dln\bar{p}_l}$ is now independent from consumption preferences, so $\frac{dV}{dln\bar{p}_l}$ is independent from consumption preferences and $\int_{\underline{\theta}}^{\bar{\theta}} g \frac{dV}{dln\bar{p}_l} \pi d\theta < 0$. □

Proof (Corollary 2): Consider G_ν such that $G_\nu' \propto \lambda(\theta)$, $-G_\nu''/G_\nu' = \nu\gamma(\theta)$. Denote $D(f)(\theta) = \int_\theta^{\bar{\theta}} g \left(\gamma(\theta') f - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma(\theta') f\pi d\theta'\right) \pi d\theta'$ and $\mathcal{D}_\nu[f](\theta) = \kappa(\theta) \frac{d}{d\theta}\{f\}(\theta) + \nu D(f)(\theta)$. We have for $\nu_1 > \nu_0 > 0$, $\mathcal{D}_{\nu_0}\left[d_{\bar{p}_l} V^{G_{\nu_1}} - d_{\bar{p}_l} V^{G_{\nu_0}}\right](\theta) = -(\nu_1 - \nu_0) D\left(d_{\bar{p}_l} V^{G_{\nu_1}}\right)(\theta) < 0$, since our previous result showed $\nu D\left(d_{\bar{p}_l} V^{G_\nu}\right)(\theta) > 0$, and $\mathbb{E}\left(g\left(d_{\bar{p}_l} V^{G_{\nu_1}} - d_{\bar{p}_l} V^{G_{\nu_0}}\right)\right) = 0$. Therefore using the same steps as in the proof of Proposition 3 (reversing signs) we have $d_{\bar{p}_l} V^{G_{\nu_1}}(\underline{\theta}) - d_{\bar{p}_l} V^{G_{\nu_0}}(\underline{\theta}) > 0$ which proves that $d_{\bar{p}_l} V^{G_\nu}(\underline{\theta})$ is increasing in ν. In addition the proof also implies $D\left(d_{\bar{p}_l} V^{G_{\nu_1}} - d_{\bar{p}_l} V^{G_{\nu_0}}\right)(\theta) < 0$: $D\left(d_{\bar{p}_l} V^{G_\nu}\right)(\theta)$ is decreasing in ν for all θ and positive.

Since $d_{\bar{p}_l} V^{G_\nu}(\underline{\theta})$ is increasing in ν and bounded above by 0, it converges to a non positive constant at infinity. We prove by contradiction that it converges to 0. First, we have $m(\theta) \leq d_{\bar{p}_l} V^{G_\nu}(\theta) \leq M(\theta)$for all ν. Indeed $d\left\{d_{\bar{p}_l} V^{G_\nu}(\theta)\right\}/d\theta < d\{d_{\bar{p}_l} V(\theta)\}/d\theta$, $d_{\bar{p}_l} V^{G_\nu}(\underline{\theta}) < 0$ so $d_{\bar{p}_l} V^{G_\nu}(\theta) < d_{\bar{p}_l} V(\theta) - d_{\bar{p}_l} V(\underline{\theta})$. In addition $0 = \int g d_{\bar{p}_l} V^{G_\nu}(\theta) \pi d\theta < \int_{\underline{\theta}}^{\theta_0} g\left(d_{\bar{p}_l} V^{lin}(\theta) - d_{\bar{p}_l} V^{lin}(\underline{\theta})\right) \pi d\theta + \int_{\theta_0}^{\bar{\theta}} g\left(d_{\bar{p}_l} V^{G_\nu}(\theta_0) + d_{\bar{p}_l} V^{lin}(\theta) - d_{\bar{p}_l} V^{lin}(\theta_0)\right) \pi d\theta$ so $d_{\bar{p}_l} V^{G_\nu}(\theta_0) \geq d_{\bar{p}_l} V^{lin}(\theta_0) + \left(\left(\int_{\theta_0}^{\bar{\theta}} g\pi d\theta\right)^{-1} - 1\right) d_{\bar{p}_l} V^{lin}(\underline{\theta})$. So $m(\theta) \leq d_{\bar{p}_l} V^{G_\nu}(\theta) \leq M(\theta)$ for all ν and $m(\theta)$, $M(\theta)$ are continuous. Next, we know from the proof of Proposition 3 that $\int_{\underline{\theta}}^{\bar{\theta}} g\gamma(\theta') d_{\bar{p}_l} V^{G_\nu} \pi d\theta' < 0$, it is direct to show that there exists a constant $C > 0$ such that $-C < \Gamma_\nu = \int_{\underline{\theta}}^{\bar{\theta}} g\gamma(\theta') d_{\bar{p}_l} V^{G_\nu} \pi d\theta'$ for all ν. If not, we can find ν_0 such that $-\Gamma_\nu > -min_{[\underline{\theta},\theta_0]} m(\theta)$. This implies $D\left(d_{\bar{p}_l} V^{G_\nu}\right)(\theta_0) < 0$ which is a contradiction. Therefore, for any θ_0, $D\left(d_{\bar{p}_l} V^{G_\nu}\right)(\theta)$ has uniformly bounded derivative on $[\underline{\theta}, \theta_0]$,since in addition $D\left(d_{\bar{p}_l} V^{G_\nu}\right)(\theta)$ is decreasing in ν for all θ and positive it is uniformly bounded on $[\underline{\theta}, \theta_0]$, a direct application of Arzela-Ascoli and Dini's theorem implies that $D\left(d_{\bar{p}_l} V^{G_\nu}\right)(\theta)$ converges uniformly on $[\underline{\theta}, \theta_0]$ as ν goes to ∞ to $D^*(\theta)$ continuous bounded and non negative. Necessarily, $D^*(\theta) = 0$. If not we have an interval $[\theta_0, \theta_1]$ in which $D^*(\theta) > \delta > 0$. But then we have $d_{\bar{p}_l} V^{G_\nu}(\theta_1) - d_{\bar{p}_l} V^{G_\nu}(\theta_0) < -\nu \int_{\theta_0}^{\theta_1} \kappa(\theta)^{-1} \delta/2 d\theta + d_{\bar{p}_l} V^{lin}(\theta_1) - d_{\bar{p}_l} V^{lin}(\theta_0)$ for ν large enough (using uniform convergence) which implies $d_{\bar{p}_l} V^{G_\nu}(\theta_1) - d_{\bar{p}_l} V^{G_\nu}(\theta_0)$ goes to $-\infty$ contradicting $d_{\bar{p}_l} V^{G_\nu}(\theta_1) - d_{\bar{p}_l} V^{G_\nu}(\theta_0) > m(\theta_1) - M(\theta_0)$.

From Bolzano-Weierstrass, we can extract a sequence ν_n s.t. Γ_{ν_n}converges to $-\Gamma \leq 0$. We show that $\Gamma = 0$. Suppose not, fix θ_0 such that $\int_{\theta_0}^{\bar{\theta}} \left(d_{\bar{p}_l} V^{lin}(\theta) - d_{\bar{p}_l} V^{lin}(\underline{\theta})\right) g\pi d\theta < \Gamma/(8\gamma(\underline{\theta}))$ and $\int_{\theta_0}^{\bar{\theta}} g\pi d\theta < 1/8$. Denote $1/\Upsilon = max_{[\underline{\theta},\theta_0]} \frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\} < \infty$ (see below for an expression) and $\omega = min_{[\underline{\theta},\theta_0]} g\pi > 0$ (by assumption). We first show that necessarily, $d_{\bar{p}_l} V^{G_{\nu_n}}(\theta) \leq -\Gamma/(4\gamma(\underline{\theta}))$ for n large enough. Suppose not, we can then find a sequence i_k going to ∞ s.t. $d_{\bar{p}_l} V^{G_{\nu_{i_k}}}(\theta_{i_k}) > -\Gamma/(4\gamma(\underline{\theta}))$. So $d_{\bar{p}_l} V^{G_{\nu_{i_k}}}(\theta) > -\Gamma/(2\gamma(\underline{\theta}))$ on $[\theta_{i_k} - \varUpsilon\Gamma/(4\gamma(\underline{\theta}));\theta_{i_k}]$ since $d_{\bar{p}_l} V^{G_{\nu_{i_k}}}$ grows slower than $d_{\bar{p}_l} V^{lin}$. This implies $D\left(d_{\bar{p}_l} V^{G_{\nu_{i_k}}}\right)(\theta_{i_k} - \varUpsilon\Gamma/4) - D\left(d_{\bar{p}_l} V^{G_{\nu_{i_k}}}\right)(\theta_{i_k}) \geq \omega\varUpsilon\Gamma/(4\gamma(\underline{\theta}))(-\Gamma/2 + (\Gamma - \epsilon_{i_k})) > 0$ (ϵ_{i_k} can be chosen arbitrarily small) which contradicts uniform convergence of $D\left(d_{\bar{p}_l} V^{G_{\nu_{i_k}}}\right)$ towards 0 below θ_0. Therefore we have $0 = \int d_{\bar{p}_l} V^{G_{\nu_{i_k}}}(\theta_{i_k}) g\pi d\theta \leq -\Gamma/(8\gamma(\underline{\theta})) + \left(1 - \frac{1}{8}\right)\Gamma/(4\gamma(\underline{\theta})) < 0$ a contradiction. Therefore $\Gamma = 0$.

Now suppose that $d_{\bar{p}_l} V^{G_\nu}(\underline{\theta})$ (increasing) converges to $-v < 0$. We have $d_{\bar{p}_l} V^{G_\nu}(\theta) \leq -v + d_{\bar{p}_l} V^{lin}(\theta) - d_{\bar{p}_l} V^{lin}(\underline{\theta})$ so we can choose θ_0 so that $d_{\bar{p}_l} V^{G_\nu}(\theta) < -v/2$ on $[\underline{\theta}, \theta_0]$. So $D\left(d_{\bar{p}_l} V^{G_{\nu_{i_k}}}\right)(\theta_0)$ is bounded away from 0 which contradicts uniform convergence to 0. The proof that $lim_0 d_{\bar{p}_l} V^{G_\nu}(\underline{\theta}) = d_{\bar{p}_l} V^{lin}(\underline{\theta})$ follows the same steps.

For the second part, since $\theta\pi'(\theta)/\pi(\theta) \leq -1 - \omega$, we have for any $\theta > \theta_0$, θ_0 large enough, $\pi(\theta) \leq \pi(\theta_0)(\theta/\theta_0)^{-1-\omega}$. This implies, since g is strictly decreasing when $G(V)$ is either CARA or CRRA, that $g < \delta < 1$ for θ large enough. Therefore, $0 \leq \frac{T'}{1-T'} \leq \frac{1}{1+\gamma}(1-\delta)\frac{1+\epsilon}{\epsilon}$, which implies that there is some $1 - \bar{T}' > 0$ such that $1 - T' > 1 - \bar{T}'$ for all θ.

Note that we have:

$$\begin{aligned} \frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\} &= -(1+\epsilon)\left(1 - T'\right)\frac{z(\theta)}{\theta}\left(\tau_l(\theta) + \partial_{z^*}E_l - \bar{s}_l\right) \\ &= -(1+\epsilon)\left(1 - T'\right)^{1+\epsilon}\theta^{\epsilon}\left(\tau_l(\theta) + \partial_{z^*}E_l - \bar{s}_l\right). \end{aligned}$$

Using again the fact that $\pi(\theta) \leq \pi(\theta_0)(\theta/\theta_0)^{-1-\gamma}$, it is direct to show that $0 < m < \tau_l(\theta) + \partial_{z^*}E_l - \bar{s}_l < M$, so we have, since $1 \geq 1 - T' > 1 - \bar{T}' > 0$,$0 < (1+\epsilon)\bar{m}\theta^{\epsilon} < \frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\} < (1+\epsilon)\bar{M}\theta^{\epsilon}$. This result implies that for any $\theta > \theta_0$, $\theta_0$large enough, $m\bar{\theta}^{1+\epsilon} < \frac{dV^{lin}}{dln\bar{p}_l}(\theta) - \frac{dV^{lin}}{dln\bar{p}_l}(\theta_0) < \bar{M}\theta^{1+\epsilon}$, $\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\}$ and $\frac{dV^{lin}}{dln\bar{p}_l}(\theta)$ grows at the same rate as θ^{ϵ} and $\theta^{1+\epsilon}$ respectively.

We now show that $F(\theta) = \frac{(1+\epsilon)^2}{\epsilon}\frac{(1-T')^2}{\theta\pi(\theta)}\frac{z(\theta)}{\theta}\int_{\theta}^{\bar{\theta}} g\left(\gamma(\theta')\frac{dV}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma(\theta')\frac{dV}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta'$ grows at a smaller rate than θ^{ϵ}, which implies that $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l}(\theta) \sim \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$ and $\frac{dV}{dln\bar{p}_l}(\theta) \sim \frac{dV^{lin}}{dln\bar{p}_l}$ for θ large enough. Indeed, we have, $\frac{d}{d\theta}\left\{\frac{dV}{dln\bar{p}_l}\right\} = -F(\theta) + \frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\}$,so if $F(\theta)$ grows at a lower rate than $\frac{d}{d\theta}\left\{\frac{dV^{lin}}{dln\bar{p}_l}\right\}$ (e.g. grows at rate $ln(\theta)$), then $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l}(\theta) \sim \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$. This also implies that tax rates are the same for high income households. Denote $X \equiv -\int_{\underline{\theta}}^{\bar{\theta}} g\gamma(\theta')\frac{dV}{dln\bar{p}_l}\pi d\theta' \geq 0$, we know from the previous result that $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l} < \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$ so $\frac{dV_1}{dln\bar{p}_l}(\theta) < \frac{dV_1}{dln\bar{p}_l}(\theta_0) + \bar{M}\left(\theta^{1+\epsilon} - \theta_0^{1+\epsilon}\right)$. We now consider θ_1 such that $\frac{dV_1}{dln\bar{p}_l}(\theta_0) + \bar{M}\left(\theta_1^{1+\epsilon} - \theta_0^{1+\epsilon}\right) > 0$ for $\theta > \theta_1$: we have

$$\begin{aligned}
F(\theta) &= \frac{(1+\epsilon)^2}{\epsilon}\frac{(1-T')^2}{\theta\pi(\theta)}\frac{z(\theta)}{\theta}\int_\theta^{\bar{\theta}} g\left(\gamma(\theta')\frac{dV}{dln\bar{p}_l} - \int_{\underline{\theta}}^{\bar{\theta}} g\gamma(\theta')\frac{dV}{dln\bar{p}_l}\pi d\theta'\right)\pi d\theta' \\
&= \frac{(1+\epsilon)^2}{\epsilon}\frac{(1-T')^{2+\epsilon}\theta^\epsilon}{\theta\pi(\theta)}\int_\theta^{\bar{\theta}}\left(g\gamma(\theta')\frac{dV}{dln\bar{p}_l} + X\right)\pi d\theta' \\
&\leq \frac{(1+\epsilon)^2}{\epsilon}(1-T')^{2+\epsilon}\theta^{\epsilon-1}\int_\theta^{\bar{\theta}}(\theta'/\theta)^{-1-\omega}\left(-G''(V(\theta'))\left(\frac{dV_1}{dln\bar{p}_l}(\theta_0) + \bar{M}\left(\theta'^{1+\epsilon} - \theta_0^{1+\epsilon}\right)\right) + G'(V(\theta'))X\right)d\theta',
\end{aligned}$$

where we used again the fact that $\pi(\theta') \leq \pi(\theta)(\theta'/\theta)^{-1-\omega}$.

Now, we have $V'(\theta) = \frac{1}{\theta}\left(\frac{z(\theta)}{\theta}\right)^{1+\frac{1}{\epsilon}} = (1-T')^{1+\epsilon}\theta^\epsilon$, which implies $0 < m_V < V(\theta)/\theta^{1+\epsilon} < M_V$, for θ large enough $V(\theta)$ grows at rate $\theta^{1+\epsilon}$. Now, if G is CARA with coefficient β, we have:

$$\begin{aligned}
F(\theta) &\leq \frac{(1+\epsilon)^2}{\epsilon}\theta^{\epsilon-1}\int_\theta^{\bar{\theta}}\left(\frac{\theta'}{\theta}\right)^{-1-\omega} e^{-\beta m_V\theta'^{1+\epsilon}}\left(\beta\left(\frac{dV_1}{dln\bar{p}_l}(\theta_0) + \bar{M}\left(\theta'^{1+\epsilon} - \theta_0^{1+\epsilon}\right)\right) + X\right)d\theta' \\
&\leq C\theta^{1+2\epsilon}e^{-\beta m_V\theta^{1+\epsilon}}
\end{aligned}$$

Since $\theta'^{1+2\epsilon}e^{-\beta m_V\theta^{1+\epsilon}} = o(\theta^\epsilon)$, we have $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l}(\theta) \sim \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$.

Next, if G is CRRA with coefficient β, we have:

$$\begin{aligned}
F(\theta) \leq{}& \frac{(1+\epsilon)^2}{\epsilon}\theta^{\epsilon-1}\int_\theta^{\bar{\theta}}\left(\frac{\theta'}{\theta}\right)^{-1-\omega} e^{-\beta m_V\theta'^{1+\epsilon}}[\beta(m_V\theta')^{-(1+\beta)(1+\epsilon)}\left(\frac{dV_1}{dln\bar{p}_l}(\theta_0) + \bar{M}\left(\theta'^{1+\epsilon} - \theta_0^{1+\epsilon}\right)\right) \\
&+ (m_V\theta')^{-\beta(1+\epsilon)}X]d\theta' \leq C\theta^{\epsilon-\beta(1+\epsilon)}.
\end{aligned}$$

Since $\theta^{\epsilon-\beta(1+\epsilon)} = o(\theta^\epsilon)$, we have $\frac{d}{d\theta}\frac{dV}{dln\bar{p}_l}(\theta) \sim \frac{d}{d\theta}\frac{dV^{lin}}{dln\bar{p}_l}$. In both cases this directly implies $\frac{dV}{dln\bar{p}_l}(\theta) \sim \frac{dV^{lin}}{dln\bar{p}_l}$.$\square$

Proof (Corollary 3): By linearity of the formula of lemma A1, we have that $V_\Delta(\theta) = dV_A^G/d\bar{p}_l(\theta) - dV_B^G/d\bar{p}_l(\theta)$ solves:

$$\mathcal{D}[V_\Delta](\theta) = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\left(\tau_{l,\Delta}(\theta) + \partial_{z^*}E_{l,A} - \bar{s}_{l,A} - (\partial_{z^*}E_{l,B} - \bar{s}_{l,B})\right), \int_{\underline{\theta}}^{\bar{\theta}} gV_\Delta(\theta)\pi d\theta = 0,$$

$$\begin{aligned}
\tau_{l,\Delta}(\theta) = (1-T')[&\frac{1+\epsilon}{\epsilon}\frac{1}{\theta\pi(\theta)}\int_\theta^{\bar{\theta}}\left((\partial_{z^*}e_{l,A} - \partial_{z^*}E_{l,A}) - (\partial_{z^*}e_{l,B} - \partial_{z^*}E_{l,B})\right)\pi d\theta' \\
&+ (\partial_{z^*}e_{l,A} - \partial_{z^*}E_{l,A}) - (\partial_{z^*}e_{l,B} - \partial_{z^*}E_{l,B})]
\end{aligned}$$

Using the same step as in Corollary 1, we have $\tau_{l,\Delta}(\theta) < 0$. Since in addition $\partial_{z^*}E_{l,A} - \bar{s}_{l,A} - (\partial_{z^*}E_{l,B} - \bar{s}_{l,B}) \leq 0$, we have following the same steps as in Proposition 3

$V_{\Delta}\left(\underline{\theta}\right)\leq 0$ which proves the result.□

Non-Linear Social Welfare Function in a Simple Example Finally we provide the derivations in our simple three-agent example. We have $0=\theta_{p}<\theta_{m}<\theta_{r}$ and agent preferences are given by$V_{i}=u(c_{l},c_{h})-\frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z}{\theta_{i}}\right)^{1+\frac{1}{\varepsilon}}$. Given the single crossing property, the IC constraint are local and downward binding, the planner's problem is therefore:

$$\begin{aligned}
&\sup_{z_i,V_i}\sum_{i}G\left(V_{i},\theta_{i}\right)\pi_{i}\\
s.t.V_{m}=V_{p},\quad &V_{r}=V_{m}+\frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z_{m}}{\theta_{m}}\right)^{1+\frac{1}{\varepsilon}}-\frac{1}{1+\frac{1}{\varepsilon}}\left(\frac{z_{m}}{\theta_{r}}\right)^{1+\frac{1}{\varepsilon}}\\
&\sum_{i}\left(z_{i}^{*}-z_{i}\right)\pi_{i}+\sum_{k}\left(p_{k}C_{k}-\chi_{k}\left(\xi_{k},C_{k}\right)\right)=0
\end{aligned}$$

We obtain that the tax rate is 0 at the top (no distortion at the top), 0 at the bottom (since $\theta_{p}=0$), while the tax rate for θ_{m}satisfies:

$$\left(g_{m}-1\right)\pi_{m}+\left(g_{p}-1\right)\pi_{p}=\pi_{m}\frac{T_{m}^{\prime}}{1-T_{m}^{\prime}}\frac{1}{1-\left(\theta_{m}/\theta_{r}\right)^{1+\frac{1}{\epsilon}}}.$$

We assume that at initial price $\partial_{z^{*}}e_{l,p}>\partial_{z^{*}}e_{l,m}=\partial_{z^{*}}E_{l}>\partial_{z^{*}}e_{l,r}$ and $\partial_{z^{*}}E_{l}<\bar{s}_{l}$. As before, we assume $v_{z^{*}}=1$ for all three agents and we consider an increase in the relative price of the necessity good, $d\bar{p}_{l}$. We first show that we can express all the welfare changes, $dV_{i}/d\bar{p}_{l}$, in terms of the labor supply change of the middle income households, $dz_{m}/d\bar{p}_{l}$.

Differentiating the IC constraints and the budget constraint, we obtain:

$$\begin{aligned}
\frac{dV_{p}}{d\bar{p}_{l}}&=\frac{dV_{m}}{d\bar{p}_{l}}\\
\frac{dV_{r}}{d\bar{p}_{l}}&=\frac{dV_{m}}{d\bar{p}_{l}}+\left(1-T_{m}^{\prime}\right)\left(1-\left(\frac{\theta_{m}}{\theta_{r}}\right)^{1+\frac{1}{\varepsilon}}\right)\frac{dz_{m}}{d\bar{p}_{l}}\\
0&=\frac{dV_{p}}{d\bar{p}_{l}}\pi_{p}+\frac{dV_{m}}{d\bar{p}_{l}}\pi_{m}+\frac{dV_{r}}{d\bar{p}_{l}}\pi_{r}-T_{m}^{\prime}\pi_{m}\frac{dz_{m}}{d\bar{p}_{l}}.
\end{aligned}$$

Solving for these equations, we obtain:

$$\begin{aligned}\frac{dV_p}{d\bar{p}_l} = \frac{dV_m}{d\bar{p}_l} &= -\left(1-T'_m\right)\left(1-\left(\frac{\theta_m}{\theta_r}\right)^{1+\frac{1}{\varepsilon}}\right) g_r\pi_r\frac{dz_m}{d\bar{p}_l}\\ \frac{dV_r}{d\bar{p}_l} &= \left(1-T'_m\right)\left(1-\left(\frac{\theta_m}{\theta_r}\right)^{1+\frac{1}{\varepsilon}}\right)\left(1-g_r\pi_r\right)\frac{dz_m}{d\bar{p}_l}\end{aligned}$$

Note that $\frac{dV_p}{d\bar{p}_l}$ and $\frac{dV_m}{d\bar{p}_l}$ have the opposite signs to $\frac{dz_m}{d\bar{p}_l}$, while $\frac{dV_r}{d\bar{p}_l}$has the same sign.

Differentiating the tax formula, we obtain, defining $\gamma_i = -\frac{G''_i}{G'_i} > 0$:

$$\begin{aligned}\frac{1}{1-T'_m}\frac{1}{1-\left(\theta_m/\theta_r\right)^{1+\frac{1}{\epsilon}}}\frac{1}{\epsilon}\pi_m\frac{dlnz_m}{dln\bar{p}_l} &= -\left(\left(\partial_{z^*}e_{l,r}-\partial_{z^*}E_l\right)\pi_r + \left(\partial_{z^*}E-\bar{s}_l\right)\pi_m\frac{1}{1-T'_m}\frac{1}{1-\left(\theta_m/\theta_r\right)^{1+\frac{1}{\epsilon}}}\right)\\ &\quad -\left(\left(1-\pi_r g_r\right)\gamma_r\frac{dV_r}{dln\bar{p}_l} - \left(\pi_p g_p\gamma_p\frac{dV_p}{dln\bar{p}_l} + \pi_m g_m\gamma_m\frac{dV_m}{dln\bar{p}_l}\right)\right)g_r\pi_r\\ &= -\left(\left(\partial_{z^*}e_{l,r}-\partial_{z^*}E_l\right)\pi_r + \left(\partial_{z^*}E-\bar{s}_l\right)\pi_m\frac{1}{1-T'_m}\frac{1}{1-\left(\theta_m/\theta_r\right)^{1+\frac{1}{\epsilon}}}\right) - \mathcal{G}\frac{dlnz_m}{dln\bar{p}_l},\end{aligned}$$

with $\mathcal{G} = z_m\left(1-T'_m\right)\left(1-\left(\frac{\theta_m}{\theta_r}\right)^{1+\frac{1}{\varepsilon}}\right)\left(\left(1-g_r\pi_r\right)^2\gamma_r + g_r\pi_r\left(\pi_p g_p\gamma_p + \pi_m g_m\gamma_m\right)\right)g_r\pi_r > 0$. When $\gamma_i = 0$ (G is linear), $\mathcal{G} = 0$, When $\gamma_i > 0$ (G is concave), $\mathcal{G} > 0$. Note in addition that, since l is a necessity,$\left(\partial_{z^*}e_{l,r}-\partial_{z^*}E_l\right)\pi_r + \left(\partial_{z^*}E-\bar{s}_l\right)\pi_m\frac{1}{1-T'_m}\frac{1}{1-\left(\theta_m/\theta_r\right)^{1+\frac{1}{\epsilon}}} < 0$. Therefore, we have $\frac{dV_p}{d\bar{p}_l} = \frac{dV_m}{d\bar{p}_l} < 0 < \frac{dV_r}{d\bar{p}_l}$,and denoting $\frac{dV_i^{lin}}{d\bar{p}_l}$ the tax rate with a linear social welfare function, $\frac{dV_i}{d\bar{p}_l} = \frac{1}{1+\mathcal{G}}\frac{dV_i^{lin}}{d\bar{p}_l}$. Using $\frac{dlnz_m}{dln\bar{p}_l} = -\epsilon\left(\frac{1}{1-T'_m}\frac{d{T'}_m^{lin}}{d\bar{p}_l} + \left(\partial_{z^*}E_l-\bar{s}_l\right)\right)$, it is direct to show:

$$\begin{gathered}\frac{dT'_m}{d\bar{p}_l} = \frac{1}{1+\mathcal{G}}\left(\frac{d{T'}_m^{lin}}{d\bar{p}_l} - \mathcal{G}\left(1-T'_m\right)\left(\partial_{z^*}E_l-\bar{s}_l\right)\right)\\ \frac{1}{1-\left(\theta_m/\theta_h\right)^{1+\frac{1}{\epsilon}}}\pi_m\frac{d}{dln\bar{p}_l}\left\{\frac{{T'}_m^{lin}}{1-{T'}_m^{lin}}\right\} = \left(\partial_{z^*}e_{l,h}-\partial_{z^*}E_l\right)\pi_h < 0.\end{gathered}$$

This proves our formulas for the three-agent example.

A.2.3 Proofs for Section 4.3

Non-Linear Production Function (Proposition 4) We now prove Proposition 4, our main result with non-linear production functions. Recall that we consider a cost shifter, $p_k^* = 1/\partial_{\xi_k}\phi_k$, which implies $\partial_{p_k^*}\phi_k = 1$ and $\partial_{p_k^*}\chi_k = (1-\alpha+t_w)^{-1} C_k$.[1] As before, we define an increase int the relative price of the necessity $d\ln\bar{p}_l$, such that $d\ln p_l^* = \bar{s}_h d\ln\bar{p}_l$ and $d\ln p_h^* = -\bar{s}_l d\ln\bar{p}_l$. We will also provide formulas for an homogeneous price change $d\ln\bar{p}$, such that $d\ln p_l^* = d\ln p_h^* = d\ln\bar{p}$.

We first prove the following Lemma, which characterizes the response of the tax rate to an increase in the price of k in partial equilibrium. Much of the derivation is similar to the derivation of Proposition 2.

Lemma A2. Under $A3$, and with a linear social welfare function the partial equilibrium response of the income tax to a change in the relative price of necessities is:

$$\frac{\partial}{\partial \ln p_k^*}\left\{\frac{T'}{1-T'}\right\} = \frac{1-t_w}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right) - \left(\frac{T'}{1-T'} + t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right).$$

Under $A1$, $\partial_{lnp_l^*}T' = -\partial_{lnp_h^*}T'$ is negative for all θ. The response to an increase in the relative price of necessities $\partial_{\bar{p}_l}T' = \partial_{p_l^*}T$ is also negative.

Proof: Using the formulas of Lemma A1, we have for an arbitrary shifter ξ_k:

$$\mathcal{D}\left[\frac{dV}{d\xi_k}\right](\theta) = \frac{\epsilon}{(1+\epsilon)^2}\frac{\theta\pi(\theta)}{(1-T')^2}\frac{\theta}{z(\theta)}\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\} = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\sum_{m=l,h}\left(\tau_m(\theta) + \partial_{z^*}E_m\right)\frac{1}{p_m}\frac{dp_m}{d\xi_k},$$

$$(1-t_w)\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{d\xi_k}\pi d\theta = -\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right).$$

Since we consider the partial equilibrium response where the shifter is p_k^* and price do not endogenously respond, we have $\frac{dp_k}{d\xi_k} = 1$, $\frac{dp_m}{d\xi_k} = 0$ for $m \neq k$. Using as before:

$$\frac{d}{d\theta}\left\{\frac{\partial V(\theta)}{\partial lnp_k^*}\right\} = -\epsilon z\left(1+\frac{1}{\epsilon}\right)\frac{1}{\theta}\left(1-T'\right)\left(\frac{1}{1-T'}\frac{\partial T'}{\partial lnp_k} + \partial_{z^*}e_k\right),$$

we obtain after the same algebra as in Proposition 2:

$$\frac{\partial}{\partial \ln p_k^*}\left\{\frac{T'}{1-T'}\right\} = \frac{1-t_w}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right) - \left(\frac{T'}{1-T'} + t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right).$$

---

[1] With monopolistic competition ($\tau_w = \alpha$), this is obvious since $\chi_k = C_k\phi_k$ so $\partial_{p_k^*}\chi_k = C_k$. With competitive firms ($\tau_w = 0$), we can rewrite the pricing function as $\phi_k(\xi_k, C_k) = \tilde{\phi}_k(\xi_k)C_k^{-\alpha} = \partial_{C_k}\chi_k(\xi_k, C_k)$ so $\chi_k(\xi_k, C_k) = \phi_k(\xi_k, C_k)C_k/(1-\alpha) + \underline{\chi_k}$, where the potential fixed cost $\underline{\chi_k}$is assumed to be independent from ξ_k.

We now want to show that $\mathcal{F}\left(z\left(\theta\right)\right) = \left(1-t_w\right)\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right) - z\tilde{\zeta}f\left(\frac{T'}{1-T'}+t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)$ is everywhere negative for a necessity good. As a direct consequence, it is everywhere positive for a luxury. Assume that $\partial_{z^*}e_k$ is decreasing (k is a necessity good) and define θ^* such that $\partial_{z^*}e_k\left(z^*\left(\theta^*\right),\mathbf{p}\right) = \partial_{z^*}E_k$. As before, we have:

$$\mathcal{F}'\left(z\left(\theta\right)\right) = -\left(1-t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)gf(z) - \left(1-t_w\right)\left(1-T'\right)\partial_{z^*z^*}e_k\int_{z(\theta)}^{z(\bar{\theta})}\left(1-g\right)f(z)dz.$$

For $\theta \geq \theta^*$, we have $\partial_{z^*}e_k < \partial_{z^*}E_k$ and since $\partial_{z^*z^*}e_k \leq 0$, we have $\mathcal{F}'\left(z\left(\theta\right)\right) > 0$ for $\theta \geq \theta^*$. Since $\mathcal{F}\left(z\left(\bar{\theta}\right)\right) = 0$, this implies $\mathcal{F}\left(z\left(\theta\right)\right) < 0$ for $\theta \geq \theta^*$.
Note in addition that we can rewrite $\mathcal{F}\left(z\left(\theta\right)\right)$ as:

$$\mathcal{F}\left(z\left(\theta\right)\right) = -\left(1-t_w\right)\int_{z(\underline{\theta})}^{z(\theta)}\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)f(z)dz + \left(1-t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\int_{z(\underline{\theta})}^{z(\theta)}\left(1-g\right)f(z)dz.$$

For $\theta \leq \theta^*$, we have $\partial_{z^*}e_k - \partial_{z^*}E_k > 0$ and decreasing in θ so:

$$\begin{aligned}\mathcal{F}\left(z\left(\theta\right)\right) &< -\left(1-t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\int_{z(\underline{\theta})}^{z(\theta)}f(z)dz + \left(1-t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\int_{z(\underline{\theta})}^{z(\theta)}\left(1-g\right)f(z)dz \\ &= -\left(1-t_w\right)\left(\partial_{z^*}e_k - \partial_{z^*}E_k\right)\int_{z(\underline{\theta})}^{z(\theta)}gf(z)dz < 0.\end{aligned}$$

So $\mathcal{F}\left(z\left(\theta\right)\right) < 0$ for $\theta < \theta^*$, which implies $\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} < 0$. By direct inspection, since $\partial_{z^*}e_l - \partial_{z^*}E_l = -\left(\partial_{z^*}e_h - \partial_{z^*}E_h\right)$, we have $\frac{p_h d}{dp_h}\left\{\frac{T'}{1-T'}\right\} = -\frac{p_l d}{dp_l}\left\{\frac{T'}{1-T'}\right\} > 0$. $\square$

We now turn to the proof of Proposition 4. We will consider both a change in the relative price of necessity and a homogeneous price increase, which together span the entire space of price changes.
Complement to Proposition 4. The response of the income tax to an homogenous increase in prices :

$$\frac{dT'}{d\bar{p}} = \left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1}\frac{1}{\bar{s}_h\bar{s}_l}\frac{\alpha}{1-\alpha}\left(\left(\partial_{z^*}E_l - \bar{s}_l\right) + \frac{\zeta}{1-t_w-\alpha\zeta}\mathbb{E}_z\left(\tau_l + \partial_{z^*}E_l - \bar{s}_l\right)\right)\frac{\partial T'}{\partial\bar{p}_l}.$$

With $\frac{dT'}{d\bar{p}} > 0$ if $\alpha > 0$, $\frac{dT'}{d\bar{p}} < 0$ if $\alpha < 0$.

Before proving Proposition 4, let us briefly discuss the change in tax rate for a homogeneous increase in prices: $dlnp_l^* = dlnp_h^* = dln\bar{p}$. In partial equilibrium, this price change has no effect on tax rates. Indeed, per Lemma $A2$, we have

$\partial_{lnp_h^*}T' + \partial_{lnp_l^*}T' = 0$. In general equilibrium, as the price increase is homogeneous, there are no direct substitution effects. With homothetic preferences, there is no change in relative prices and relative quantities. With non-homothetic preference, a homogeneous increase in prices endogenously increases the relative price of luxuries. In the proof of Proposition 4, we show that, when $\alpha \geq 0$, the increase in the relative price of h is given by:

$$\frac{d\log\left(p_h/p_l\right)}{d\log p^*} = -\left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1}\frac{1}{\bar{s}_h\bar{s}_l}\frac{\alpha}{1-\alpha}\left(\left(\partial_{z^*}E_l-\bar{s}_l\right)+\frac{\zeta}{1-t_w-\alpha\zeta}\mathbb{E}_z(\tau_l+\partial_{z^*}E_l-\bar{s}_l)\right) \geq 0.$$

An increase in inflation reduces real income and therefore decreases the share of h. As a result, the relative price of h increases through market size effects ($\alpha > 0$), and this increase is amplified through the substitution and income effects described in the main text.

Thus, while homogeneous exogenous price increases have no impact on tax rates in partial equilibrium, they lead to more redistribution in general equilibrium. Households reduce their labor supply and therefore reallocate their income towards the necessity product, which increases the relative price of the luxury product. It then becomes optimal to redistribute to lower income households.

Proof (Proposition 4 and Complement): We have already proved the first part of Proposition 4 in Lemma $A.2$. We now derive formulas for the general equilibrium response. For an arbitrary cost shifter ξ_k, we have:

$$\frac{\epsilon}{(1+\epsilon)^2}\frac{\theta\pi(\theta)}{(1-T')^2}\frac{\theta}{z(\theta)}\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\} = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\sum_{m=l,h}\left(\tau_m\left(\theta\right)+\partial_{z^*}E_m\right)\frac{1}{p_m}\frac{dp_m}{d\xi_k}.$$

$$\left(1-t_w\right)\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{d\xi_k}\pi d\theta = -\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right).$$

Using the same algebra as in Lemma $A.2$, we obtain:

$$\frac{d}{d\xi_k}\left\{\frac{T'}{1-T'}\right\} = \sum_{m=l,h}\frac{\partial}{\partial \text{ln} p_k^*}\left\{\frac{T'}{1-T'}\right\}\frac{1}{p_m}\frac{dp_m}{d\xi_k}$$

$$\frac{\partial}{\partial \text{ln} p_k^*}\left\{\frac{T'}{1-T'}\right\} = \frac{1-t_w}{z\tilde{\zeta}f(z(\theta))}\mathbb{E}_{z>z(\theta)}\left(\partial_{z^*}e_k-\partial_{z^*}E_k\right) - \left(\frac{T'}{1-T'}+t_w\right)\left(\partial_{z^*}e_k-\partial_{z^*}E_k\right).$$

Using the fact, from Lemma $A.2$ that $\partial_{\ln p_h^*} T' = -\partial_{\ln p_l^*} T' = -\partial_{\ln \bar{p}_l} T'$, we have

$$\frac{d}{d\xi_k}\left\{\frac{T'}{1-T'}\right\} = \frac{\partial}{\partial \ln \bar{p}_l}\left\{\frac{T'}{1-T'}\right\}\left(\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k}\right),$$

so the general equilibrium response of the tax rate only depends on the endogenous increase of the relative price of necessities. Next, we have using our pricing function:

$$\begin{aligned}
\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k} &= \frac{1}{p_l}\frac{\partial p_l}{\partial \xi_k} - \frac{1}{p_h}\frac{\partial p_h}{\partial \xi_k} - \alpha\left(\frac{1}{C_l}\frac{dC_l}{d\xi_k} - \frac{1}{C_h}\frac{dC_h}{d\xi_k}\right) \\
\bar{s}_l\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \bar{s}_h\frac{1}{p_h}\frac{dp_h}{d\xi_k} &= \bar{s}_l\frac{1}{p_l}\frac{\partial p_l}{\partial \xi_k} + \bar{s}_h\frac{1}{p_h}\frac{\partial p_h}{\partial \xi_k} - \alpha\left(\bar{s}_l\frac{1}{C_l}\frac{dC_l}{d\xi_k} + \bar{s}_h\frac{1}{C_h}\frac{dC_h}{d\xi_k}\right).
\end{aligned}$$

To complete the proof of Proposition 4, we therefore need to determine the response of aggregate demand to price shifter ξ_k. We record the derivation in the following Lemma:

Lemma A3. The response of aggregate consumption to an arbitrary cost shift $\xi_k$is given by:

$$\begin{aligned}
\frac{1}{C_l}\frac{dC_l}{d\xi_k} - \frac{1}{C_h}\frac{dC_h}{d\xi_k} =& -\frac{\zeta}{1-t_w}\frac{1}{\bar{s}_h\bar{s}_l}\int_{\underline{\theta}}^{\bar{\theta}}\left(\tau_l + \left(\partial_{z^*}E_l - \bar{s}_l\right)\right)^2\frac{z(\theta)}{Z}\pi d\theta\left(\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) \\
& - \frac{\zeta}{1-t_w}\frac{1}{\bar{s}_h\bar{s}_l}\int_{\underline{\theta}}^{\bar{\theta}}\left(\tau_l + \left(\partial_{z^*}E_l - \bar{s}_l\right)\right)\frac{z(\theta)}{Z}\pi d\theta\left(\bar{s}_l\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \bar{s}_h\frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) \\
& - \sigma\left(\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) - \frac{1}{1-t_w}\frac{1}{\bar{s}_h\bar{s}_l}\left(\partial_{z^*}E_l - \bar{s}_l\right)\frac{1}{Z}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right), \\
\bar{s}_l\frac{1}{C_l}\frac{dC_l}{d\xi_k} + \bar{s}_h\frac{1}{C_h}\frac{dC_h}{d\xi_k} =& -\frac{\zeta}{1-t_w}\int_{\underline{\theta}}^{\bar{\theta}}\left(\tau_l\left(\theta\right) + \partial_{z^*}E_l - \bar{s}_l\right)\frac{z(\theta)}{Z}\pi d\theta\left(\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) \\
& - \frac{\zeta}{1-t_w}\left(\bar{s}_l\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \bar{s}_h\frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) \\
& - \frac{1}{1-t_w}\frac{1}{Z}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right),
\end{aligned}$$

with $Z = E_l + E_h = \int_{\underline{\theta}}^{\bar{\theta}} z(\theta)\pi d\theta$. We have $\frac{1}{C_l}\frac{\partial C_l}{\partial ln\bar{p}_l} - \frac{1}{C_h}\frac{\partial C_h}{\partial ln\bar{p}_l}, \bar{s}_l\frac{1}{C_l}\frac{\partial C_l}{\partial ln\bar{p}} + \bar{s}_h\frac{1}{C_h}\frac{\partial C_h}{\partial ln\bar{p}} \leq 0$, $\frac{1}{C_l}\frac{\partial C_l}{\partial ln\bar{p}} - \frac{1}{C_h}\frac{\partial C_h}{\partial ln\bar{p}}, \bar{s}_l\frac{1}{C_l}\frac{\partial \bar{C}_l}{\partial ln\bar{p}_l} + \bar{s}_h\frac{1}{C_h}\frac{\partial C_h}{\partial ln\bar{p}_l} \geq 0$.

Proof: We have, using Slutsky's formula:

$$\begin{aligned}
\frac{1}{C_m}\frac{dC_m}{d\xi_k} &= \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \frac{dc_m}{d\xi_k}\pi d\theta \\
&= \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \partial_{z^*}c_m\left(\frac{dz^*}{d\xi_k} - c_l\frac{dp_l}{d\xi_k} - c_h\frac{1}{p_h}\frac{dp_h}{d\xi_k}\right)\pi d\theta + \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \partial_{p_l}c_m^h\frac{dp_l}{d\xi_k}\pi d\theta + \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \partial_{p_h}c_m^h\frac{dp_h}{d\xi_k}\pi d\theta \\
&= \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \partial_{z^*}c_m\left(\frac{1}{v_{z^*}}\frac{dV}{d\xi_k} + \left(1-T'\right)\frac{dz}{d\xi_k}\right)\pi d\theta + \mathcal{S}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \mathcal{S}_{mh}\frac{1}{p_h}\frac{dp_h}{d\xi_k} \\
&= \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\int_{\theta}^{\bar{\theta}} \left(\partial_{z^*}c_m - \partial_{z^*}C_m\right)\pi d\theta + \frac{\partial_{z^*}C_m}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \frac{1}{v_{z^*}}\frac{dV}{d\xi_k}\pi d\theta \\
&+ \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \partial_{z^*}c_m\left(1-T'\right)\frac{dz}{d\xi_k}\pi d\theta + \mathcal{S}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \mathcal{S}_{mh}\frac{1}{p_h}\frac{dp_h}{d\xi_k},
\end{aligned}$$

where the last line is the third line integrated by part.
Next, using from the proof of Lemma $A1$:

$$\begin{aligned}
\int_{\underline{\theta}}^{\bar{\theta}} \left(\left(\frac{1}{1-T'} - \left(1-t_w\right)\right)\frac{\epsilon}{1+\epsilon}\theta\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\} - \left(1-t_w\right)\frac{1}{v_{z^*}}\frac{dV(\theta)}{d\xi_k}\right)\pi\left(\theta\right)d\theta &= -\left(1-t_w\right)\int_{\underline{\theta}}^{\bar{\theta}} g\left(\theta\right)\frac{dV(\theta)}{d\xi_k}\pi\left(\theta\right)d\theta \\
&= \partial_{\xi_k}\chi_k\left(\xi_k, C_k\right),
\end{aligned}$$

we obtain:

$$\begin{aligned}
\frac{1}{C_m}\frac{dC_m}{d\xi_k} &= \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\int_{\theta}^{\bar{\theta}} \left(\partial_{z^*}c_m - \partial_{z^*}C_m\right)\pi d\theta' d\theta \\
&+ \frac{1}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \partial_{z^*}c_m\left(1-T'\right)\frac{dz}{d\xi_k}\pi d\theta + \mathcal{S}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \mathcal{S}_{mh}\frac{1}{p_h}\frac{dp_h}{d\xi_k} \\
&+ \frac{1}{1-t_w}\frac{\partial_{z^*}C_m}{C_m}\int_{\underline{\theta}}^{\bar{\theta}} \frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\left(\frac{1}{1-T'} - \left(1-t_w\right)\right)\frac{\epsilon}{1+\epsilon}\theta\pi\left(\theta\right)d\theta - \frac{1}{1-t_w}\frac{\partial_{z^*}C_m}{C_m}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right).
\end{aligned}$$

Then, using $\tau_h\left(\theta\right) = -\tau_l\left(\theta\right)$, $\partial_{z^*}E_l - \bar{s}_l = -\left(\partial_{z^*}E_h - \bar{s}_h\right)$ we can rewrite the formula of Lemma $A1$ as:

$$\begin{aligned}
\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\} &= -\left(1+\epsilon\right)\left(1-T'\right)\frac{z(\theta)}{\theta}\left(\left(\tau_l\left(\theta\right) + \partial_{z^*}E_l - \bar{s}_l\right)\left(\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) + \bar{s}_l\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \bar{s}_h\frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) \\
\frac{dz}{d\xi_k} &= \frac{\epsilon}{1+\epsilon}\frac{\theta}{1-T'}\frac{d}{d\theta}\left\{\frac{dV(\theta)}{d\xi_k}\right\} = -\epsilon z(\theta)\left(\left(\tau_l\left(\theta\right) + \partial_{z^*}E_l - \bar{s}_l\right)\left(\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{p_h}\frac{dp_h}{d\xi_k}\right) + \bar{s}_l\frac{1}{p_l}\frac{dp_l}{d\xi_k} + \bar{s}_h\frac{1}{p_h}\frac{dp_h}{d\xi_k}\right),
\end{aligned}$$

so we have:

$$\begin{aligned}
\frac{1}{C_m}\frac{dC_m}{d\xi_k} &= \frac{1}{E_m}\int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\{\frac{1+\epsilon}{\epsilon}\frac{1}{\theta\pi}\int_{\theta}^{\bar{\theta}}\left(\partial_{z^*}e_m-\partial_{z^*}E_m\right)\pi d\theta'+\left(\partial_{z^*}e_m-\partial_{z^*}E_m\right) \\
&\quad +\frac{1}{1-T'}\frac{1}{1-t_w}\partial_{z^*}E_m\}\frac{\epsilon}{1+\epsilon}\theta\pi d\theta \\
&\quad +\mathcal{S}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k}+\mathcal{S}_{mh}\frac{1}{p_h}\frac{dp_h}{d\xi_k}-\frac{1}{1-t_w}\frac{\partial_{z^*}E_m}{E_m}\partial_{\xi_k}\chi_k\left(\xi_k,C_k\right), \\
&= \frac{1}{1-t_w}\frac{1}{E_m}\int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\{\tau_m+\left(\partial_{z^*}E_m-\bar{s}_m\right)\}\frac{1}{1-T'}\frac{\epsilon}{1+\epsilon}\theta\pi\left(\theta\right)d\theta \\
&\quad +\frac{\bar{s}_m}{E_m}\frac{1}{1-t_w}\int_{\underline{\theta}}^{\bar{\theta}}\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\frac{1}{1-T'}\frac{\epsilon}{1+\epsilon}\theta\pi\left(\theta\right)d\theta \\
&\quad +\mathcal{S}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k}+\mathcal{S}_{mh}\frac{1}{p_h}\frac{dp_h}{d\xi_k}-\frac{1}{1-t_w}\frac{\partial_{z^*}E_m}{E_m}\partial_{\xi_k}\chi_k\left(\xi_k,C_k\right).
\end{aligned}$$

Using $\tau_l+\frac{1}{1-T'}\left(\partial_{z^*}E_l-\bar{s}_l\right)=-\tau_h-\frac{1}{1-T'}\left(\partial_{z^*}E_h-\bar{s}_h\right)$ and the expressions for $\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}$ and $\frac{dz}{d\xi_k}$ we obtain the formulas of the Lemma. The sign of the response is a direct consequence of Corollary 1 which shows $\tau_l+\left(\partial_{z^*}E_l-\bar{s}_l\right)\leq 0$. $\square$

Coming back to the proof of Proposition 4 and using the formulas of Lemma $A3$, we have:

$$\begin{aligned}
\frac{1}{p_l}\frac{dp_l}{d\xi_k}-\frac{1}{p_h}\frac{dp_h}{d\xi_k} &= \left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1}\left(\frac{1}{p_l}\frac{\partial p_l}{\partial\xi_k}-\frac{1}{p_h}\frac{\partial p_h}{\partial\xi_k}+\alpha\frac{1}{1-t_w}\frac{1}{\bar{s}_h\bar{s}_l}\left(\partial_{z^*}E_l-\bar{s}_l\right)\frac{1}{Z}\partial_{\xi_k}\chi_k\left(\xi_k,C_k\right)\right) \\
+\left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1}\frac{\frac{\alpha\zeta}{1-t_w}}{1-\frac{\alpha\zeta}{1-t_w}}\frac{1}{\bar{s}_h\bar{s}_l}&\int_{\underline{\theta}}^{\bar{\theta}}\left(\tau_l+\left(\partial_{z^*}E_l-\bar{s}_l\right)\right)\frac{z(\theta)}{Z}\pi d\theta\left(\bar{s}_l\frac{1}{p_l}\frac{\partial p_l}{\partial\xi_k}+\bar{s}_h\frac{1}{p_h}\frac{\partial p_h}{\partial\xi_k}+\alpha\frac{1}{1-t_w}\frac{1}{Z}\partial_{\xi_k}\chi_k\left(\xi_k,C_k\right)\right),
\end{aligned}$$

with $\Omega=\frac{\zeta}{1-t_w}\frac{1}{\bar{s}_h\bar{s}_l}\left(\int_{\underline{\theta}}^{\bar{\theta}}\left(\tau_l+\left(\partial_{z^*}E_l-\bar{s}_l\right)\right)^2\frac{z(\theta)}{Z}\pi d\theta+\frac{\alpha\zeta}{1-t_w-\alpha\zeta}\left(\alpha\frac{\zeta}{1-t_w}\int_{\underline{\theta}}^{\bar{\theta}}\left(\tau_l+\left(\partial_{z^*}E_l-\bar{s}_l\right)\right)\frac{z(\theta)}{Z}\pi d\theta\right)^2\right)\geq 0$, with strict inequality if preferences are non-homothetic.

For an increase in the relative price of necessities, we have $\frac{\partial lnp_l}{\partial ln\bar{p}_l}=\bar{s}_h$, $\frac{\partial lnp_h}{\partial ln\bar{p}_l}=-\bar{s}_l$ $\frac{\partial lnp_l}{\partial ln\bar{p}_l}=\bar{s}_h$ and $\partial_{ln\bar{p}_l}\chi_l=\left(1-\alpha+t_w\right)^{-1}\bar{s}_hE_l$, $\partial_{ln\bar{p}_l}\chi_h=\left(1-\alpha+t_w\right)^{-1}\bar{s}_lE_h$, so we have:

$$\begin{aligned}
&\frac{1}{p_l}\frac{dp_l}{dln\bar{p}_l}-\frac{1}{p_h}\frac{dp_h}{dln\bar{p}_l}=\left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1} \\
&\quad\frac{d}{dln\bar{p}_l}\left\{\frac{T'}{1-T'}\right\}=\left(1-\alpha\left(\sigma+\Omega\right)\right)^{-1}\frac{\partial}{\partial\ln\bar{p}_l}\left\{\frac{T'}{1-T'}\right\}<\frac{\partial}{\partial\ln\bar{p}_l}\left\{\frac{T'}{1-T'}\right\}<0\quad if\ \alpha>0,
\end{aligned}$$

which shows the first formula of Proposition 4 and that partial equilibrium effects are amplified in general equilibrium.

Finally for a homogenous price increase $\frac{\partial lnp_l}{\partial ln\bar{p}}=\frac{\partial lnp_h}{\partial ln\bar{p}}=1$, $\partial_{ln\bar{p}_l}\chi_l=\left(1-\alpha+t_w\right)^{-1}E_l$,

$\partial_{ln\bar{p}_l}\chi_h = (1-\alpha+t_w)^{-1} E_h$, so we have:

$$\begin{aligned}\frac{1}{p_l}\frac{dp_l}{dln\bar{p}} - \frac{1}{p_h}\frac{dp_h}{dln\bar{p}} &= (1-\alpha(\sigma+\Omega))^{-1}\frac{\alpha}{1-\alpha}\frac{1}{\bar{s}_h\bar{s}_l}\left((\partial_{z^*}E_l - \bar{s}_l) + \frac{\zeta}{1-t_w-\alpha\zeta}\frac{1}{\bar{s}_h\bar{s}_l}\int_{\underline{\theta}}^{\bar{\theta}}(\tau_l + (\partial_{z^*}E_l - \bar{s}_l))\frac{z(\theta)}{Z}\pi d\theta\right)\\ \frac{d}{dln\bar{p}}\left\{\frac{T'}{1-T'}\right\} &= (1-\alpha(\sigma+\Omega))^{-1}\frac{\alpha}{1-\alpha}\frac{1}{\bar{s}_h\bar{s}_l}\left((\partial_{z^*}E_l - \bar{s}_l) + \frac{\zeta}{1-t_w-\alpha\zeta}\frac{1}{\bar{s}_h\bar{s}_l}\int_{\underline{\theta}}^{\bar{\theta}}(\tau_l + (\partial_{z^*}E_l - \bar{s}_l))\frac{z(\theta)}{Z}\pi d\theta\right)\frac{\partial}{\partial \ln\bar{p}_l}\left\{\frac{T'}{1-T'}\right\}.\end{aligned}$$

Since l is a necessity, we have $(\partial_{z^*}E_l - \bar{s}_l) + \frac{\zeta}{1-t_w-\alpha\zeta}\frac{1}{\bar{s}_h\bar{s}_l}\int_{\underline{\theta}}^{\bar{\theta}}(\tau_l + (\partial_{z^*}E_l - \bar{s}_l))\frac{z(\theta)}{Z}\pi d\theta < 0$, $\partial_{\ln\bar{p}_l}T' < 0$ so $\frac{d}{dln\bar{p}}\left\{\frac{T'}{1-T'}\right\}$ has the same sign as α.$\square$

A.2.4 Formulas for Section 5.2

In this section, we provide the formulas that underpin the quantitative results of Section 5.2. The formulas allow us to compute the change in taxes in response to an arbitrary cost shock ξ_k in an n-sector economy. While the formulas are expressed in terms of $\frac{dV}{d\xi_k}$, they allow us to recover tax rates using:

$$\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\} = -(1+\epsilon)\frac{z}{\theta}(1-T')\left(\frac{1}{1-T'}\frac{dT'}{d\xi_k} + \sum_{m=1}^{n}\partial_{z^*}e_m\frac{1}{p_m}\frac{dp_m}{d\xi_k}\right).$$

The main advantage of the result below is that it provides a simple procedure to compute the change in the tax schedule in response to supply curve shifters. As a first step, we can compute $N+1$ "partial equilibrium" responses which do not depend on the endogenous adjustment of prices $\frac{dp_m}{d\xi_k}$. Once they are computed, the general equilibrium response of prices and the full response of the tax schedule is simply computed by linear algebra.

Proposition A1. Under assumption $A3$, the change in welfare for agent θ, $dV(\theta)/d\xi_k$ in response to an exogenous supply shift $d\xi_k$ is given by:

$$\frac{dV}{d\xi_k} = \sum_{m=1}^{n}\frac{\partial V}{\partial p_m}\frac{1}{p_m}\frac{dp_m}{d\xi_k} - \frac{\partial V}{\partial B}\partial_{\xi_k}\chi_k(\xi_k, C_k),$$

where $\frac{\partial V}{\partial p_m}$and $\frac{\partial V}{\partial B}$ solve:

$$\mathcal{D}\left[\frac{\partial V}{\partial p_m}\right](\theta) = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}(\tau_m(\theta) + \partial_{z^*}E_m),\ (1-t_w)\int_{\underline{\theta}}^{\bar{\theta}} g\frac{\partial V}{\partial p_m}\pi d\theta = 0.$$

$$\mathcal{D}\left[\frac{\partial V}{\partial B}\right](\theta) = 0,\ (1-t_w)\int_{\underline{\theta}}^{\bar{\theta}} g\frac{\partial V}{\partial B}\pi d\theta = 1.$$

To express the equilibrium change in prices, define the vector $\mathbf{C^B}$and the matrix $\mathcal{C}$ with elements:

$$C_l^B = \frac{1}{1-t_w}\frac{\partial_{z^*}E_l}{E_l} + \frac{1}{E_l}\int_{\underline{\theta}}^{\bar{\theta}} \left\{\tau_l\left(\theta\right) + \partial_{z^*}E_l\right\} \frac{1}{1-T'}\frac{1}{1-t_w}\frac{d}{d\theta}\left\{\frac{\partial V}{\partial B}\right\}\frac{\epsilon}{1+\epsilon}\theta\pi\left(\theta\right)d\theta$$

$$\mathcal{C}_{l,m} = \mathcal{S}_{l,m} + \frac{1}{E_l}\int_{\underline{\theta}}^{\bar{\theta}} \left\{\tau_l\left(\theta\right) + \partial_{z^*}E_l\right\} \frac{1}{1-T'}\frac{1}{1-t_w}\frac{d}{d\theta}\left\{\frac{\partial V}{\partial p_m}\right\}\frac{\epsilon}{1+\epsilon}\theta\pi\left(\theta\right)d\theta.$$

Then the equilibrium change in prices $\frac{\mathbf{dlnp}}{\mathbf{d\xi_k}} = \left\{\frac{1}{p_1}\frac{dp_1}{d\xi_k}, ..., \frac{1}{p_N}\frac{dp_N}{d\xi_k}\right\}'$ solves:

$$\frac{\mathbf{dlnp}}{\mathbf{d\xi_k}} = \left(Id + \alpha\mathcal{C}\right)^{-1}\left(\delta_{\mathbf{k}}\partial_{\xi_k}ln\phi_k\left(\xi_k, C_k\right) + \alpha\mathbf{C^B}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right)\right),$$

where $\delta_{\mathbf{k}}$ is the column vector with 1 on its k^{th} row and 0 otherwise.

Proof: Recall from Lemma $A1$ that $\frac{dV}{d\xi_k}$ solves equation:

$$\mathcal{D}\left[\frac{dV}{d\xi_k}\right](\theta) = -\frac{\epsilon}{1+\epsilon}\frac{\theta\pi(\theta)}{1-T'}\sum_{l=1}^{n}\left(\tau_l\left(\theta\right) + \partial_{z^*}E_l\right)\frac{1}{p_l}\frac{dp_l}{d\xi_k},\ (1-t_w)\int_{\underline{\theta}}^{\bar{\theta}} g\frac{dV}{d\xi_k}\pi d\theta \quad = -\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right).$$

Since the equation is linear in $\frac{dV}{d\xi_k}$, it is direct that $\frac{dV}{d\xi_k} = \sum_{m=1}^{N}\frac{\partial V}{\partial p_m}\frac{1}{p_m}\frac{dp_m}{d\xi_k} - \frac{\partial V}{\partial B}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right)$ with $\frac{\partial V}{\partial p_m}$, $\frac{\partial V}{\partial B}$ defined in the Proposition. Next, we have, using the pricing equation:

$$\frac{dlnp_m}{d\xi_k} = \frac{\partial ln\phi_m}{\partial\xi_k} - \alpha\frac{1}{C_m}\frac{dC_m}{d\xi_k},$$

with $\frac{\partial ln\phi_m}{\partial\xi_k} = 0$ if $m \neq k$. Using the same steps in Lemma $A3$, we have:

$$\frac{1}{C_m}\frac{dC_m}{d\xi_k} = \frac{1}{1-t_w}\frac{1}{E_m}\int_{\underline{\theta}}^{\bar{\theta}}\left\{\tau_m + \partial_{z^*}E_m\right\}\frac{1}{1-T'}\frac{d}{d\theta}\left\{\frac{dV}{d\xi_k}\right\}\frac{\epsilon}{1+\epsilon}\theta\pi\left(\theta\right)d\theta$$
$$+ \sum_{l=1}^{n}\mathcal{S}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k} - \frac{1}{1-t_w}\frac{\partial_{z^*}E_m}{E_m}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right).$$

Using $\frac{dV}{d\xi_k} = \sum_{m=1}^{n}\frac{\partial V}{\partial p_m}\frac{1}{p_m}\frac{dp_m}{d\xi_k} - \frac{\partial V}{\partial B}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right)$, we obtain:

$$\frac{1}{C_m}\frac{dC_m}{d\xi_k} = \sum_{l=1}^{n}\mathcal{C}_{ml}\frac{1}{p_l}\frac{dp_l}{d\xi_k} - C_m^B\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right),$$
$$\frac{\mathbf{dlnp}}{\mathbf{d\xi_k}} = \left(Id + \alpha\mathcal{C}\right)^{-1}\left(\delta_{\mathbf{k}}\partial_{\xi_k}ln\phi_k\left(\xi_k, C_k\right) + \alpha\mathbf{C^B}\partial_{\xi_k}\chi_k\left(\xi_k, C_k\right)\right).\square$$